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[المكتبة التخصصية]

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SOLUTION MANUAL CONTENTS

12–1.

A baseball is thrown downward from a 50-ft tower with an initial speed of 18 ft/s. Determine the speed at which it hits the ground and the time of travel.

SOLUTION

$v_2^2 = v_1^2 + 2a_c(s_2 - s_1)$

$v_2^2 = (18)^2 + 2(32.2)(50 - 0)$

$v_2 = 59.532 = 59.5 \text{ ft/s}$ **Ans.**

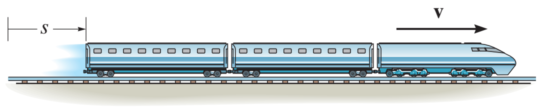
$v_2 = v_1 + a_c t$

$59.532 = 18 + 32.2(t)$

$t = 1.29 \text{ s}$ **Ans.**

12–2.

When a train is traveling along a straight track at 2 m/s, it begins to accelerate at $a = (60\, v^{-4})$ m/s^2, where v is in m/s. Determine its velocity v and the position 3 s after the acceleration.



SOLUTION

$$a = \frac{dv}{dt}$$

$$dt = \frac{dv}{a}$$

$$\int_0^3 dt = \int_2^v \frac{dv}{60v^{-4}}$$

$$3 = \frac{1}{300}(v^5 - 32)$$

$$v = 3.925 \text{ m/s} = 3.93 \text{ m/s} \qquad \textbf{Ans.}$$

$$ads = vdv$$

$$ds = \frac{vdv}{a} = \frac{1}{60} v^5\, dv$$

$$\int_0^s ds = \frac{1}{60}\int_2^{3.925} v^5\, dv$$

$$s = \frac{1}{60}\left(\frac{v^6}{6}\right)\Bigg|_2^{3.925}$$

$$= 9.98 \text{ m} \qquad \textbf{Ans.}$$

12–3.

From approximately what floor of a building must a car be dropped from an at-rest position so that it reaches a speed of 80.7 ft/s (55 mi/h) when it hits the ground? Each floor is 12 ft higher than the one below it. (*Note:* You may want to remember this when traveling 55 mi/h.)

SOLUTION

$(+\downarrow) \qquad v^2 = v_0^2 + 2a_c(s - s_0)$

$$80.7^2 = 0 + 2(32.2)(s - 0)$$

$$s = 101.13 \text{ ft}$$

$$\#\text{ of floors} = \frac{101.13}{12} = 8.43$$

The car must be dropped from the 9th floor. **Ans.**

***12–4.**

Traveling with an initial speed of 70 km/h, a car accelerates at 6000 km/h^2 along a straight road. How long will it take to reach a speed of 120 km/h? Also, through what distance does the car travel during this time?

SOLUTION

$v = v_1 + a_c t$

$120 = 70 + 6000(t)$

$t = 8.33(10^{-3}) \text{ hr} = 30 \text{ s}$ **Ans.**

$v^2 = v_1^2 + 2\,a_c(s - s_1)$

$(120)^2 = 70^2 + 2(6000)(s - 0)$

$s = 0.792 \text{ km} = 792 \text{ m}$ **Ans.**

12–5.

A bus starts from rest with a constant acceleration of 1 m/s^2. Determine the time required for it to attain a speed of 25 m/s and the distance traveled.

SOLUTION

***Kinematics*:**

$v_0 = 0$, $v = 25$ m/s, $s_0 = 0$, and $a_c = 1\ m/s^2$.

$(\overset{+}{\rightarrow})$ $$v = v_0 + a_c t$$

$$25 = 0 + (1)t$$

$$t = 25\ s$$ **Ans.**

$(\overset{+}{\rightarrow})$ $$v^2 = v_0^2 + 2a_c(s - s_0)$$

$$25^2 = 0 + 2(1)(s - 0)$$

$$s = 312.5\ m$$ **Ans.**

12–6.

A stone A is dropped from rest down a well, and in 1 s another stone B is dropped from rest. Determine the distance between the stones another second later.

SOLUTION

$$+\downarrow s = s_1 + v_1 t + \frac{1}{2}a_c t^2$$

$$s_A = 0 + 0 + \frac{1}{2}(32.2)(2)^2$$

$$s_A = 64.4 \text{ ft}$$

$$s_A = 0 + 0 + \frac{1}{2}(32.2)(1)^2$$

$$s_B = 16.1 \text{ ft}$$

$$\Delta s = 64.4 - 16.1 = 48.3 \text{ ft}$$ **Ans.**

12–7.

A bicyclist starts from rest and after traveling along a straight path a distance of 20 m reaches a speed of 30 km/h. Determine his acceleration if it is *constant*. Also, how long does it take to reach the speed of 30 km/h?

SOLUTION

$$v_2 = 30\text{ km/h} = 8.33\text{ m/s}$$

$$v_2^2 = v_1^2 + 2\,a_c(s_2 - s_1)$$

$$(8.33)^2 = 0 + 2\,a_c(20 - 0)$$

$$a_c = 1.74\text{ m/s}^2 \qquad \textbf{Ans.}$$

$$v_2 = v_1 + a_c\,t$$

$$8.33 = 0 + 1.74(t)$$

$$t = 4.80\text{ s} \qquad \textbf{Ans.}$$

***■12–8.**

A particle moves along a straight line with an acceleration of $a = 5/(3s^{1/3} + s^{5/2})$ m/s^2, where s is in meters. Determine the particle's velocity when $s = 2$ m, if it starts from rest when $s = 1$ m. Use Simpson's rule to evaluate the integral.

SOLUTION

$$a = \frac{5}{\left(3s^{\frac{1}{3}} + s^{\frac{5}{2}}\right)}$$

$$a\,ds = v\,dv$$

$$\int_1^2 \frac{5\,ds}{\left(3s^{\frac{1}{3}} + s^{\frac{5}{2}}\right)} = \int_0^v v\,dv$$

$$0.8351 = \frac{1}{2}v^2$$

$$v = 1.29 \text{ m/s}$$ **Ans.**

12–9.

If it takes 3 s for a ball to strike the ground when it is released from rest, determine the height in meters of the building from which it was released. Also, what is the velocity of the ball when it strikes the ground?

SOLUTION

Kinematics:

$v_0 = 0,\ a_c = g = 9.81\ \text{m/s}^2,\ t = 3\ \text{s, and}\ s = h.$

$$(+\downarrow) \qquad v = v_0 + a_c t$$

$$= 0 + (9.81)(3)$$

$$= 29.4\ \text{m/s} \qquad \textbf{Ans.}$$

$$(+\downarrow) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$h = 0 + 0 + \frac{1}{2}(9.81)(3^2)$$

$$= 44.1\ \text{m} \qquad \textbf{Ans.}$$

12–10.

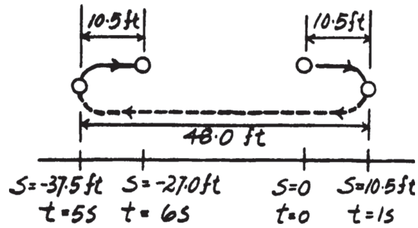
The position of a particle along a straight line is given by $s = (1.5t^3 - 13.5t^2 + 22.5t)$ ft, where t is in seconds. Determine the position of the particle when $t = 6$ s and the total distance it travels during the 6-s time interval. *Hint:* Plot the path to determine the total distance traveled.

SOLUTION

Position: The position of the particle when $t = 6$ s is

$$s|_{t=6s} = 1.5(6^3) - 13.5(6^2) + 22.5(6) = -27.0 \text{ ft} \qquad \textbf{Ans.}$$

Total DistanceTraveled: The velocity of the particle can be determined by applying Eq. 12–1.



$$v = \frac{ds}{dt} = 4.50t^2 - 27.0t + 22.5$$

The times when the particle stops are

$$4.50t^2 - 27.0t + 22.5 = 0$$

$$t = 1 \text{ s} \qquad \text{and} \qquad t = 5 \text{ s}$$

The position of the particle at $t = 0$ s, 1 s and 5 s are

$$s|_{t=0\text{ s}} = 1.5(0^3) - 13.5(0^2) + 22.5(0) = 0$$

$$s|_{t=1\text{ s}} = 1.5(1^3) - 13.5(1^2) + 22.5(1) = 10.5 \text{ ft}$$

$$s|_{t=5\text{ s}} = 1.5(5^3) - 13.5(5^2) + 22.5(5) = -37.5 \text{ ft}$$

From the particle's path, the total distance is

$$s_{\text{tot}} = 10.5 + 48.0 + 10.5 = 69.0 \text{ ft} \qquad \textbf{Ans.}$$

12–11.

If a particle has an initial velocity of $v_0 = 12$ ft/s to the right, at $s_0 = 0$, determine its position when $t = 10$ s, if $a = 2$ ft/s^2 to the left.

SOLUTION

$$(\overset{+}{\rightarrow}) \qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$= 0 + 12(10) + \frac{1}{2}(-2)(10)^2$$

$$= 20 \text{ ft} \qquad \textbf{Ans.}$$

***12–12.**

Determine the time required for a car to travel 1 km along a road if the car starts from rest, reaches a maximum speed at some intermediate point, and then stops at the end of the road. The car can accelerate at 1.5 m/s^2 and decelerate at 2 m/s^2.

SOLUTION

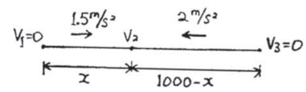
Using formulas of constant acceleration:

$$v_2 = 1.5\, t_1$$

$$x = \frac{1}{2}(1.5)(t_1^2)$$

$$0 = v_2 - 2\, t_2$$

$$1000 - x = v_2 t_2 - \frac{1}{2}(2)(t_2^2)$$



Combining equations:

$$t_1 = 1.33\, t_2; \quad v_2 = 2\, t_2$$

$$x = 1.33\, t_2^2$$

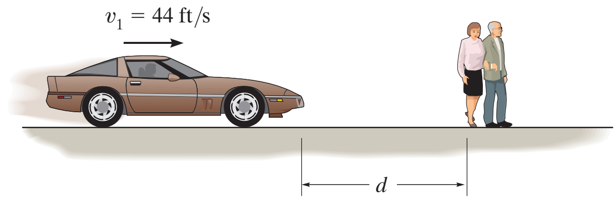
$$1000 - 1.33\, t_2^2 = 2\, t_2^2 - t_2^2$$

$$t_2 = 20.702 \text{ s}; \qquad t_1 = 27.603 \text{ s}$$

$$t = t_1 + t_2 = 48.3 \text{ s}$$ **Ans.**

12–13.

Tests reveal that a normal driver takes about 0.75 s before he or she can *react* to a situation to avoid a collision. It takes about 3 s for a driver having 0.1% alcohol in his system to do the same. If such drivers are traveling on a straight road at 30 mph (44 ft/s) and their cars can decelerate at 2 ft/s^2, determine the shortest stopping distance d for each from the moment they see the pedestrians. *Moral*: If you must drink, please don't drive!



SOLUTION

Stopping Distance: For normal driver, the car moves a distance of $d' = vt = 44(0.75) = 33.0$ ft before he or she reacts and decelerates the car. The stopping distance can be obtained using Eq. 12–6 with $s_0 = d' = 33.0$ ft and $v = 0$.

$$(\overset{+}{\rightarrow}) \qquad v^2 = v_0^2 + 2a_c\,(s - s_0)$$

$$0^2 = 44^2 + 2(-2)(d - 33.0)$$

$$d = 517 \text{ ft} \qquad \textbf{Ans.}$$

For a drunk driver, the car moves a distance of $d' = vt = 44(3) = 132$ ft before he or she reacts and decelerates the car. The stopping distance can be obtained using Eq. 12–6 with $s_0 = d' = 132$ ft and $v = 0$.

$$(\overset{+}{\rightarrow}) \qquad v^2 = v_0^2 + 2a_c\,(s - s_0)$$

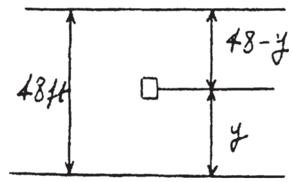
$$0^2 = 44^2 + 2(-2)(d - 132)$$

$$d = 616 \text{ ft} \qquad \textbf{Ans.}$$

12–14.

A car is to be hoisted by elevator to the fourth floor of a parking garage, which is 48 ft above the ground. If the elevator can accelerate at 0.6 ft/s^2, decelerate at 0.3 ft/s^2, and reach a maximum speed of 8 ft/s, determine the shortest time to make the lift, starting from rest and ending at rest.

SOLUTION



$+\uparrow \quad v^2 = v_0^2 + 2\,a_c\,(s - s_0)$

$v_{max}^2 = 0 + 2(0.6)(y - 0)$

$0 = v_{max}^2 + 2(-0.3)(48 - y)$

$0 = 1.2\,y - 0.6(48 - y)$

$y = 16.0 \text{ ft}, \qquad v_{max} = 4.382 \text{ ft/s} < 8 \text{ ft/s}$

$+\uparrow \quad v = v_0 + a_c\,t$

$4.382 = 0 + 0.6\,t_1$

$t_1 = 7.303 \text{ s}$

$0 = 4.382 - 0.3\,t_2$

$t_2 = 14.61 \text{ s}$

$t = t_1 + t_2 = 21.9 \text{ s}$ **Ans.**

12–15.

A train starts from rest at station A and accelerates at $0.5\ \text{m/s}^2$ for 60 s. Afterwards it travels with a constant velocity for 15 min. It then decelerates at $1\ \text{m/s}^2$ until it is brought to rest at station B. Determine the distance between the stations.

SOLUTION

Kinematics: For stage (1) motion, $v_0 = 0$, $s_0 = 0$, $t = 60$ s, and $a_c = 0.5\ \text{m/s}^2$. Thus,

$$(\xrightarrow{+}) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_1 = 0 + 0 + \frac{1}{2}(0.5)(60^2) = 900\ \text{m}$$

$$(\xrightarrow{+}) \qquad v = v_0 + a_c t$$

$$v_1 = 0 + 0.5(60) = 30\ \text{m/s}$$

For stage (2) motion, $v_0 = 30$ m/s, $s_0 = 900$ m, $a_c = 0$ and $t = 15(60) = 900$ s. Thus,

$$(\xrightarrow{+}) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_2 = 900 + 30(900) + 0 = 27\,900\ \text{m}$$

For stage (3) motion, $v_0 = 30$ m/s, $v = 0$, $s_0 = 27\,900$ m and $a_c = -1\ \text{m/s}^2$. Thus,

$$(\xrightarrow{+}) \qquad v = v_0 + a_c t$$

$$0 = 30 + (-1)t$$

$$t = 30\ \text{s}$$

$$\xrightarrow{+} \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_3 = 27\,900 + 30(30) + \frac{1}{2}(-1)(30^2)$$

$$= 28\,350\ \text{m} = 28.4\ \text{km}$$ **Ans.**

***12–16.**

A particle travels along a straight line such that in 2 s it moves from an initial position $s_A = +0.5$ m to a position $s_B = -1.5$ m. Then in another 4 s it moves from s_B to $s_C = +2.5$ m. Determine the particle's average velocity and average speed during the 6-s time interval.

SOLUTION

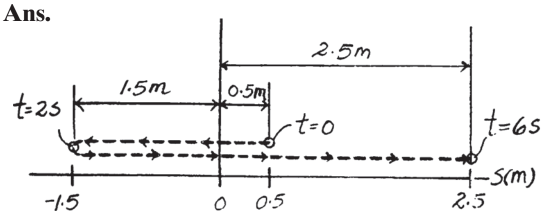
$$\Delta s = (s_C - s_A) = 2 \text{ m}$$

$$s_T = (0.5 + 1.5 + 1.5 + 2.5) = 6 \text{ m}$$

$$t = (2 + 4) = 6 \text{ s}$$

$$v_{avg} = \frac{\Delta s}{t} = \frac{2}{6} = 0.333 \text{ m/s}$$ **Ans.**

$$(v_{sp})_{avg} = \frac{s_T}{t} = \frac{6}{6} = 1 \text{ m/s}$$ **Ans.**



12–17.

The acceleration of a particle as it moves along a straight line is given by $a = (2t - 1)\ \text{m/s}^2$, where t is in seconds. If $s = 1\ \text{m}$ and $v = 2\ \text{m/s}$ when $t = 0$, determine the particle's velocity and position when $t = 6\ \text{s}$. Also, determine the total distance the particle travels during this time period.

SOLUTION

$$\int_2^v dv = \int_0^t (2\,t - 1)\,dt$$

$$v = t^2 - t + 2$$

$$\int_1^s ds = \int_0^t (t^2 - t + 2)\,dt$$

$$s = \frac{1}{3}t^3 - \frac{1}{2}t^2 + 2\,t + 1$$

When $t = 6$ s,

$$v = 32\ \text{m/s} \qquad \textbf{Ans.}$$

$$s = 67\ \text{m} \qquad \textbf{Ans.}$$

Since $v \neq 0$ then

$$d = 67 - 1 = 66\ \text{m} \qquad \textbf{Ans.}$$

12–18.

A freight train travels at $v = 60(1 - e^{-t})$ ft/s, where t is the elapsed time in seconds. Determine the distance traveled in three seconds, and the acceleration at this time.

SOLUTION

$v = 60(1 - e^{-t})$

$$\int_0^s ds = \int v\,dt = \int_0^3 60(1 - e^{-t})dt$$

$s = 60(t + e^{-t})\big|_0^3$

$s = 123$ ft **Ans.**

$$a = \frac{dv}{dt} = 60(e^{-t})$$

At $t = 3$ s

$a = 60e^{-3} = 2.99\ \text{ft/s}^2$ **Ans.**

12–19.

A particle travels to the right along a straight line with a velocity $v = [5/(4 + s)]$ m/s, where s is in meters. Determine its position when $t = 6$ s if $s = 5$ m when $t = 0$.

SOLUTION

$$\frac{ds}{dt} = \frac{5}{4 + s}$$

$$\int_5^s (4 + s)\, ds = \int_0^t 5\, dt$$

$$4\,s + 0.5\,s^2 - 32.5 = 5\,t$$

When $t = 6$ s,

$$s^2 + 8\,s - 125 = 0$$

Solving for the positive root

$s = 7.87$ m **Ans.**

***12–20.**

The velocity of a particle traveling along a straight line is $v = (3t^2 - 6t)$ ft/s, where t is in seconds. If $s = 4$ ft when $t = 0$, determine the position of the particle when $t = 4$ s. What is the total distance traveled during the time interval $t = 0$ to $t = 4$ s? Also, what is the acceleration when $t = 2$ s?

SOLUTION

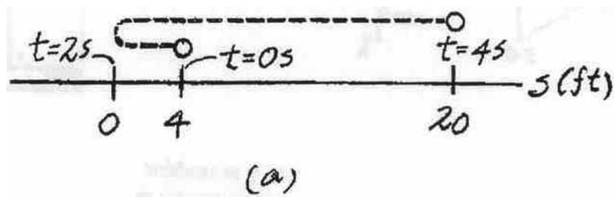
Position: The position of the particle can be determined by integrating the kinematic equation $ds = v\,dt$ using the initial condition $s = 4$ ft when $t = 0$ s. Thus,

$(\overset{+}{\rightarrow})$ $$ds = v\,dt$$

$$\int_{4\text{ ft}}^{s} ds = \int_0^t \left(3t^2 - 6t\right)dt$$

$$s\Big|_{4\text{ ft}}^{s} = (t^3 - 3t^2)\Big|_0^t$$

$$s = \left(t^3 - 3t^2 + 4\right)\text{ ft}$$



When $t = 4$ s,

$$s|_{4\text{ s}} = 4^3 - 3(4^2) + 4 = 20\text{ ft}$$ **Ans.**

The velocity of the particle changes direction at the instant when it is momentarily brought to rest. Thus,

$$v = 3t^2 - 6t = 0$$

$$t(3t - 6) = 0$$

$$t = 0 \text{ and } t = 2\text{ s}$$

The position of the particle at $t = 0$ and 2 s is

$$s|_{0\text{ s}} = 0 - 3\left(0^2\right) + 4 = 4\text{ ft}$$

$$s|_{2\text{ s}} = 2^3 - 3\left(2^2\right) + 4 = 0$$

Using the above result, the path of the particle shown in Fig. *a* is plotted. From this figure,

$$s_{\text{Tot}} = 4 + 20 = 24\text{ ft}$$ **Ans.**

Acceleration:

$(\overset{+}{\rightarrow})$ $$a = \frac{dv}{dt} = \frac{d}{dt}(3t^2 - 6t)$$

$$a = (6t - 6)\text{ ft/s}^2$$

When $t = 2$ s,

$$a|_{t=2\text{ s}} = 6(2) - 6 = 6\text{ ft/s}^2 \rightarrow$$ **Ans.**

12–21.

If the effects of atmospheric resistance are accounted for, a falling body has an acceleration defined by the equation $a = 9.81[1 - v^2(10^{-4})]$ m/s^2, where v is in m/s and the positive direction is downward. If the body is released from rest at a *very high altitude*, determine (a) the velocity when $t = 5$ s, and (b) the body's terminal or maximum attainable velocity (as $t \rightarrow \infty$).

SOLUTION

Velocity: The velocity of the particle can be related to the time by applying Eq. 12–2.

$$(+\downarrow) \qquad dt = \frac{dv}{a}$$

$$\int_0^t dt = \int_0^v \frac{dv}{9.81[1 - (0.01v)^2]}$$

$$t = \frac{1}{9.81}\left[\int_0^v \frac{dv}{2(1 + 0.01v)} + \int_0^v \frac{dv}{2(1 - 0.01v)}\right]$$

$$9.81t = 50\ln\left(\frac{1 + 0.01v}{1 - 0.01v}\right)$$

$$v = \frac{100(e^{0.1962t} - 1)}{e^{0.1962t} + 1} \qquad \textbf{(1)}$$

a) When $t = 5$ s, then, from Eq. (1)

$$v = \frac{100[e^{0.1962(5)} - 1]}{e^{0.1962(5)} + 1} = 45.5 \text{ m/s} \qquad \textbf{Ans.}$$

b) If $t \rightarrow \infty$, $\dfrac{e^{0.1962t} - 1}{e^{0.1962t} + 1} \rightarrow 1$. Then, from Eq. (1)

$$v_{max} = 100 \text{ m/s} \qquad \textbf{Ans.}$$

12–22.

The position of a particle on a straight line is given by $s = (t^3 - 9t^2 + 15t)$ ft, where t is in seconds. Determine the position of the particle when $t = 6$ s and the total distance it travels during the 6-s time interval. *Hint*: Plot the path to determine the total distance traveled.

SOLUTION

$s = t^3 - 9t^2 + 15t$

$v = \dfrac{ds}{dt} = 3t^2 - 18t + 15$

$v = 0$ when $t = 1$ s and $t = 5$ s

$t = 0, \ s = 0$

$t = 1 \text{ s}, \quad s = 7 \text{ ft}$

$t = 5 \text{ s}, \ s = -25 \text{ ft}$

$t = 6 \text{ s}, \ s = -18 \text{ ft}$ **Ans.**

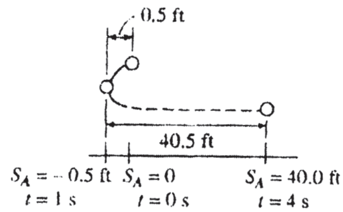
$s_T = 7 + 7 + 25 + (25 - 18) = 46 \text{ ft}$ **Ans.**

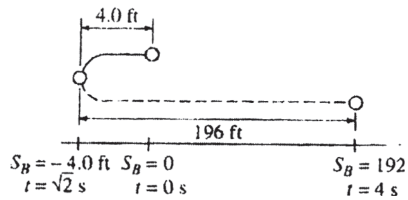
12–23.

Two particles A and B start from rest at the origin $s = 0$ and move along a straight line such that $a_A = (6t - 3)\ \text{ft/s}^2$ and $a_B = (12t^2 - 8)\ \text{ft/s}^2$, where t is in seconds. Determine the distance between them when $t = 4$ s and the total distance each has traveled in $t = 4$ s.

SOLUTION

Velocity: The velocity of particles A and B can be determined using Eq. 12-2.





$$dv_A = a_A dt$$

$$\int_0^{v_A} dv_A = \int_0^t (6t - 3)dt$$

$$v_A = 3t^2 - 3t$$

$$dv_B = a_B dt$$

$$\int_0^{v_B} dv_B = \int_0^t (12t^2 - 8)dt$$

$$v_B = 4t^3 - 8t$$

The times when particle A stops are

$3t^2 - 3t = 0 \qquad t = 0$ s and $= 1$ s

The times when particle B stops are

$4t^3 - 8t = 0 \quad t = 0$ s and $t = \sqrt{2}$ s

Position: The position of particles A and B can be determined using Eq. 12-1.

$$ds_A = v_A dt$$

$$\int_0^{s_A} ds_A = \int_0^t (3t^2 - 3t)dt$$

$$s_A = t^3 - \frac{3}{2}t^2$$

$$ds_B = v_B dt$$

$$\int_0^{s_B} ds_B = \int_0^t (4t^3 - 8t)dt$$

$$s_B = t^4 - 4t^2$$

The positions of particle A at $t = 1$ s and 4 s are

$$s_A|_{t=1\,s} = 1^3 - \frac{3}{2}(1^2) = -0.500\ \text{ft}$$

$$s_A|_{t=4\,s} = 4^3 - \frac{3}{2}(4^2) = 40.0\ \text{ft}$$

Particle A has traveled

$$d_A = 2(0.5) + 40.0 = 41.0\ \text{ft} \qquad \textbf{Ans.}$$

The positions of particle B at $t = \sqrt{2}$ s and 4 s are

$$s_B|_{t=\sqrt{2}} = (\sqrt{2})^4 - 4(\sqrt{2})^2 = -4\ \text{ft}$$

$$s_B|_{t=4} = (4)^4 - 4(4)^2 = 192\ \text{ft}$$

Particle B has traveled

$$d_B = 2(4) + 192 = 200\ \text{ft} \qquad \textbf{Ans.}$$

At $t = 4$ s the distance beween A and B is

$$\Delta s_{AB} = 192 - 40 = 152\ \text{ft} \qquad \textbf{Ans.}$$

***12–24.**

A particle is moving along a straight line such that its velocity is defined as $v = (-4s^2)$ m/s, where s is in meters. If $s = 2$ m when $t = 0$, determine the velocity and acceleration as functions of time.

SOLUTION

$$v = -4s^2$$

$$\frac{ds}{dt} = -4s^2$$

$$\int_2^s s^{-2}\,ds = \int_0^t -4\,dt$$

$$-s^{-1}\big|_2^s = -4t\big|_0^t$$

$$t = \frac{1}{4}(s^{-1} - 0.5)$$

$$s = \frac{2}{8t + 1}$$

$$v = -4\left(\frac{2}{8t + 1}\right)^2 = -\frac{16}{(8t + 1)^2}\ \text{m/s}$$ **Ans.**

$$a = \frac{dv}{dt} = \frac{16(2)(8t + 1)(8)}{(8t + 1)^4} = \frac{256}{(8t + 1)^3}\ \text{m/s}^2$$ **Ans.**

12–25.

A sphere is fired downwards into a medium with an initial speed of 27 m/s. If it experiences a deceleration of $a = (-6t)\ \text{m/s}^2$, where t is in seconds, determine the distance traveled before it stops.

SOLUTION

Velocity: $v_0 = 27$ m/s at $t_0 = 0$ s. Applying Eq. 12–2, we have

$$(+\downarrow) \qquad dv = a\,dt$$

$$\int_{27}^{v} dv = \int_0^t -6t\,dt$$

$$v = \left(27 - 3t^2\right)\ \text{m/s} \qquad \textbf{(1)}$$

At $v = 0$, from Eq. (1)

$$0 = 27 - 3t^2 \qquad t = 3.00\ \text{s}$$

Distance Traveled: $s_0 = 0$ m at $t_0 = 0$ s. Using the result $v = 27 - 3t^2$ and applying Eq. 12–1, we have

$$(+\downarrow) \qquad ds = v\,dt$$

$$\int_0^s ds = \int_0^t \left(27 - 3t^2\right) dt$$

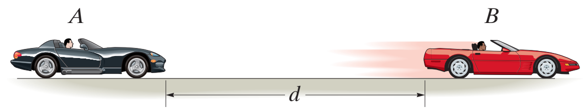
$$s = \left(27t - t^3\right)\ \text{m} \qquad \textbf{(2)}$$

At $t = 3.00$ s, from Eq. (2)

$$s = 27(3.00) - 3.00^3 = 54.0\ \text{m} \qquad \textbf{Ans.}$$

12–26.

When two cars A and B are next to one another, they are traveling in the same direction with speeds v_A and v_B, respectively. If B maintains its constant speed, while A begins to decelerate at a_A, determine the distance d between the cars at the instant A stops.



SOLUTION

Motion of car A:

$$v = v_0 + a_c t$$

$$0 = v_A - a_A t \qquad t = \frac{v_A}{a_A}$$

$$v^2 = v_0^2 + 2a_c(s - s_0)$$

$$0 = v_A^2 + 2(-a_A)(s_A - 0)$$

$$s_A = \frac{v_A^2}{2a_A}$$

Motion of car B:

$$s_B = v_B t = v_B\left(\frac{v_A}{a_A}\right) = \frac{v_A v_B}{a_A}$$

The distance between cars A and B is

$$s_{BA} = |s_B - s_A| = \left|\frac{v_A v_B}{a_A} - \frac{v_A^2}{2a_A}\right| = \left|\frac{2v_A v_B - v_A^2}{2a_A}\right| \qquad \textbf{Ans.}$$

12–27.

A particle is moving along a straight line such that when it is at the origin it has a velocity of 4 m/s. If it begins to decelerate at the rate of $a = (-1.5v^{1/2})$ m/s^2, where v is in m/s, determine the distance it travels before it stops.

SOLUTION

$$a = \frac{dv}{dt} = -1.5v^{\frac{1}{2}}$$

$$\int_4^v v^{-\frac{1}{2}}\,dv = \int_0^t -1.5\,dt$$

$$2v^{\frac{1}{2}}\Big|_4^v = -1.5t\Big|_0^t$$

$$2\left(v^{\frac{1}{2}} - 2\right) = -1.5t$$

$$v = (2 - 0.75t)^2 \text{ m/s} \qquad \textbf{(1)}$$

$$\int_0^s ds = \int_0^t (2 - 0.75t)^2\,dt = \int_0^t (4 - 3t + 0.5625t^2)\,dt$$

$$s = 4t - 1.5t^2 + 0.1875t^3 \qquad \textbf{(2)}$$

From Eq. (1), the particle will stop when

$$0 = (2 - 0.75t)^2$$

$$t = 2.667 \text{ s}$$

$$s\big|_{t=2.667} = 4(2.667) - 1.5(2.667)^2 + 0.1875(2.667)^3 = 3.56 \text{ m} \qquad \textbf{Ans.}$$

***12–28.**

A particle travels to the right along a straight line with a velocity $v = [5/(4 + s)]$ m/s, where s is in meters. Determine its deceleration when $s = 2$ m.

SOLUTION

$$v = \frac{5}{4 + s}$$

$$v\,dv = a\,ds$$

$$dv = \frac{-5\,ds}{(4 + s)^2}$$

$$\frac{5}{(4 + s)}\left(\frac{-5\,ds}{(4 + s)^2}\right) = a\,ds$$

$$a = \frac{-25}{(4 + s)^3}$$

When $s = 2$ m

$a = -0.116 \text{ m/s}^2$ **Ans.**

12–29.

A particle moves along a straight line with an acceleration $a = 2v^{1/2}\ \text{m/s}^2$, where v is in m/s. If $s = 0$, $v = 4$ m/s when $t = 0$, determine the time for the particle to achieve a velocity of 20 m/s. Also, find the displacement of particle when $t = 2$ s.

SOLUTION

Velocity:

$$(\xrightarrow{+}) \qquad dt = \frac{dv}{a}$$

$$\int_0^t dt = \int_0^v \frac{dv}{2v^{1/2}}$$

$$t\Big|_0^t = v^{1/2}\Big|_4^v$$

$$t = v^{1/2} - 2$$

$$v = (t + 2)^2$$

When $v = 20$ m/s,

$$20 = (t + 2)^2$$

$$t = 2.47 \text{ s} \qquad \textbf{Ans.}$$

Position:

$$(\xrightarrow{+}) \qquad ds = v\,dt$$

$$\int_0^s ds = \int_0^t (t + 2)^2\,dt$$

$$s\Big|_0^s = \frac{1}{3}(t + 2)^3\Big|_0^t$$

$$s = \frac{1}{3}[(t + 2)^3 - 2^3]$$

$$= \frac{1}{3}t(t^2 + 6t + 12)$$

When $t = 2$ s,

$$s = \frac{1}{3}(2)[(2)^2 + 6(2) + 12]$$

$$= 18.7 \text{ m} \qquad \textbf{Ans.}$$

12–30.

As a train accelerates uniformly it passes successive kilometer marks while traveling at velocities of 2 m/s and then 10 m/s. Determine the train's velocity when it passes the next kilometer mark and the time it takes to travel the 2-km distance.

SOLUTION

Kinematics: For the first kilometer of the journey, $v_0 = 2$ m/s, $v = 10$ m/s, $s_0 = 0$, and $s = 1000$ m. Thus,

$$(\overset{+}{\rightarrow}) \qquad v^2 = v_0{}^2 + 2a_c(s - s_0)$$

$$10^2 = 2^2 + 2a_c(1000 - 0)$$

$$a_c = 0.048 \text{ m/s}^2$$

For the second kilometer, $v_0 = 10$ m/s, $s_0 = 1000$ m, $s = 2000$ m, and $a_c = 0.048$ m/s^2. Thus,

$$(\overset{+}{\rightarrow}) \qquad v^2 = v_0{}^2 + 2a_c(s - s_0)$$

$$v^2 = 10^2 + 2(0.048)(2000 - 1000)$$

$$v = 14 \text{ m/s} \qquad \textbf{Ans.}$$

For the whole journey, $v_0 = 2$ m/s, $v = 14$ m/s, and $a_c = 0.048$ m/s^2 Thus,

$$(\overset{+}{\rightarrow}) \qquad v = v_0 + a_c t$$

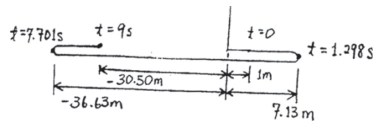
$$14 = 2 + 0.048t$$

$$t = 250 \text{ s} \qquad \textbf{Ans.}$$

12–31.

The acceleration of a particle along a straight line is defined by $a = (2t - 9)\ \text{m/s}^2$, where t is in seconds. At $t = 0$, $s = 1$ m and $v = 10$ m/s. When $t = 9$ s, determine (a) the particle's position, (b) the total distance traveled, and (c) the velocity.

SOLUTION



$a = 2t - 9$

$$\int_{10}^{v} dv = \int_{0}^{t} (2t - 9)\, dt$$

$v - 10 = t^2 - 9\,t$

$v = t^2 - 9\,t + 10$

$$\int_{1}^{s} ds = \int_{0}^{t} (t^2 - 9t + 10)\, dt$$

$s - 1 = \frac{1}{3}t^3 - 4.5\,t^2 + 10\,t$

$s = \frac{1}{3}t^3 - 4.5\,t^2 + 10\,t + 1$

Note when $v = t^2 - 9\,t + 10 = 0$:

$$t = 1.298 \text{ s and } t = 7.701 \text{ s}$$

When $t = 1.298$ s, $\quad s = 7.13$ m

When $t = 7.701$ s, $\quad s = -36.63$ m

When $t = 9$ s, $\quad s = -30.50$ m

(a) $\quad s = -30.5$ m **Ans.**

(b) $\quad s_{Tot} = (7.13 - 1) + 7.13 + 36.63 + (36.63 - 30.50)$

$\quad s_{Tot} = 56.0$ m **Ans.**

(c) $\quad v = 10$ m/s **Ans.**

***12–32.**

The acceleration of a particle traveling along a straight line is $a = \frac{1}{4}s^{1/2}$ m/s^2, where s is in meters. If $v = 0$, $s = 1$ m when $t = 0$, determine the particle's velocity at $s = 2$ m.

SOLUTION

Velocity:

$(\overset{+}{\rightarrow})$
$$v\,dv = a\,ds$$

$$\int_0^v v\,dv = \int_1^s \frac{1}{4}s^{1/2}ds$$

$$\frac{v^2}{2}\bigg|_0^v = \frac{1}{6}s^{3/2}\bigg|_1^s$$

$$v = \frac{1}{\sqrt{3}}(s^{3/2} - 1)^{1/2} \text{ m/s}$$

When $s = 2$ m, $v = 0.781$ m/s. **Ans.**

12–33.

At $t = 0$ bullet A is fired vertically with an initial (muzzle) velocity of 450 m/s. When $t = 3$ s, bullet B is fired upward with a muzzle velocity of 600 m/s. Determine the time t, after A is fired, as to when bullet B passes bullet A. At what altitude does this occur?

SOLUTION

$$+\uparrow s_A = (s_A)_0 + (v_A)_0 t + \frac{1}{2}a_c t^2$$

$$s_A = 0 + 450 t + \frac{1}{2}(-9.81) t^2$$

$$+\uparrow s_B = (s_B)_0 + (v_B)_0 t + \frac{1}{2} a_c t^2$$

$$s_B = 0 + 600(t - 3) + \frac{1}{2}(-9.81)(t - 3)^2$$

Require $s_A = s_B$

$$450 t - 4.905 t^2 = 600 t - 1800 - 4.905 t^2 + 29.43 t - 44.145$$

$t = 10.3$ s **Ans.**

$h = s_A = s_B = 4.11$ km **Ans.**

12–34.

A boy throws a ball straight up from the top of a 12-m high tower. If the ball falls past him 0.75 s later, determine the velocity at which it was thrown, the velocity of the ball when it strikes the ground, and the time of flight.

SOLUTION

***Kinematics*:** When the ball passes the boy, the displacement of the ball in equal to zero.

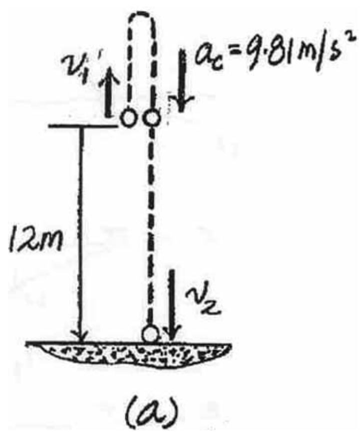
Thus, $s = 0$. Also, $s_0 = 0$, $v_0 = v_1$, $t = 0.75$ s, and $a_c = -9.81 \text{ m/s}^2$.

$(+\uparrow) \qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$$0 = 0 + v_1(0.75) + \frac{1}{2}(-9.81)(0.75^2)$$

$$v_1 = 3.679 \text{ m/s} = 3.68 \text{ m/s} \qquad \textbf{Ans.}$$

When the ball strikes the ground, its displacement from the roof top is $s = -12$ m. Also, $v_0 = v_1 = 3.679$ m/s, $t = t_2$, $v = v_2$, and $a_c = -9.81 \text{ m/s}^2$.



$(+\uparrow) \qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$$-12 = 0 + 3.679t_2 + \frac{1}{2}(-9.81)t_2^2$$

$$4.905t_2^2 - 3.679t_2 - 12 = 0$$

$$t_2 = \frac{3.679 \pm \sqrt{(-3.679)^2 - 4(4.905)(-12)}}{2(4.905)}$$

Choosing the positive root, we have

$$t_2 = 1.983 \text{ s} = 1.98 \text{ s} \qquad \textbf{Ans.}$$

Using this result,

$(+\uparrow) \qquad v = v_0 + a_c t$

$$v_2 = 3.679 + (-9.81)(1.983)$$

$$= -15.8 \text{ m/s} = 15.8 \text{ m/s} \downarrow \qquad \textbf{Ans.}$$

12–35.

When a particle falls through the air, its initial acceleration $a = g$ diminishes until it is zero, and thereafter it falls at a constant or terminal velocity v_f. If this variation of the acceleration can be expressed as $a = (g/v^2{}_f)(v^2{}_f - v^2)$, determine the time needed for the velocity to become $v = v_f/2$. Initially the particle falls from rest.

SOLUTION

$$\frac{dv}{dt} = a = \left(\frac{g}{v_f^2}\right)\left(v_f^2 - v^2\right)$$

$$\int_0^v \frac{dv}{v_f^2 - v^{2'}} = \frac{g}{v_f^2}\int_0^t dt$$

$$\frac{1}{2v_f}\ln\left(\frac{v_f + v}{v_f - v}\right)\Bigg|_0^v = \frac{g}{v_f^2}t$$

$$t = \frac{v_f}{2g}\ln\left(\frac{v_f + v}{v_f - v}\right)$$

$$t = \frac{v_f}{2g}\ln\left(\frac{v_f + v_f/2}{v_f - v_f/2}\right)$$

$$t = 0.549\left(\frac{v_f}{g}\right)$$ **Ans.**

***12–36.**

A particle is moving with a velocity of v_0 when $s = 0$ and $t = 0$. If it is subjected to a deceleration of $a = -kv^3$, where k is a constant, determine its velocity and position as functions of time.

SOLUTION

$$a = \frac{dv}{dt} = -kv^3$$

$$\int_{v_0}^{v} v^{-3}\,dv = \int_0^t -k\,dt$$

$$-\frac{1}{2}(v^{-2} - v_0^{-2}) = -kt$$

$$v = \left(2kt + \left(\frac{1}{v_0^2}\right)\right)^{-\frac{1}{2}} \qquad \textbf{Ans.}$$

$$ds = v\,dt$$

$$\int_0^s ds = \int_0^t \frac{dt}{\left(2kt + \left(\frac{1}{v_0^2}\right)\right)^{\frac{1}{2}}}$$

$$s = \left.\frac{2\left(2kt + \left(\frac{1}{v_0^2}\right)\right)^{\frac{1}{2}}}{2k}\right|_0^t$$

$$s = \frac{1}{k}\left[\left(2kt + \left(\frac{1}{v_0^2}\right)\right)^{\frac{1}{2}} - \frac{1}{v_0}\right] \qquad \textbf{Ans.}$$

12–37.

As a body is projected to a high altitude above the earth's *surface*, the variation of the acceleration of gravity with respect to altitude y must be taken into account. Neglecting air resistance, this acceleration is determined from the formula $a = -g_0[R^2/(R + y)^2]$, where g_0 is the constant gravitational acceleration at sea level, R is the radius of the earth, and the positive direction is measured upward. If $g_0 = 9.81\ \text{m/s}^2$ and $R = 6356$ km, determine the minimum initial velocity (escape velocity) at which a projectile should be shot vertically from the earth's surface so that it does not fall back to the earth. *Hint:* This requires that $v = 0$ as $y \rightarrow \infty$.

SOLUTION

$$v\,dv = a\,dy$$

$$\int_v^0 v\,dv = -g_0 R^2 \int_0^\infty \frac{dy}{(R + y)^2}$$

$$\left.\frac{v^2}{2}\right|_v^0 = \left.\frac{g_0 R^2}{R + y}\right|_0^\infty$$

$$v = \sqrt{2g_0 R}$$

$$= \sqrt{2(9.81)(6356)(10)^3}$$

$$= 11167\ \text{m/s} = 11.2\ \text{km/s}$$ **Ans.**

12–38.

Accounting for the variation of gravitational acceleration a with respect to altitude y (see Prob. 12–37), derive an equation that relates the velocity of a freely falling particle to its altitude. Assume that the particle is released from rest at an altitude y_0 from the earth's surface. With what velocity does the particle strike the earth if it is released from rest at an altitude $y_0 = 500$ km? Use the numerical data in Prob. 12–37.

SOLUTION

From Prob. 12–37,

$$(+\uparrow) \qquad a = -g_0 \frac{R^2}{(R + y)^2}$$

Since $a\,dy = v\,dv$

then

$$-g_0 R^2 \int_{y_0}^{y} \frac{dy}{(R + y)^2} = \int_0^v v\,dv$$

$$g_0 R^2 \left[\frac{1}{R + y}\right]_{y_0}^{y} = \frac{v^2}{2}$$

$$g_0 R^2 [\frac{1}{R + y} - \frac{1}{R + y_0}] = \frac{v^2}{2}$$

Thus

$$v = -R\sqrt{\frac{2g_0 (y_0 - y)}{(R + y)(R + y_0)}}$$

When $y_0 = 500$ km, $\quad y = 0$,

$$v = -6356(10^3)\sqrt{\frac{2(9.81)(500)(10^3)}{6356(6356 + 500)(10^6)}}$$

$v = -3016 \text{ m/s} = 3.02 \text{ km/s} \downarrow$ **Ans.**

12–39.

A freight train starts from rest and travels with a constant acceleration of 0.5 ft/s^2. After a time t' it maintains a constant speed so that when $t = 160$ s it has traveled 2000 ft. Determine the time t' and draw the v–t graph for the motion.

SOLUTION

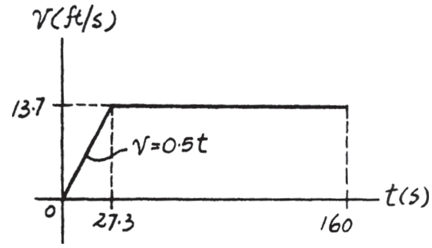
Total Distance Traveled: The distance for part one of the motion can be related to time $t = t'$ by applying Eq. 12–5 with $s_0 = 0$ and $v_0 = 0$.

$$(\overset{+}{\rightarrow}) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_1 = 0 + 0 + \frac{1}{2}(0.5)(t')^2 = 0.25(t')^2$$

The velocity at time t can be obtained by applying Eq. 12–4 with $v_0 = 0$.

$$(\overset{+}{\rightarrow}) \qquad v = v_0 + a_c t = 0 + 0.5t = 0.5t \qquad \textbf{(1)}$$



The time for the second stage of motion is $t_2 = 160 - t'$ and the train is traveling at a constant velocity of $v = 0.5t'$ (Eq. (1)). Thus, the distance for this part of motion is

$$(\overset{+}{\rightarrow}) \qquad s_2 = vt_2 = 0.5t'(160 - t') = 80t' - 0.5(t')^2$$

If the total distance traveled is $s_{Tot} = 2000$, then

$$s_{Tot} = s_1 + s_2$$

$$2000 = 0.25(t')^2 + 80t' - 0.5(t')^2$$

$$0.25(t')^2 - 80t' + 2000 = 0$$

Choose a root that is less than 160 s, then

$$t' = 27.34 \text{ s} = 27.3 \text{ s} \qquad \textbf{Ans.}$$

$v - t$ Graph: The equation for the velocity is given by Eq. (1). When $t = t' = 27.34$ s, $v = 0.5(27.34) = 13.7$ ft/s.

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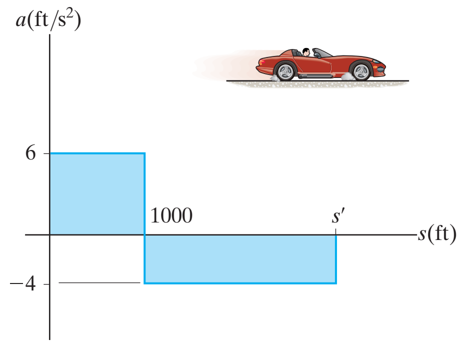
***12–40.**

A sports car travels along a straight road with an acceleration-deceleration described by the graph. If the car starts from rest, determine the distance s' the car travels until it stops. Construct the $v-s$ graph for $0 \le s \le s'$.



SOLUTION

v – s Graph: For $0 \le s < 1000$ ft, the initial condition is $v = 0$ at $s = 0$.

$$(\overset{+}{\rightarrow}) \qquad vdv = ads$$

$$\int_0^v vdv = \int_0^s 6ds$$

$$\frac{v^2}{2} = 6s$$

$$v = \left(\sqrt{12}s^{1/2}\right)\text{ ft/s}$$

When $s = 1000$ ft,

$$v = \sqrt{12}(1000)^{1/2} = 109.54\text{ ft/s} = 110\text{ ft/s}$$

For 1000 ft $< s \le s'$, the initial condition is $v = 109.54$ ft/s at $s = 1000$ ft.

$$(\overset{+}{\rightarrow}) \qquad vdv = ads$$

$$\int_{109.54\text{ ft/s}}^v vdv = \int_{1000\text{ ft}}^s -4ds$$

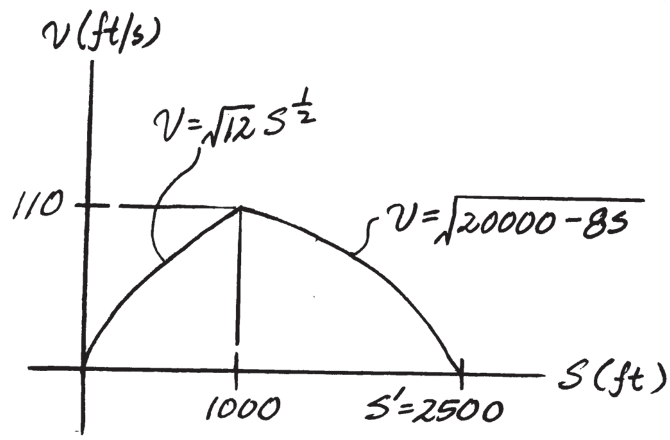
$$\left.\frac{v^2}{2}\right|_{109.54\text{ ft/s}}^{v} = -4s\Big|_{1000\text{ ft}}^{s}$$

$$v = \left(\sqrt{20\,000 - 8s}\right)\text{ ft/s}$$

When $v = 0$,

$$0 = \sqrt{20\,000 - 8s'} \qquad\qquad s' = 2500\text{ ft} \qquad\qquad \textbf{Ans.}$$

The v–s graph is shown in Fig. a.



(a)

12–41.

A train starts from station A and for the first kilometer, it travels with a uniform acceleration. Then, for the next two kilometers, it travels with a uniform speed. Finally, the train decelerates uniformly for another kilometer before coming to rest at station B. If the time for the whole journey is six minutes, draw the $v-t$ graph and determine the maximum speed of the train.

SOLUTION

For stage (1) motion,

$$\left(\xrightarrow{+}\right) \qquad v_1 = v_0 + (a_c)_1 t$$

$$v_{\max} = 0 + (a_c)_1 t_1$$

$$v_{\max} = (a_c)_1 t_1 \qquad \textbf{(1)}$$

$$\left(\xrightarrow{+}\right) \qquad v_1^2 = v_0^2 + 2(a_c)_1(s_1 - s_0)$$

$$v_{\max}{}^2 = 0 + 2(a_c)_1(1000 - 0)$$

$$(a_c)_1 = \frac{v_{\max}{}^2}{2000} \qquad \textbf{(2)}$$

Eliminating $(a_c)_1$ from Eqs. (1) and (2), we have

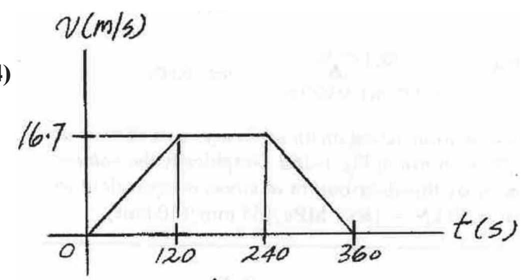
$$t_1 = \frac{2000}{v_{\max}} \qquad \textbf{(3)}$$

For stage (2) motion, the train travels with the constant velocity of $v_{\max}$ for $t = (t_2 - t_1)$. Thus,

$$\left(\xrightarrow{+}\right) \qquad s_2 = s_1 + v_1 t + \frac{1}{2}(a_c)_2 t^2$$

$$1000 + 2000 = 1000 + v_{\max}(t_2 - t_1) + 0$$

$$t_2 - t_1 = \frac{2000}{v_{\max}} \qquad \textbf{(4)}$$



(a)

For stage (3) motion, the train travels for $t = 360 - t_2$. Thus,

$$\left(\xrightarrow{+}\right) \qquad v_3 = v_2 + (a_c)_3 t$$

$$0 = v_{\max} - (a_c)_3(360 - t_2)$$

$$v_{\max} = (a_c)_3(360 - t_2) \qquad \textbf{(5)}$$

$$\left(\xrightarrow{+}\right) \qquad v_3^2 = v_2^2 + 2(a_c)_3(s_3 - s_2)$$

$$0 = v_{\max}{}^2 + 2[-(a_c)_3](4000 - 3000)$$

$$(a_c)_3 = \frac{v_{\max}{}^2}{2000} \qquad \textbf{(6)}$$

Eliminating $(a_c)_3$ from Eqs. (5) and (6) yields

$$360 - t_2 = \frac{2000}{v_{\max}} \qquad \textbf{(7)}$$

Solving Eqs. (3), (4), and (7), we have

$$t_1 = 120\text{ s} \qquad t_2 = 240\text{ s}$$

$$v_{\max} = 16.7\text{ m/s} \qquad \textbf{Ans.}$$

Based on the above results, the $v-t$ graph is shown in Fig. a.

12–42.

A particle starts from $s = 0$ and travels along a straight line with a velocity $v = (t^2 - 4t + 3)$ m/s, where t is in seconds. Construct the $v-t$ and $a-t$ graphs for the time interval $0 \leq t \leq 4$ s.

SOLUTION

a–t Graph:

$$a = \frac{dv}{dt} = \frac{d}{dt}(t^2 - 4t + 3)$$

$$a = (2t - 4)\,\text{m/s}^2$$

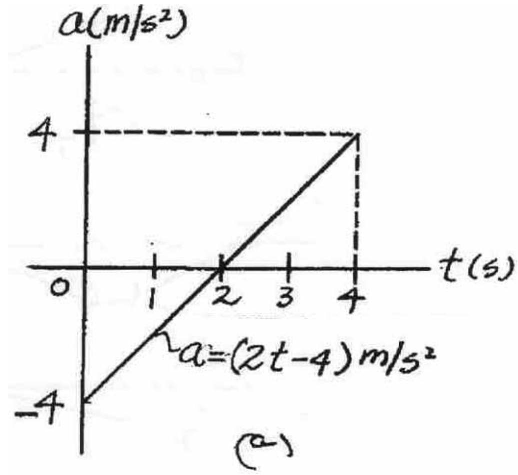
Thus,

$$a|_{t=0} = 2(0) - 4 = -4\ \text{m/s}^2$$

$$a|_{t=2} = 0$$

$$a|_{t=4\text{ s}} = 2(4) - 4 = 4\ \text{m/s}^2$$

The $a-t$ graph is shown in Fig. a.



v–t Graph: The slope of the $v-t$ graph is zero when $a = \frac{dv}{dt} = 0$. Thus,

$$a = 2t - 4 = 0 \qquad t = 2\text{ s}$$

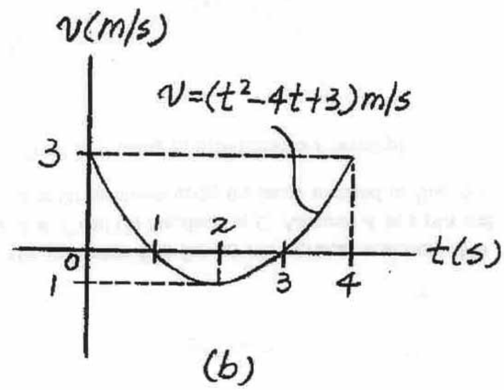
The velocity of the particle at $t = 0$ s, 2 s, and 4 s are

$$v|_{t=0\text{ s}} = 0^2 - 4(0) + 3 = 3\text{ m/s}$$

$$v|_{t=2\text{ s}} = 2^2 - 4(2) + 3 = -1\text{ m/s}$$

$$v|_{t=4\text{ s}} = 4^2 - 4(4) + 3 = 3\text{ m/s}$$

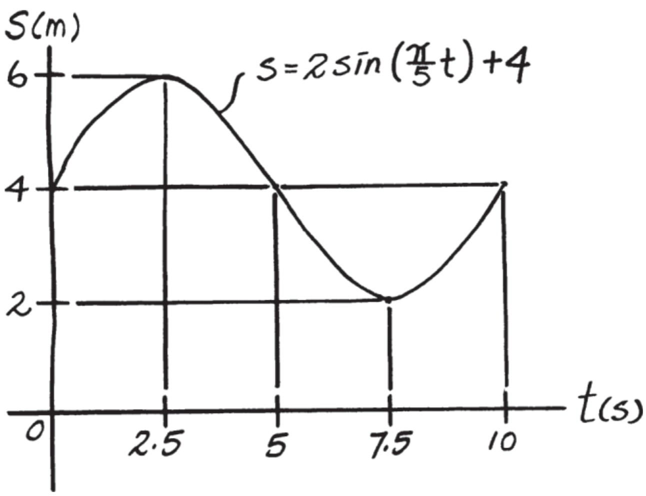
The $v-t$ graph is shown in Fig. b.

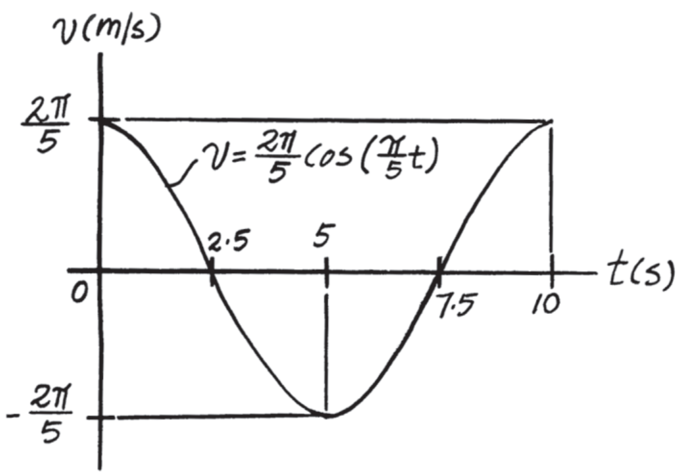


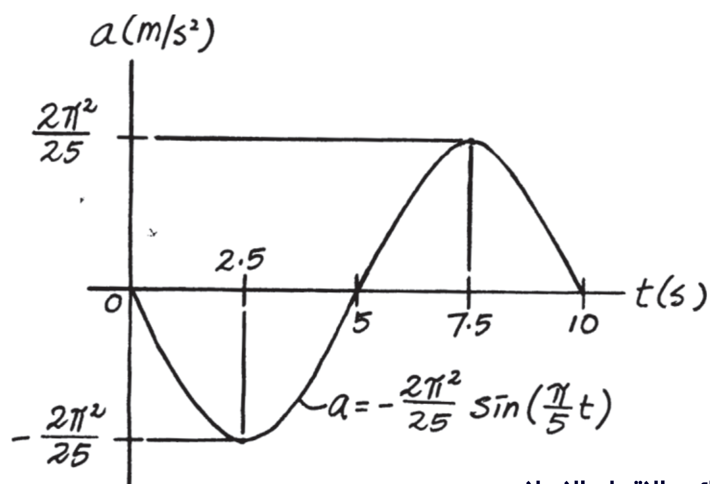
12–43.

If the position of a particle is defined by $s = [2 \sin [(\pi/5)t] + 4]$ m, where t is in seconds, construct the $s-t$, $v-t$, and $a-t$ graphs for $0 \le t \le 10$ s.

SOLUTION







***12–44.**

An airplane starts from rest, travels 5000 ft down a runway, and after uniform acceleration, takes off with a speed of 162 mi/h. It then climbs in a straight line with a uniform acceleration of 3 ft/s^2 until it reaches a constant speed of 220 mi/h. Draw the *s–t*, *v–t*, and *a–t* graphs that describe the motion.

SOLUTION

$$v_1 = 0$$

$$v_2 = 162\frac{\text{mi}}{\text{h}}\frac{(1\text{h})\ 5280\ \text{ft}}{(3600\ \text{s})(1\ \text{mi})} = 237.6\ \text{ft/s}$$

$$v_2^2 = v_1^2 + 2\,a_c(s_2 - s_1)$$

$$(237.6)^2 = 0^2 + 2(a_c)(5000 - 0)$$

$$a_c = 5.64538\ \text{ft/s}^2$$

$$v_2 = v_1 + a_c t$$

$$237.6 = 0 + 5.64538\,t$$

$$t = 42.09 = 42.1\ \text{s}$$

$$v_3 = 220\,\frac{\text{mi}}{\text{h}}\frac{(1\text{h})\ 5280\ \text{ft}}{(3600\ \text{s})(1\ \text{mi})} = 322.67\ \text{ft/s}$$

$$v_3^2 = v_2^2 + 2a_c(s_3 - s_2)$$

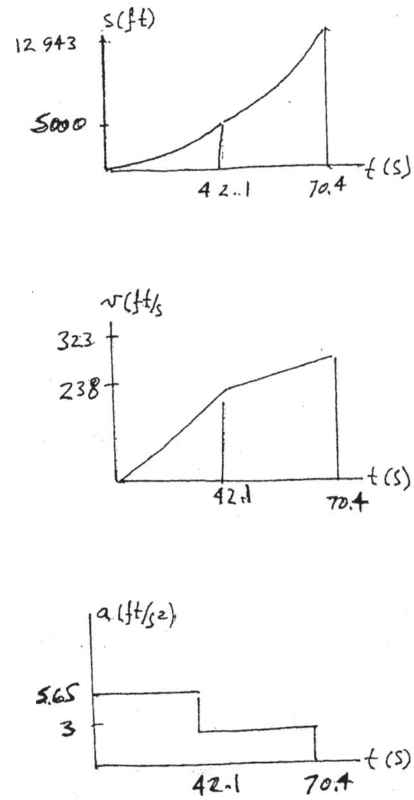
$$(322.67)^2 = (237.6)^2 + 2(3)(s - 5000)$$

$$s = 12\,943.34\ \text{ft}$$

$$v_3 = v_2 + a_c t$$

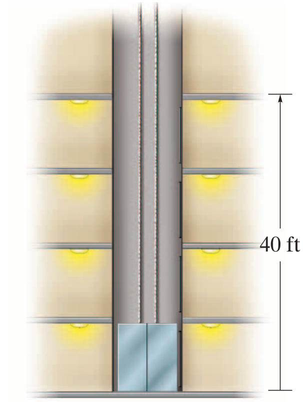
$$322.67 = 237.6 + 3\,t$$

$$t = 28.4\ \text{s}$$



12–45.

The elevator starts from rest at the first floor of the building. It can accelerate at 5 $\mathrm{ft/s^2}$ and then decelerate at 2 $\mathrm{ft/s^2}$. Determine the shortest time it takes to reach a floor 40 ft above the ground. The elevator starts from rest and then stops. Draw the a–t, v–t, and s–t graphs for the motion.



SOLUTION

$(+\uparrow)\quad v_2 = v_1 + a_c t_1$

$v_{max} = 0 + 5\,t_1$

$(+\uparrow)\quad v_3 = v_2 + a_c\,t$

$0 = v_{max} - 2\,t_2$

Thus

$t_1 = 0.4\,t_2$

$(+\uparrow)\quad s_2 = s_1 + v_1 t_1 + \frac{1}{2} a_c t_1^2$

$h = 0 + 0 + \frac{1}{2}(5)(t_1^2) = 2.5\,t_1^2$

$(+\uparrow)\quad 40 - h = 0 + v_{max} t_2 - \frac{1}{2}(2)\,t_2^2$

$(+\uparrow)\quad v^2 = v_1^2 + 2\,a_c(s - s_1)$

$v_{max}^2 = 0 + 2(5)(h - 0)$

$v_{max}^2 = 10h$

$0 = v_{max}^2 + 2(-2)(40 - h)$

$v_{max}^2 = 160 - 4h$

Thus,

$10\,h = 160 - 4h$

$h = 11.429\ \text{ft}$

$v_{max} = 10.69\ \text{ft/s}$

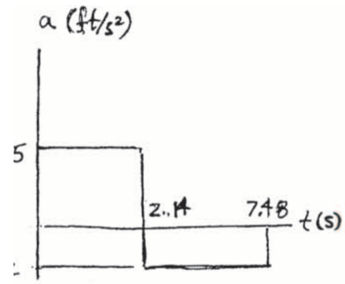
$t_1 = 2.138\ \text{s}$

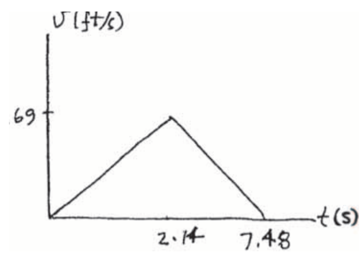
$t_2 = 5.345\ \text{s}$

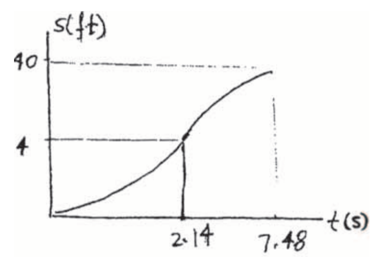
$t = t_1 + t_2 = 7.48\ \text{s}$ **Ans.**

When $t = 2.145$, $v = v_{max} = 10.7$ ft/s

and $h = 11.4$ ft.

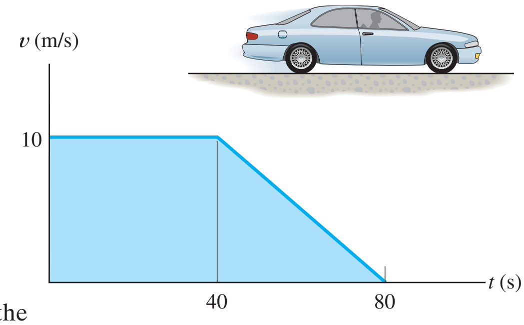






12–46.

The velocity of a car is plotted as shown. Determine the total distance the car moves until it stops ($t = 80$ s). Construct the a–t graph.



SOLUTION

Distance Traveled: The total distance traveled can be obtained by computing the area under the $v - t$ graph.

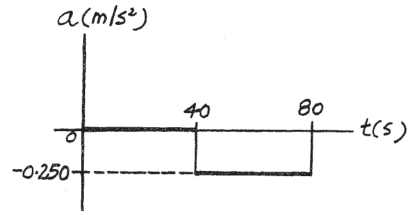
$$s = 10(40) + \frac{1}{2}(10)(80 - 40) = 600 \text{ m} \qquad \textbf{Ans.}$$

a – t Graph: The acceleration in terms of time t can be obtained by applying $a = \frac{dv}{dt}$. For time interval $0 \text{ s} \leq t < 40$ s,

$$a = \frac{dv}{dt} = 0$$

For time interval $40 \text{ s} < t \leq 80$ s, $\frac{v - 10}{t - 40} = \frac{0 - 10}{80 - 40}$, $v = \left(-\frac{1}{4}t + 20\right)$ m/s.

$$a = \frac{dv}{dt} = -\frac{1}{4} = -0.250 \text{ m/s}^2$$

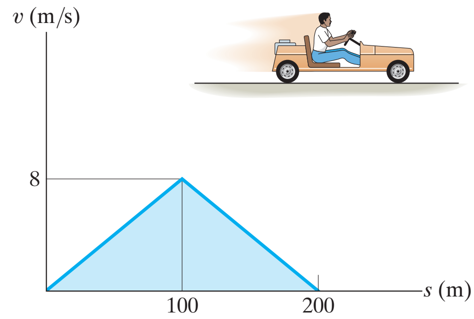


For $0 \leq t < 40$ s, $a = 0$.

For $40 \text{ s} < t \leq 80$, $a = -0.250 \text{ m/s}^2$.

12–47.

The v–s graph for a go-cart traveling on a straight road is shown. Determine the acceleration of the go-cart at $s = 50$ m and $s = 150$ m. Draw the a–s graph.



SOLUTION

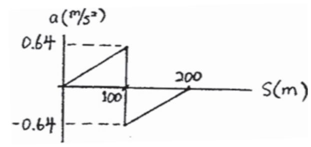
For $0 \leq s < 100$

$v = 0.08\,s, \qquad dv = 0.08\,ds$

$a\,ds = (0.08\,s)(0.08\,ds)$

$a = 6.4(10^{-3})\,s$

At $s = 50$ m, $\quad a = 0.32\text{ m/s}^2$ **Ans.**



For $100 < s < 200$

$v = -0.08\,s + 16,$

$dv = -0.08\,ds$

$a\,ds = (-0.08\,s + 16)(-0.08\,ds)$

$a = 0.08(0.08\,s - 16)$

At $s = 150$ m, $\quad a = -0.32\text{ m/s}^2$ **Ans.**

Also,

$v\,dv = a\,ds$

$a = v(\frac{dv}{ds})$

At $s = 50$ m,

$a = 4(\frac{8}{100}) = 0.32\text{ m/s}^2$ **Ans.**

At $s = 150$ m,

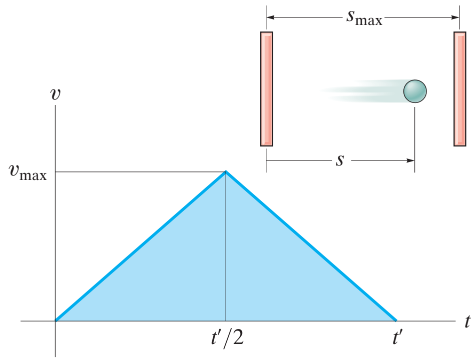
$a = 4(\frac{-8}{100}) = -0.32\text{ m/s}^2$ **Ans.**

At $s = 100$ m, a changes from $a_{max} = 0.64\text{ m/s}^2$

to $a_{min} = -0.64\text{ m/s}^2$.

***12–48.**

The v–t graph for a particle moving through an electric field from one plate to another has the shape shown in the figure. The acceleration and deceleration that occur are constant and both have a magnitude of $4\ \text{m/s}^2$. If the plates are spaced 200 mm apart, determine the maximum velocity $v_{\max}$ and the time t' for the particle to travel from one plate to the other. Also draw the s–t graph. When $t = t'/2$ the particle is at $s = 100$ mm.



SOLUTION

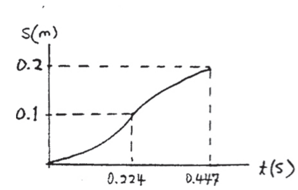
$a_c = 4\ \text{m/s}^2$

$$\frac{s}{2} = 100\ \text{mm} = 0.1\ \text{m}$$

$$v^2 = v_0^2 + 2\,a_c(s - s_0)$$

$$v_{max}^2 = 0 + 2(4)(0.1 - 0)$$

$$v_{max} = 0.89442\ \text{m/s} \qquad = 0.894\ \text{m/s}$$ **Ans.**



$$v = v_0 + a_c\,t'$$

$$0.89442 = 0 + 4\left(\frac{t'}{2}\right)$$

$$t' = 0.44721\ \text{s} \qquad = 0.447\ \text{s}$$ **Ans.**

$$s = s_0 + v_0\,t + \frac{1}{2}\,a_c\,t^2$$

$$s = 0 + 0 + \frac{1}{2}(4)(t)^2$$

$$s = 2\,t^2$$

When $t = \dfrac{0.44721}{2} = 0.2236 = 0.224$ s,

$$s = 0.1\ \text{m}$$

$$\int_{0.894}^{v} ds = -\int_{0.2235}^{t} 4\,dt$$

$$v = -4\,t + 1.788$$

$$\int_{0.1}^{s} ds = \int_{0.2235}^{t} (-4t + 1.788)\,dt$$

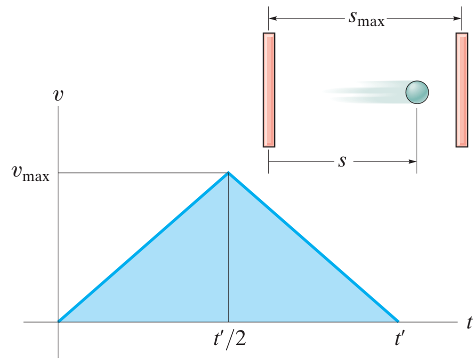
$$s = -2\,t^2 + 1.788\,t - 0.2$$

When $t = 0.447$ s,

$$s = 0.2\ \text{m}$$

12–49.

The v–t graph for a particle moving through an electric field from one plate to another has the shape shown in the figure, where $t' = 0.2$ s and $v_{max} = 10$ m/s. Draw the s–t and a–t graphs for the particle. When $t = t'/2$ the particle is at $s = 0.5$ m.



SOLUTION

For $0 < t < 0.1$ s,

$v = 100\,t$

$$a = \frac{dv}{dt} = 100$$

$ds = v\,dt$

$$\int_0^s ds = \int_0^t 100\,t\,dt$$

$s = 50\,t^2$

When $t = 0.1$ s,

$s = 0.5$ m

For 0.1 s $< t < 0.2$ s,

$v = -100\,t + 20$

$$a = \frac{dv}{dt} = -100$$

$ds = v\,dt$

$$\int_{0.5}^s ds = \int_{0.1}^t (-100t + 20)\mathrm{d}t$$

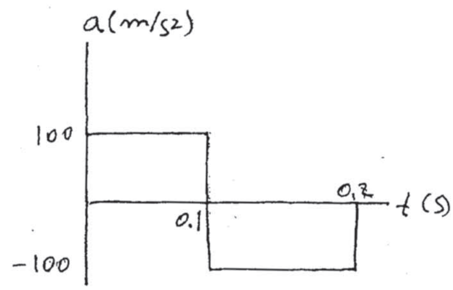
$s - 0.5 = (-50\,t^2 + 20\,t - 1.5)$

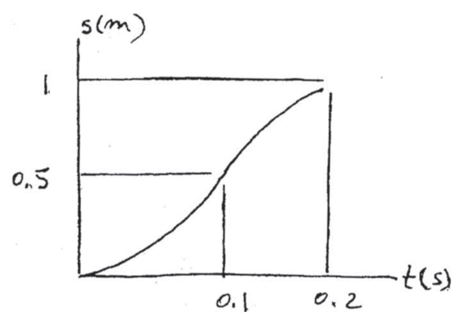
$s = -50\,t^2 + 20\,t - 1$

When $t = 0.2$ s,

$s = 1$ m

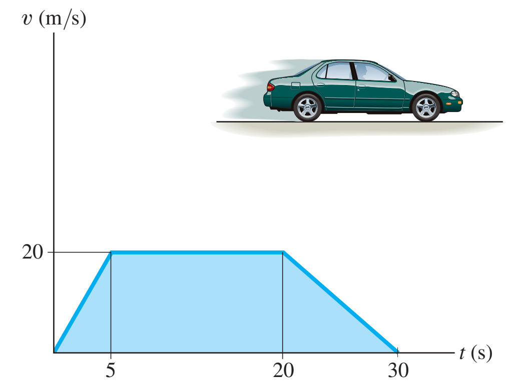
When $t = 0.1$ s, $s = 0.5$ m and a changes from 100 m/s^2 to -100 m/s^2. When t = 0.2 s, $s = 1$ m.





12–50.

The v–t graph of a car while traveling along a road is shown. Draw the s–t and a–t graphs for the motion.



SOLUTION

$0 \le t \le 5 \qquad a = \dfrac{\Delta v}{\Delta t} = \dfrac{20}{5} = 4\ \text{m/s}^2$

$5 \le t \le 20 \qquad a = \dfrac{\Delta v}{\Delta t} = \dfrac{20 - 20}{20 - 5} = 0\ \text{m/s}^2$

$20 \le t \le 30 \qquad a = \dfrac{\Delta v}{\Delta t} = \dfrac{0 - 20}{30 - 20} = -2\ \text{m/s}^2$

From the v–t graph at $t_1 = 5$ s, $t_2 = 20$ s, and $t_3 = 30$ s,

$$s_1 = A_1 = \frac{1}{2}(5)(20) = 50\ \text{m}$$

$$s_2 = A_1 + A_2 = 50 + 20(20 - 5) = 350\ \text{m}$$

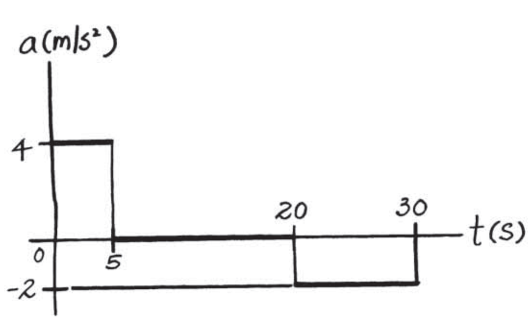
$$s_3 = A_1 + A_2 + A_3 = 350 + \frac{1}{2}(30 - 20)(20) = 450\ \text{m}$$

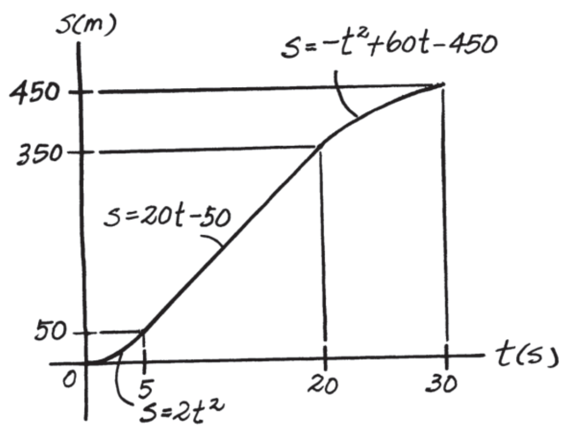
The equations defining the portions of the s–t graph are

$0 \le t \le 5\ \text{s} \qquad v = 4t; \qquad ds = v\,dt; \qquad \displaystyle\int_0^s ds = \int_0^t 4t\,dt; \qquad s = 2t^2$

$5 \le t \le 20\ \text{s} \qquad v = 20; \qquad ds = v\,dt; \qquad \displaystyle\int_{50}^s ds = \int_5^t 20\,dt; \qquad s = 20t - 50$

$20 \le t \le 30\ \text{s} \qquad v = 2(30 - t); \qquad ds = v\,dt; \qquad \displaystyle\int_{350}^s ds = \int_{20}^t 2(30 - t)\,dt; \qquad s = -t^2 + 60t - 450$





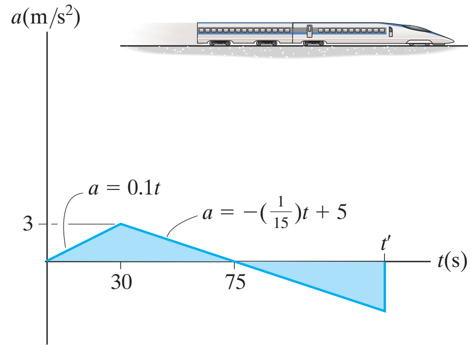
For $0 \le t < 5$ s, $a = 4\ \text{m/s}^2$.

For $20\ \text{s} < t \le 30$ s, $a = -2\ \text{m/s}^2$.

At $t = 5$ s, $s = 50$ m. At $t = 20$ s, $s = 350$ m.

12–51.

The a–t graph of the bullet train is shown. If the train starts from rest, determine the elapsed time t' before it again comes to rest. What is the total distance traveled during this time interval? Construct the v–t and s–t graphs.



SOLUTION

$v-t$ Graph: For the time interval $0 \le t < 30$ s, the initial condition is $v = 0$ when $t = 0$ s.

$$(\overset{+}{\rightarrow}) \qquad dv = a\,dt$$

$$\int_0^v dv = \int_0^t 0.1t\,dt$$

$$v = \left(0.05t^2\right)\text{ m/s}$$

When $t = 30$ s,

$$v\big|_{t=30\text{ s}} = 0.05\left(30^2\right) = 45\text{ m/s}$$

or the time interval $30\text{ s} < t \le t'$, the initial condition is $v = 45$ m/s at $t = 30$ s.

$$(\overset{+}{\rightarrow}) \qquad dv = a\,dt$$

$$\int_{45\text{ m/s}}^v dv = \int_{30\text{ s}}^t \left(-\frac{1}{15}t + 5\right)dt$$

$$v = \left(-\frac{1}{30}t^2 + 5t - 75\right)\text{ m/s}$$

Thus, when $v = 0$,

$$0 = -\frac{1}{30}t'^2 + 5t' - 75$$

Choosing the root $t' > 75$ s,

$$t' = 133.09\text{ s} = 133\text{ s} \qquad \textbf{Ans.}$$

Also, the change in velocity is equal to the area under the a–t graph. Thus,

$$\Delta v = \int a\,dt$$

$$0 = \frac{1}{2}(3)(75) + \frac{1}{2}\left[\left(-\frac{1}{15}t' + 5\right)(t' - 75)\right]$$

$$0 = -\frac{1}{30}t'^2 + 5t' - 75$$

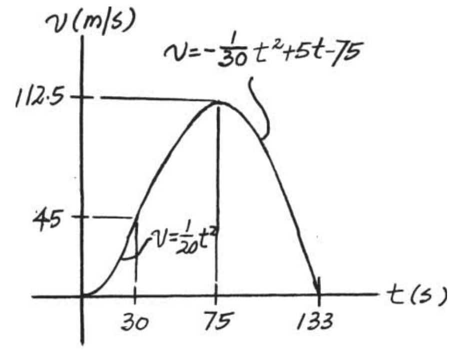
This equation is the same as the one obtained previously.

The slope of the v–$t$graph is zero when $t = 75$ s, which is the instant $a = \dfrac{dv}{dt} = 0$. Thus,

$$v\big|_{t=75\text{ s}} = -\frac{1}{30}\left(75^2\right) + 5(75) - 75 = 112.5\text{ m/s}$$

12–51. continued

The v–t graph is shown in Fig. *a*.



(a)

$s - t$ Graph: Using the result of v, the equation of the s–t graph can be obtained by integrating the kinematic equation $ds = vdt$. For the time interval $0 \leq t < 30$ s, the initial condition $s = 0$ at $t = 0$ s will be used as the integration limit. Thus,

$(\overset{+}{\rightarrow})$ $$ds = vdt$$

$$\int_0^s ds = \int_0^t 0.05t^2\,dt$$

$$s = \left(\frac{1}{60}t^3\right) \text{ m}$$

When $t = 30$ s,

$$s\big|_{t=30\text{ s}} = \frac{1}{60}\left(30^3\right) = 450 \text{ m}$$

For the time interval $30\text{ s} < t \leq t' = 133.09$ s, the initial condition is $s = 450$ m when $t = 30$ s.

$(\overset{+}{\rightarrow})$ $$ds = vdt$$

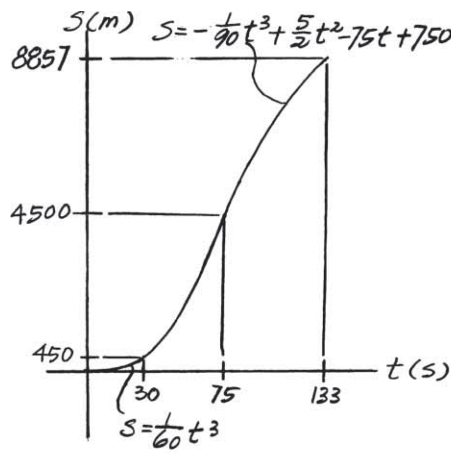
$$\int_{450\text{ m}}^s ds = \int_{30\text{ s}}^t \left(-\frac{1}{30}t^2 + 5t - 75\right)dt$$

$$s = \left(-\frac{1}{90}t^3 + \frac{5}{2}t^2 - 75t + 750\right) \text{ m}$$

When $t = 75$ s and $t' = 133.09$ s,

$$s\big|_{t=75\text{ s}} = -\frac{1}{90}\left(75^3\right) + \frac{5}{2}\left(75^2\right) - 75(75) + 750 = 4500 \text{ m}$$

$$s\big|_{t=133.09\text{ s}} = -\frac{1}{90}\left(133.09^3\right) + \frac{5}{2}\left(133.09^2\right) - 75(133.09) + 750 = 8857 \text{ m} \qquad \textbf{Ans.}$$



(b)

The s–t graph is shown in Fig. *b*.

When $t = 30$ s,

$v = 45$ m/s and $s = 450$ m.

When $t = 75$ s,

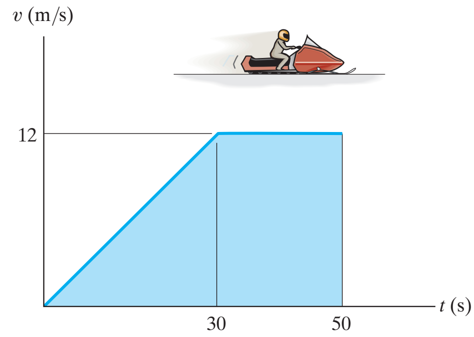
$v = v_{\max} = 112.5$ m/s and $s = 4500$ m.

When $t = 133$ s,

$v = 0$ and $s = 8857$ m.

***12–52.**

The snowmobile moves along a straight course according to the v–t graph. Construct the s–t and a–t graphs for the same 50-s time interval. When $t = 0$, $s = 0$.



SOLUTION

***s–t* Graph:** The position function in terms of time t can be obtained by applying $v = \frac{ds}{dt}$. For time interval $0\text{ s} \le t < 30\text{ s}$, $v = \frac{12}{30}t = \left(\frac{2}{5}t\right)$ m/s.

$$ds = vdt$$

$$\int_0^s ds = \int_0^t \frac{2}{5}tdt$$

$$s = \left(\frac{1}{5}t^2\right)\text{ m}$$

At $t = 30$ s, $$s = \frac{1}{5}\left(30^2\right) = 180\text{ m}$$

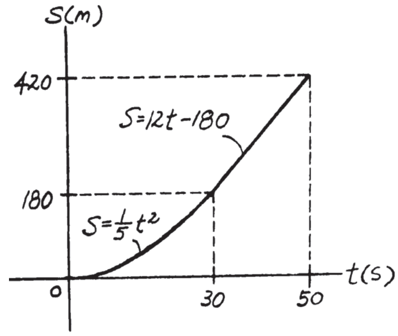
For time interval **$30\text{ s} < t \le 50\text{ s}$**,

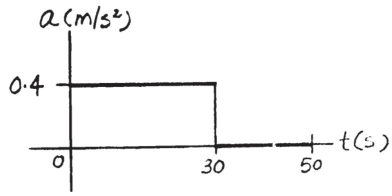
$$ds = vdt$$

$$\int_{180\text{ m}}^s ds = \int_{30\,s}^t 12dt$$

$$s = (12t - 180)\text{ m}$$

At $t = 50$ s, $$s = 12(50) - 180 = 420\text{ m}$$

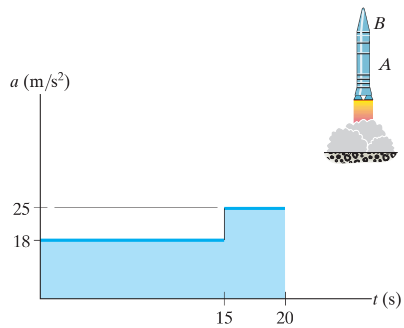




***a–t* Graph:** The acceleration function in terms of time t can be obtained by applying $a = \frac{dv}{dt}$. For time interval **$0\text{ s} \le t < 30\text{ s}$** and **$30\text{ s} < t \le 50\text{ s}$**, $a = \frac{dv}{dt} = \frac{2}{5} = 0.4\text{ m/s}^2$ and $a = \frac{dv}{dt} = 0$, respectively.

12–53.

A two-stage missile is fired vertically from rest with the acceleration shown. In 15 s the first stage A burns out and the second stage B ignites. Plot the $v-t$ and $s-t$ graphs which describe the two-stage motion of the missile for $0 \le t \le 20$ s.



SOLUTION

Since $v = \int a\,dt$, the constant lines of the a–t graph become sloping lines for the v–t graph.

The numerical values for each point are calculated from the total area under the a–t graph to the point.

At $t = 15$ s, $\quad v = (18)(15) = 270$ m/s

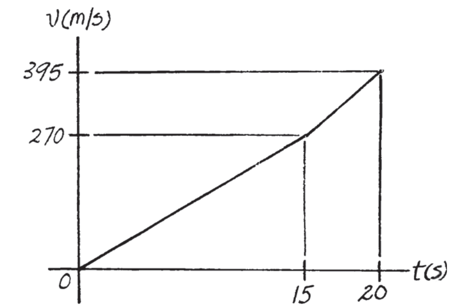
At $t = 20$ s, $\quad v = 270 + (25)(20 - 15) = 395$ m/s

Since $s = \int v\,dt$, the sloping lines of the v–t graph become parabolic curves for the s–t graph.

The numerical values for each point are calculated from the total area under the v–t graph to the point.

At $t = 15$ s, $\quad s = \frac{1}{2}(15)(270) = 2025$ m

At $t = 20$ s, $\quad s = 2025 + 270(20 - 15) + \frac{1}{2}(395 - 270)(20 - 15) = 3687.5 \text{ m} = 3.69$ km



Also:

$0 \le t \le 15$:

$$a = 18 \text{ m/s}^2$$

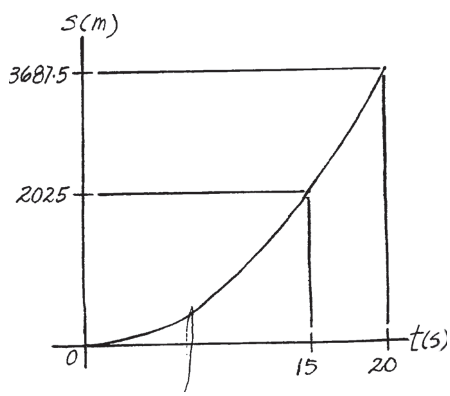
$$v = v_0 + a_c t = 0 + 18t$$

$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2 = 0 + 0 + 9t^2$$

When $t = 15$:

$$v = 18(15) = 270 \text{ m/s}$$

$$s = 9(15)^2 = 2025 \text{ m} = 2.025 \text{ km}$$



$15 \le t \le 20$:

$$a = 25 \text{ m/s}^2$$

$$v = v_0 + a_c t = 270 + 25(t - 15)$$

$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2 = 2025 + 270(t - 15) + \frac{1}{2}(25)(t - 15)^2$$

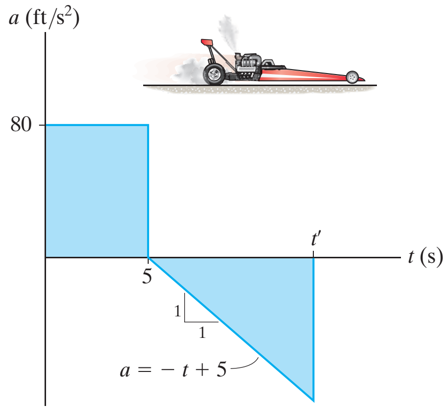
When $t = 20$:

$$v = 395 \text{ m/s}$$

$$s = 3687.5 \text{ m} = 3.69 \text{ km}$$

12–54.

The dragster starts from rest and has an acceleration described by the graph. Determine the time t' for it to stop. Also, what is its maximum speed? Construct the $v-t$ and $s-t$ graphs for the time interval $0 \leq t \leq t'$.



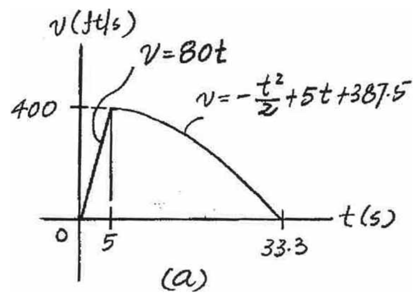
SOLUTION

v–t Graph: For the time interval $0 \leq t < 5$ s, the initial condition is $v = 0$ when $t = 0$ s.

$$(\overset{+}{\rightarrow}) \qquad dv = a\,dt$$

$$\int_0^v dv = \int_0^t 80\,dt$$

$$v = (80t) \text{ ft/s}$$



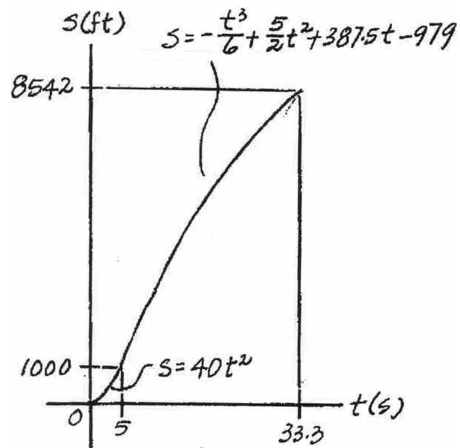
(a)

The maximum speed occurs at the instant when the acceleration changes sign when $t = 5$ s. Thus,

$$v_{\max} = v|_{t=5\,\text{s}} = 80(5) = 400 \text{ ft/s} \qquad \textbf{Ans.}$$

For the time interval $5 < t \leq t'$, the initial condition is $v = 400$ ft/s when $t = 5$ s.

$$(\overset{+}{\rightarrow}) \qquad dv = a\,dt$$



(b)

$$\int_{400\text{ ft/s}}^{v} dv = \int_{5\text{ s}}^{t} (-t + 5)\,dt$$

$$v = \left(-\frac{t^2}{2} + 5t + 387.5\right) \text{ft/s}$$

Thus when $v = 0$,

$$0 = -\frac{t'^2}{2} + 5t' + 387.5$$

Choosing the positive root,

$$t' = 33.28 \text{ s} = 33.3 \text{ s} \qquad \textbf{Ans.}$$

Also, the change in velocity is equal to the area under the $a-t$ graph. Thus

$$\Delta v = \int a\,dt$$

$$0 = 80(5) + \left\{\frac{1}{2}[(-t' + 5)(t' - 5)]\right\}$$

$$0 = -\frac{t'^2}{2} + 5t' + 387.5$$

This quadratic equation is the same as the one obtained previously. The $v-t$ graph is shown in Fig. a.

s–t Graph: For the time interval $0 \le t < 5\,\text{s}$, the initial condition is $s = 0$ when $t = 0\,\text{s}$.

$$(\overset{+}{\rightarrow}) \qquad ds = v\,dt$$

$$\int_0^s ds = \int_0^t 80\,dt$$

$$s = (40t^2)\text{ ft}$$

When $t = 5\,\text{s}$,

$$s|_{t=5\,\text{s}} = 40(5^2) = 1000\text{ ft}$$

For the time interval $5\,\text{s} < t \le t' = 45\,\text{s}$, the initial condition is $s = 1000\text{ ft}$ when $t = 5\,\text{s}$.

$$(\overset{+}{\rightarrow}) \qquad ds = v\,dt$$

$$\int_{1000\,\text{ft}}^s ds = \int_{5\,\text{s}}^t \left(-\frac{t^2}{2} + 5t + 387.5\right)dt$$

$$s = \left(-\frac{t^3}{6} + \frac{5}{2}t^2 + 387.5t - 979.17\right)\text{ft}$$

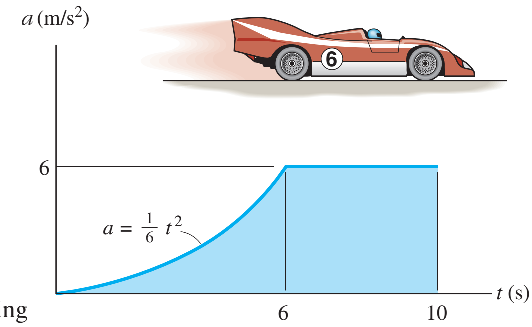
When $t = t' = 33.28\text{ s}$,

$$s|_{t=33.28\,\text{s}} = -\frac{33.28^3}{6} + \frac{5}{2}(33.28^2) + 387.5(33.28) - 979.17 = 8542\text{ ft}$$

The s–t graph is shown in Fig. b.

12–55.

A race car starting from rest travels along a straight road and for 10 s has the acceleration shown. Construct the v–t graph that describes the motion and find the distance traveled in 10 s.



SOLUTION

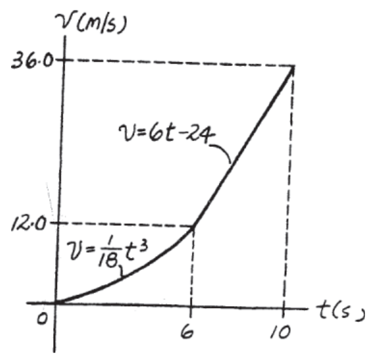
v–t Graph: The velocity function in terms of time t can be obtained by applying formula $a = \dfrac{dv}{dt}$. For time interval $0\text{ s} \le t < 6\text{ s}$,

$$dv = adt$$

$$\int_0^v dv = \int_0^t \frac{1}{6}t^2 dt$$

$$v = \left(\frac{1}{18}t^3\right)\text{ m/s}$$

At $t = 6$ s, $\qquad v = \dfrac{1}{18}\left(6^3\right) = 12.0\text{ m/s},$



For time interval $6\text{ s} < t \le 10\text{ s}$,

$$dv = adt$$

$$\int_{12.0\text{m/s}}^v dv = \int_{6s}^t 6dt$$

$$v = (6t - 24)\text{ m/s}$$

At $t = 10$ s, $\qquad v = 6(10) - 24 = 36.0\text{ m/s}$

Position: The position in terms of time t can be obtained by applying $v = \dfrac{ds}{dt}$. For time interval $0\text{ s} \le t < 6\text{ s}$,

$$ds = vdt$$

$$\int_0^s ds = \int_0^t \frac{1}{18}t^3\, dt$$

$$s = \left(\frac{1}{72}t^4\right)\text{m}$$

When $t = 6$ s, $v = 12.0$ m/s and $s = \dfrac{1}{72}\left(6^4\right) = 18.0$ m.

For time interval $6\text{ s} < t \le 10\text{ s}$,

$$ds = vdt$$

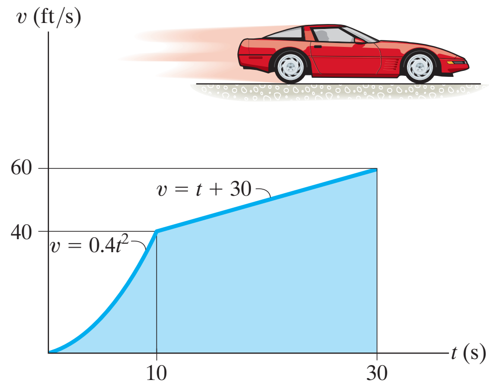
$$\int_{18.0\text{ m}}^s dv = \int_{6s}^t (6t - 24)dt$$

$$s = \left(3t^2 - 24t + 54\right)\text{m}$$

When $t = 10$ s, $v = 36.0$ m/s and $s = 3\left(10^2\right) - 24(10) + 54 = 114$ m **Ans.**

***12–56.**

The v–t graph for the motion of a car as it moves along a straight road is shown. Draw the a–t graph and determine the maximum acceleration during the 30-s time interval. The car starts from rest at $s = 0$.



SOLUTION

For $t < 10$ s:

$$v = 0.4t^2$$

$$a = \frac{dv}{dt} = 0.8t$$

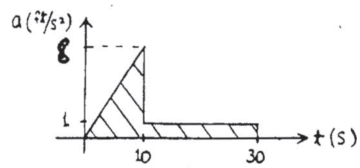
At $t = 10$ s:

$$a = 8 \text{ ft/s}^2$$

For $10 < t \leq 30$ s:

$$v = t + 30$$

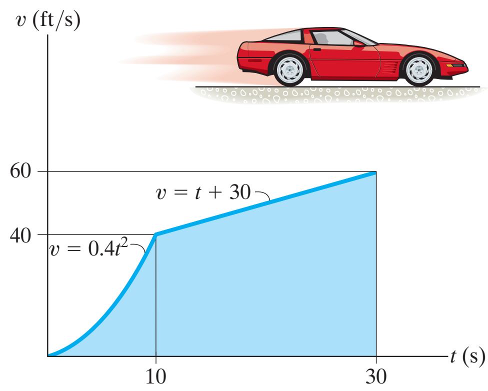
$$a = \frac{dv}{dt} = 1$$



$$a_{max} = 8 \text{ ft/s}^2$$ **Ans.**

12–57.

The v–t graph for the motion of a car as it moves along a straight road is shown. Draw the s–t graph and determine the average speed and the distance traveled for the 30-s time interval. The car starts from rest at $s = 0$.



SOLUTION

For $t < 10$ s,

$$v = 0.4t^2$$

$$ds = v\,dt$$

$$\int_0^s ds = \int_0^t 0.4t^2\,dt$$

$$s = 0.1333t^3$$

At $t = 10$ s,

$$s = 133.3 \text{ ft}$$

For $10 < t < 30$ s,

$$v = t + 30$$

$$ds = v\,dt$$

$$\int_{133.3}^s ds = \int_{10}^t (t + 30)\,dt$$

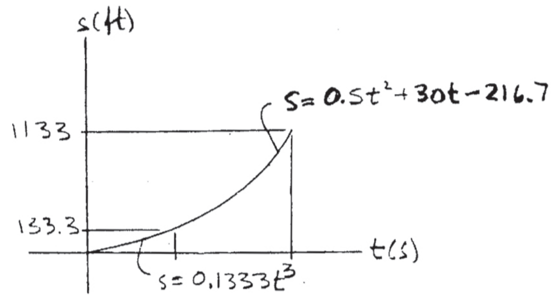
$$s = 0.5t^2 + 30t - 216.7$$

At $t = 30$ s,

$$s = 1133 \text{ ft}$$

$$(v_{sp})_{\text{Avg}} = \frac{\Delta s}{\Delta t} = \frac{1133}{30} = 37.8 \text{ ft/s}$$ **Ans.**

$$s_T = 1133 \text{ ft} = 1.13(10^3) \text{ ft}$$ **Ans.**

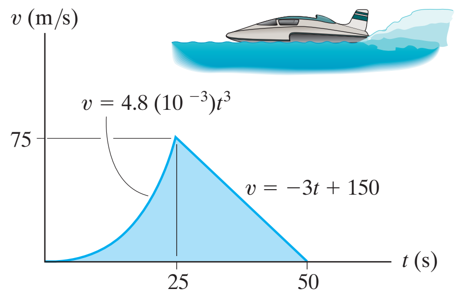


When $t = 0$ s, $s = 133$ ft.

When $t = 30$ s, $s = s_I = 1.33 \ (10^3)$ ft

12–58.

The jet-powered boat starts from rest at $s = 0$ and travels along a straight line with the speed described by the graph. Construct the $s-t$ and $a-t$ graph for the time interval $0 \le t \le 50$ s.



SOLUTION

s–t Graph: The initial condition is $s = 0$ when $t = 0$.

$$\left(\overset{+}{\rightarrow}\right) \qquad ds = v\,dt$$

$$\int_0^s ds = \int_0^t 4.8(10^{-3})t^3\,dt$$

$$s = \left[1.2(10^{-3})t^4\right]\text{m}$$

At $t = 25$ s,

$$s|_{t=25\text{ s}} = 1.2(10^{-3})(25^4) = 468.75\text{ m}$$

For the time interval $25\text{ s} < t \le 50\text{ s}$, the initial condition $s = 468.75$ m when $t = 25$ s will be used.

$$\left(\overset{+}{\rightarrow}\right) \qquad ds = v\,dt$$

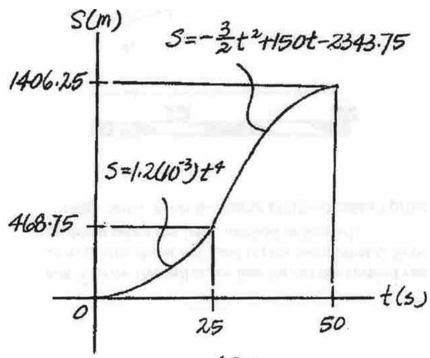
$$\int_{468.75\text{ m}}^s ds = \int_{25\text{ s}}^t (-3t + 150)\,dt$$

$$s = \left(-\frac{3}{2}t^2 + 150t - 2343.75\right)\text{m}$$

When $t = 50$ s,

$$s|_{t=50\text{ s}} = -\frac{3}{2}(50^2) + 150(50) - 2343.75 = 1406.25\text{ m}$$

The $s-t$ graph is shown in Fig. a.



(a)

a–t Graph: For the time interval $0 \le t < 25$ s,

$$a = \frac{dv}{dt} = \frac{d}{dt}\left[4.8(10^{-3})t^3\right] = (0.0144t^2)\text{ m/s}^2$$

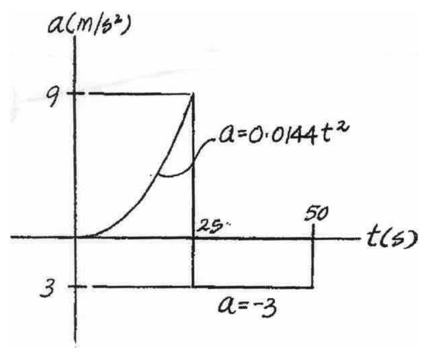
When $t = 25$ s,

$$a|_{t=25\text{ s}} = 0.0144(25^2)\text{ m/s}^2 = 9\text{ m/s}^2$$

For the time interval $25\text{ s} < t \le 50\text{ s}$,

$$a = \frac{dv}{dt} = \frac{d}{dt}(-3t + 150) = -3\text{ m/s}^2$$

The $a-t$ graph is shown in Fig. b.



(b)

When $t = 25$ s,

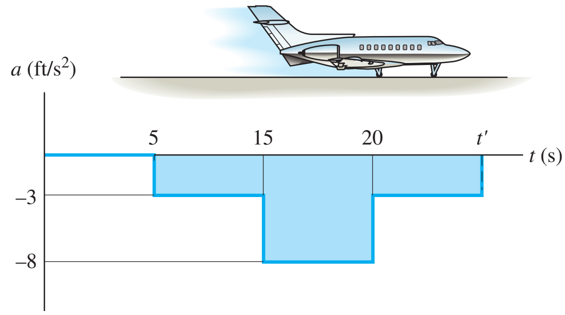
$$a = a_{max} = 9\text{ m/s}^2 \text{ and } s = 469\text{ m.}$$

When $t = 50$ s,

$$s = 1406\text{ m.}$$

12–59.

An airplane lands on the straight runway, originally traveling at 110 ft/s when $s = 0$. If it is subjected to the decelerations shown, determine the time t' needed to stop the plane and construct the s–t graph for the motion.



SOLUTION

$v_0 = 110\text{ ft/s}$

$\Delta v = \int a\,dt$

$0 - 110 = -3(15 - 5) - 8(20 - 15) - 3(t' - 20)$

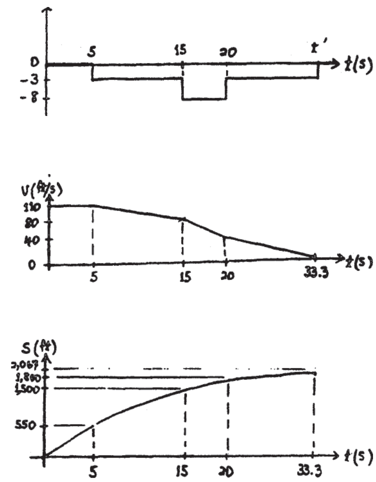
$t' = 33.3\text{ s}$ **Ans.**

$s\big|_{t = 5\text{s}} = 550\text{ ft}$

$s\big|_{t = 15\text{s}} = 1500\text{ ft}$

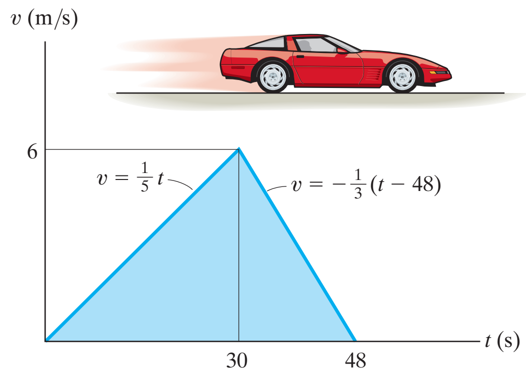
$s\big|_{t = 20\text{s}} = 1800\text{ ft}$

$s\big|_{t = 33.3\text{s}} = 2067\text{ ft}$



***12–60.**

A car travels along a straight road with the speed shown by the $v-t$ graph. Plot the $a-t$ graph.



SOLUTION

a–t Graph: For $0 \leq t < 30\,\text{s}$,

$$v = \frac{1}{5}t$$

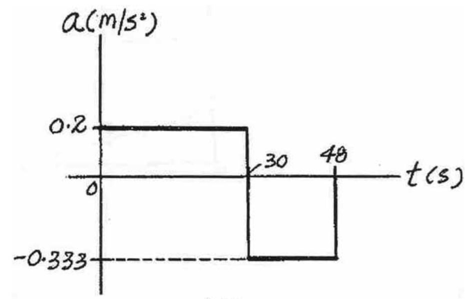
$$a = \frac{dv}{dt} = \frac{1}{5} = 0.2\ \text{m/s}^2$$

For $30\ \text{s} < t \leq 48\ \text{s}$

$$v = -\frac{1}{3}(t - 48)$$

$$a = \frac{dv}{dt} = -\frac{1}{3}(1) = -0.333\ \text{m/s}^2$$

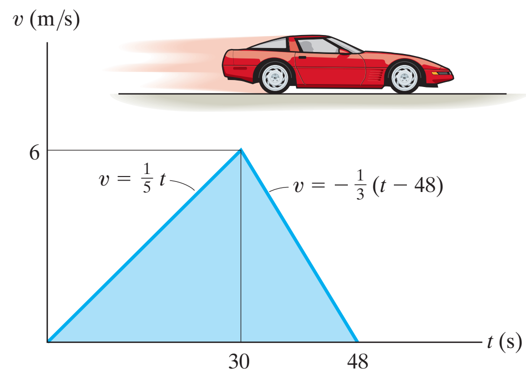
Using these results, $a-t$ graph shown in Fig. a can be plotted.



(a)

12–61.

A car travels along a straight road with the speed shown by the v–t graph. Determine the total distance the car travels until it stops when $t = 48$ s. Also plot the s–t and a–t graphs.



SOLUTION

For $0 \leq t \leq 30$ s,

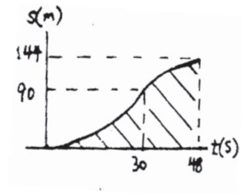
$$v = \frac{1}{5}t$$

$$a = \frac{dv}{dt} = \frac{1}{5}$$

$$ds = v\,dt$$

$$\int_0^s ds = \int_0^t \frac{1}{5}t\,dt$$

$$s = \frac{1}{10}t^2$$



When $t = 30$ s, $\quad s = 90$ m,

$$v = -\frac{1}{3}(t - 48)$$

$$a = \frac{dv}{dt} = -\frac{1}{3}$$

$$ds = v\,dt$$

$$\int_{90}^s ds = \int_{30}^t -\frac{1}{3}(t - 48)dt$$

$$s = -\frac{1}{6}t^2 + 16t - 240$$

When $t = 48$ s,

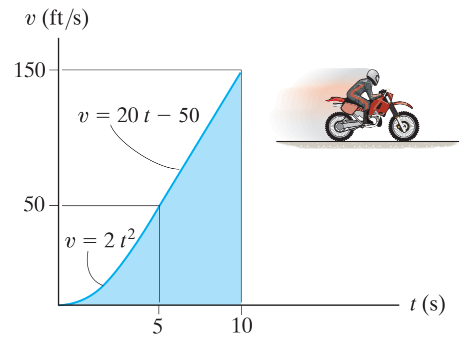
$$s = 144 \text{ m}$$ **Ans.**

Also, from the v–t graph

$$\Delta s = \int v\,dt \quad s - 0 = \frac{1}{2}(6)(48) = 144 \text{ m}$$ **Ans.**

12–62.

A motorcyclist travels along a straight road with the velocity described by the graph. Construct the $s-t$ and $a-t$ graphs.



SOLUTION

s–t Graph: For the time interval $0 \le t < 5\,\text{s}$, the initial condition is $s = 0$ when $t = 0$.

$$\left(\xrightarrow{+}\right) \qquad ds = vdt$$

$$\int_0^s ds = \int_0^t 2t^2 dt$$

$$s = \left(\frac{2}{3}t^3\right)\text{ft}$$

When $t = 5\,\text{s}$,

$$s = \frac{2}{3}(5^3) = 83.33\text{ ft} = 83.3\text{ ft and } a = 20\text{ ft/s}^2$$

For the time interval $5\,\text{s} < t \le 10\,\text{s}$, the initial condition is $s = 83.33$ ft when $t = 5$ s.

$$\left(\xrightarrow{+}\right) \qquad ds = vdt$$

$$\int_{83.33\,\text{ft}}^s ds = \int_{5\,\text{s}}^t (20t - 50)dt$$

$$s\Big|_{83.33\,\text{ft}}^{s} = (10t^2 - 50t)\Big|_{5\,\text{s}}^{t}$$

$$s = (10t^2 - 50t + 83.33)\text{ ft}$$

When $t = 10$ s,

$$s\big|_{t=10\,\text{s}} = 10(10^2) - 50(10) + 83.33 = 583\text{ ft}$$

The $s-t$ graph is shown in Fig. a.

a–t Graph: For the time interval $0 \le t < 5$ s,

$$\left(\xrightarrow{+}\right) \qquad a = \frac{dv}{dt} = \frac{d}{dt}(2t^2) = (4t)\,\text{ft/s}^2$$

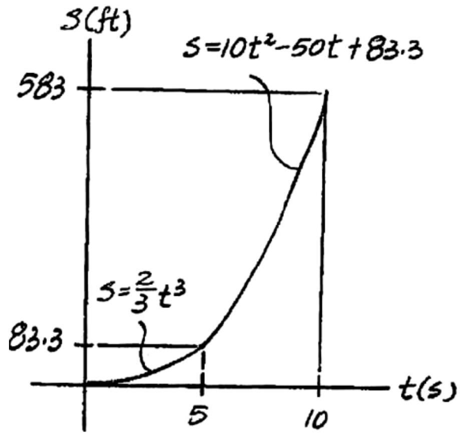
When $t = 5\,\text{s}$,

$$a = 4(5) = 20\text{ ft/s}^2$$

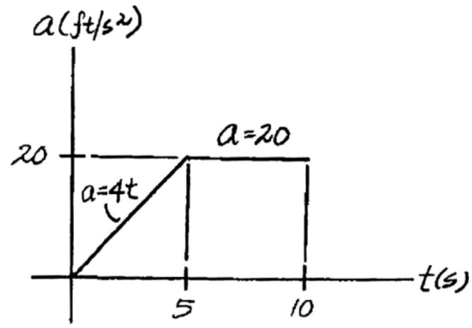
For the time interval $5\,\text{s} < t \le 10$ s,

$$\left(\xrightarrow{+}\right) \qquad a = \frac{dv}{dt} = \frac{d}{dt}(20t - 50) = 20\text{ ft/s}^2$$

The $a-t$ graph is shown in Fig. b.



(a)



(b)

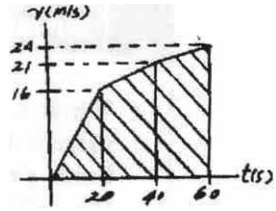
12–63.

The speed of a train during the first minute has been recorded as follows:

t (s)	0	20	40	60
v (m/s)	0	16	21	24

Plot the $v-t$ graph, approximating the curve as straight-line segments between the given points. Determine the total distance traveled.

SOLUTION



The total distance traveled is equal to the area under the graph.

$$s_T = \frac{1}{2}(20)(16) + \frac{1}{2}(40 - 20)(16 + 21) + \frac{1}{2}(60 - 40)(21 + 24) = 980 \text{ m} \qquad \textbf{Ans.}$$

***12–64.**

A man riding upward in a freight elevator accidentally drops a package off the elevator when it is 100 ft from the ground. If the elevator maintains a constant upward speed of 4 ft/s, determine how high the elevator is from the ground the instant the package hits the ground. Draw the v–t curve for the package during the time it is in motion. Assume that the package was released with the same upward speed as the elevator.

SOLUTION

For package:

$(+\uparrow) \qquad v^2 = v_0^2 + 2a_c(s_2 - s_0)$

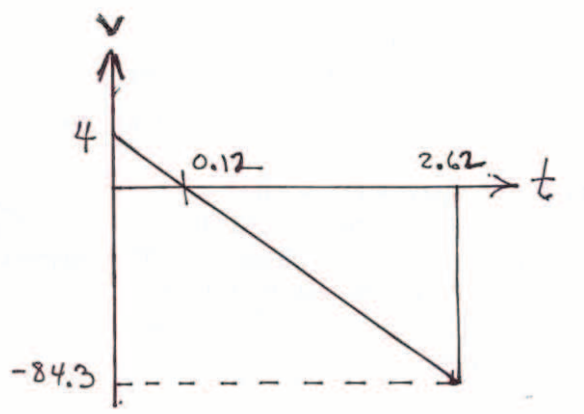
$$v^2 = (4)^2 + 2(-32.2)(0 - 100)$$

$$v = 80.35 \text{ ft/s} \downarrow$$

$(+\uparrow) \qquad v = v_0 + a_c t$

$$-80.35 = 4 + (-32.2)t$$

$$t = 2.620 \text{ s}$$



For elevator:

$(+\uparrow) \qquad s_2 = s_0 + vt$

$$s = 100 + 4(2.620)$$

$$s = 110 \text{ ft}$$ **Ans.**

12–65.

Two cars start from rest side by side and travel along a straight road. Car A accelerates at $4\ \text{m/s}^2$ for 10 s and then maintains a constant speed. Car B accelerates at $5\ \text{m/s}^2$ until reaching a constant speed of 25 m/s and then maintains this speed. Construct the a–t, v–t, and s–t graphs for each car until $t = 15$ s. What is the distance between the two cars when $t = 15$ s?

SOLUTION

Car A:

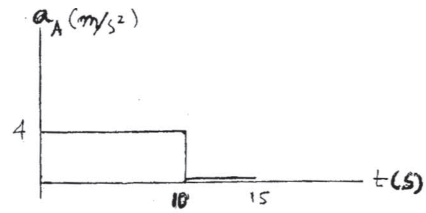
$v = v_0 + a_c t$

$v_A = 0 + 4t$

At $t = 10$ s, $\quad v_A = 40$ m/s

$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$s_A = 0 + 0 + \frac{1}{2}(4)t^2 = 2t^2$

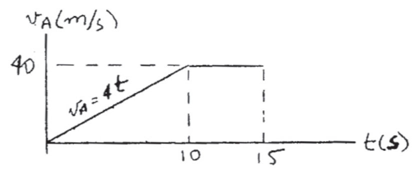


At $t = 10$ s, $\qquad s_A = 200$ m

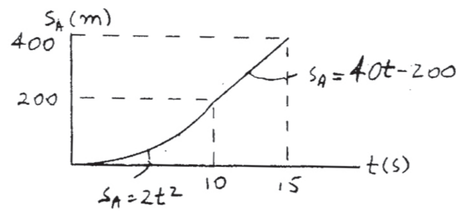
$t > 10$ s, $\qquad ds = v\,dt$

$$\int_{200}^{s_A} ds = \int_{10}^{t} 40\,dt$$

$$s_A = 40t - 200$$



At $t = 15$ s, $\qquad s_A = 400$ m



Car B:

$$v = v_0 + a_c t$$

$$v_B = 0 + 5t$$

When $v_B = 25$ m/s, $\qquad t = \frac{25}{5} = 5$ s

$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_B = 0 + 0 + \frac{1}{2}(5)t^2 = 2.5t^2$$

When $t = 10$ s, $v_A = (v_A)_{max} = 40$ m/s and $s_A = 200$ m.

When $t = 5$ s, $s_B = 62.5$ m.

When $t = 15$ s, $s_A = 400$ m and $s_B = 312.5$ m.

12–65. continued

At $t = 5$ s, $\quad s_B = 62.5$ m

$t > 5$ s, $\quad ds = v\,dt$

$$\int_{62.5}^{s_B} ds = \int_5^t 25\,dt$$

$$s_B - 62.5 = 25t - 125$$

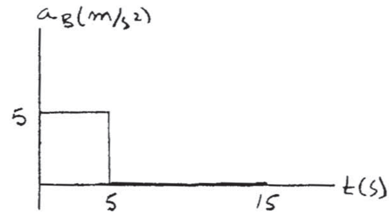
$$s_B = 25t - 62.5$$

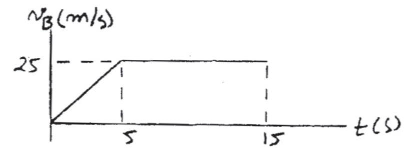
When $t = 15$ s, $\quad s_B = 312.5$

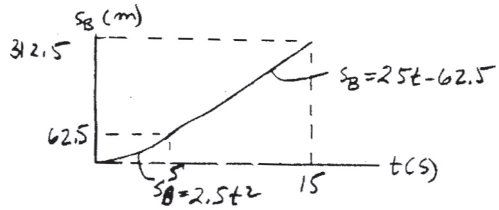
Distance between the cars is

$\Delta s = s_A - s_B = 400 - 312.5 = 87.5$ m **Ans.**

Car A is ahead of car B.

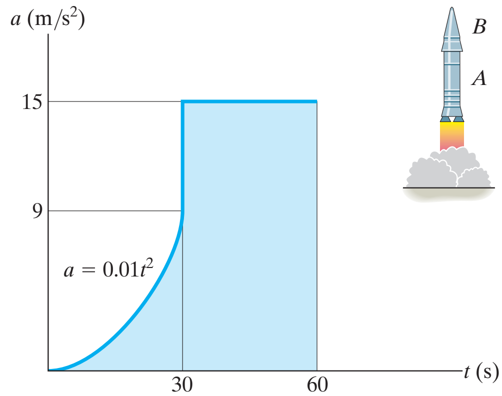






12–66.

A two-stage rocket is fired vertically from rest at $s = 0$ with an acceleration as shown. After 30 s the first stage A burns out and the second stage B ignites. Plot the v–t and s–t graphs which describe the motion of the second stage for $0 \leq t \leq 60$ s.



SOLUTION

For $0 \leq t \leq 30$ s

$$\int_0^v dv = \int_0^t 0.01\, t^2\, dt$$

$$v = 0.00333t^3$$

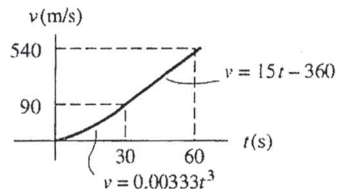
When $t = 30$ s, $\quad v = 90$ m/s

For $30 \text{ s} \leq t \leq 60$ s

$$\int_{90}^v dv = \int_{30}^t 15dt$$

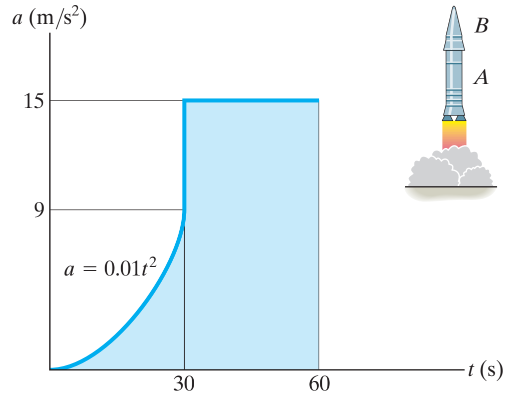
$$v = 15t - 360$$

When $t = 60$ s, $\quad v = 540$ m/s



12–67.

A two-stage rocket is fired vertically from rest at $s = 0$ with an acceleration as shown. After 30 s the first stage A burns out and the second stage B ignites. Plot the s–t graph which describes the motion of the second stage for $0 \le t \le 60$ s.



SOLUTION

***v*–*t* Graph:** When $t = 0$, $v = 0$. For $0 \le t \le 30$ s,

$(+\uparrow) \qquad dv = a\,dt$

$$\int_0^v dv = \int_0^t 0.01t^2 dt$$

$$v\Big|_0^v = \frac{0.01}{3}t^3\Big|_0^t$$

$$v = \{0.003333t^3\} \text{ m/s}$$

When $t = 30$ s, $v = 0.003333(30^3) = 90$ m/s

For 30 s $< t \le 60$ s,

$(+\uparrow) \qquad dv = a\,dt$

$$\int_{90 \text{ m/s}}^v dv = \int_{30 \text{ s}}^t 15\,dt$$

$$v\Big|_{90 \text{ m/s}}^v = 15t\Big|_{30 \text{ s}}^t$$

$$v - 90 = 15t - 450$$

$$v = \{15t - 360\} \text{ m/s}$$

When $t = 60$ s, $v = 15(60) - 360 = 540$ m/s

***s*–*t* Graph:** When $t = 0$, $s = 0$. For $0 \le t \le 30$ s,

$(+\uparrow) \qquad ds = vdt$

$$\int_0^s ds = \int_0^t 0.003333t^3 dt$$

$$s\Big|_0^s = 0.0008333t^4\Big|_0^t$$

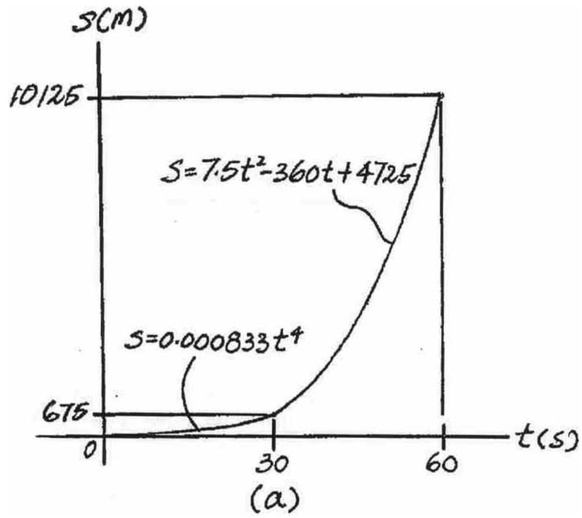
$$s = \{0.0008333\,t^4\} \text{ m}$$

When $t = 30$ s, $s = 0.0008333(30^4) = 675$ m

For 30 s $< t \le 60$ s,

$(+\uparrow) \qquad ds = vdt$

$$\int_{675 \text{ m}}^s ds = \int_{30 \text{ s}}^t (15t - 360)\,dt$$



12–67. continued

$$s\Big|_{675\text{ m}}^{s} = (7.5t^2 - 360t)\Big|_{30\text{ s}}^{t}$$

$$s - 675 = (7.5t^2 - 360t) - [7.5(30^2) - 360(30)]$$

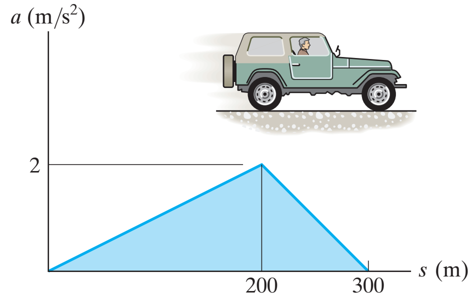
$$s = \{7.5t^2 - 360t + 4725\}\text{ m}$$

When $t = 60$ s, $s = 7.5(60^2) - 360(60) + 4725 = 10\,125$ m

Using these results, the s–t graph shown in Fig. *a* can be plotted.

***12–68.**

The a–s graph for a jeep traveling along a straight road is given for the first 300 m of its motion. Construct the v–s graph. At $s = 0$, $v = 0$.



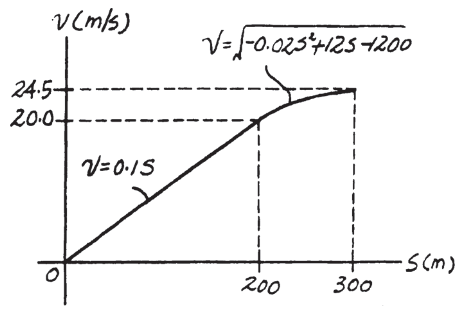
SOLUTION

***a*−*s* Graph**: The function of acceleration a in terms of s for the interval $0 \text{ m} \le s < 200 \text{ m}$ is

$$\frac{a - 0}{s - 0} = \frac{2 - 0}{200 - 0} \qquad a = (0.01s) \text{ m/s}^2$$

For the interval $200 \text{ m} < s \le 300 \text{ m}$,

$$\frac{a - 2}{s - 200} = \frac{0 - 2}{300 - 200} \qquad a = (-0.02s + 6) \text{ m/s}^2$$



***v*−*s* Graph**: The function of velocity v in terms of s can be obtained by applying $vdv = ads$. For the interval $\mathbf{0 \text{ m} \le s < 200 \text{ m}}$,

$$vdv = ads$$

$$\int_0^v vdv = \int_0^s 0.01s\,ds$$

$$v = (0.1s) \text{ m/s}$$

At $s = 200$ m, $\quad v = 0.100(200) = 20.0 \text{ m/s}$

For the interval $\mathbf{200 \text{ m} < s \le 300 \text{ m}}$,

$$vdv = ads$$

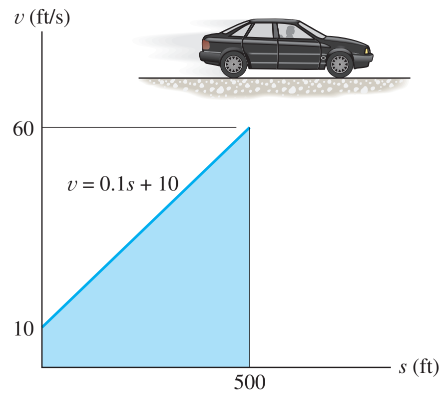
$$\int_{20.0 \text{ m/s}}^{v} vdv = \int_{200 \text{ m}}^{s} (-0.02s + 6)ds$$

$$v = \left(\sqrt{-0.02s^2 + 12s - 1200}\right) \text{ m/s}$$

At $s = 300$ m, $\quad v = \sqrt{-0.02(300^2) + 12(300) - 1200} = 24.5 \text{ m/s}$

12–69.

The v–s graph for the car is given for the first 500 ft of its motion. Construct the a–s graph for $0 \leq s \leq 500$ ft. How long does it take to travel the 500-ft distance? The car starts at $s = 0$ when $t = 0$.

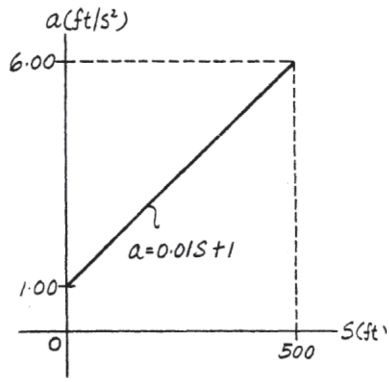


SOLUTION

a – s Graph: The acceleration a in terms of s can be obtained by applying $v dv = a ds$.

$$a = v\frac{dv}{ds} = (0.1s + 10)(0.1) = (0.01s + 1)\ \text{ft/s}^2$$

At $s = 0$ and $s = 500$ ft, $a = 0.01(0) + 1 = 1.00\ \text{ft/s}^2$ and $a = 0.01(500) + 1 = 6.00\ \text{ft/s}^2$, respectively.



Position: The position s in terms of time t can be obtained by applying $v = \dfrac{ds}{dt}$.

$$dt = \frac{ds}{v}$$

$$a\big|_{s=0} = 100\ \text{ft/s}^2$$

$$a\big|_{s=500\text{ft}} = 6.00\ \text{ft/s}^2$$

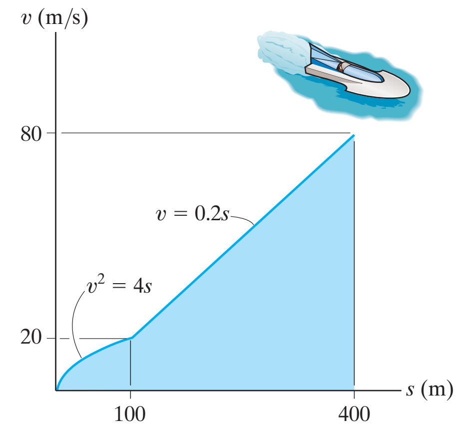
$$\int_0^t dt = \int_0^s \frac{ds}{0.1s + 10}$$

$$t = 10\ln(0.01s + 1)$$

When $s = 500$ ft, $\quad t = 10\ln[0.01(500) + 1] = 17.9$ s **Ans.**

12–70.

The boat travels along a straight line with the speed described by the graph. Construct the s–t and a–s graphs. Also, determine the time required for the boat to travel a distance $s = 400$ m if $s = 0$ when $t = 0$.



SOLUTION

***s–t* Graph:** For $0 \leq s < 100$ m, the initial condition is $s = 0$ when $t = 0$ s.

$$(\stackrel{+}{\rightarrow}) \qquad dt = \frac{ds}{v}$$

$$\int_0^t dt = \int_0^s \frac{ds}{2s^{1/2}}$$

$$t = s^{1/2}$$

$$s = \left(t^2\right) \text{ m}$$

When $s = 100$ m,

$$100 = t^2 \qquad t = 10 \text{ s}$$

For $100 \text{ m} < s \leq 400$ m, the initial condition is $s = 100$ m when $t = 10$ s.

$$(\stackrel{+}{\rightarrow}) \qquad dt = \frac{ds}{v}$$

$$\int_{10\text{ s}}^t dt = \int_{100\text{ m}}^s \frac{ds}{0.2s}$$

$$t - 10 = 5\ln\frac{s}{100}$$

$$\frac{t}{5} - 2 = \ln\frac{s}{100}$$

$$e^{t/5-2} = \frac{s}{100}$$

$$\frac{e^{t/5}}{e^2} = \frac{s}{100}$$

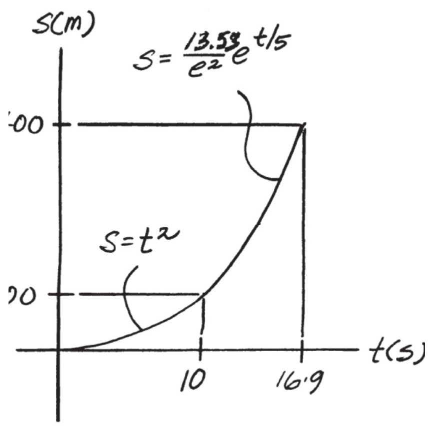
$$s = \left(13.53e^{t/5}\right) \text{ m}$$

When $s = 400$ m,

$$400 = 13.53e^{t/5}$$

$$t = 16.93 \text{ s} = 16.9 \text{ s} \qquad \textbf{Ans.}$$

The s–t graph is shown in Fig. a.



(a)

***a–s* Graph:** For $0 \text{ m} \leq s < 100$ m,

$$a = v\frac{dv}{ds} = \left(2s^{1/2}\right)\left(s^{-1/2}\right) = 2 \text{ m/s}^2$$

For $100 \text{ m} < s \leq 400$ m,

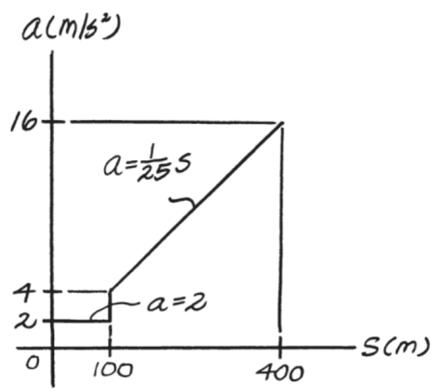
$$a = v\frac{dv}{ds} = (0.2s)(0.2) = 0.04s$$

When $s = 100$ m and 400 m,

$$a\big|_{s=100\text{ m}} = 0.04(100) = 4 \text{ m/s}^2$$

$$a\big|_{s=400\text{ m}} = 0.04(400) = 16 \text{ m/s}^2$$

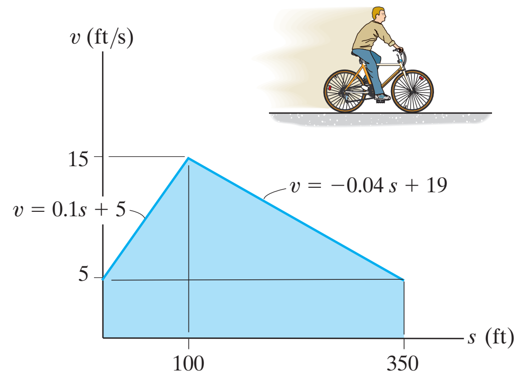
The a–s graph is shown in Fig. b.



(b)

12–71.

The $v-s$ graph of a cyclist traveling along a straight road is shown. Construct the $a-s$ graph.



SOLUTION

a–s Graph: For $0 \leq s < 100$ ft,

$$\left(\overset{+}{\rightarrow}\right) \qquad a = v\frac{dv}{ds} = \left(0.1s + 5\right)\left(0.1\right) = \left(0.01s + 0.5\right) \text{ ft/s}^2$$

Thus at $s = 0$ and 100 ft

$$a\big|_{s=0} = 0.01\left(0\right) + 0.5 = 0.5 \text{ ft/s}^2$$

$$a\big|_{s=100\text{ ft}} = 0.01\left(100\right) + 0.5 = 1.5 \text{ ft/s}^2$$

For 100 ft $< s \leq 350$ ft,

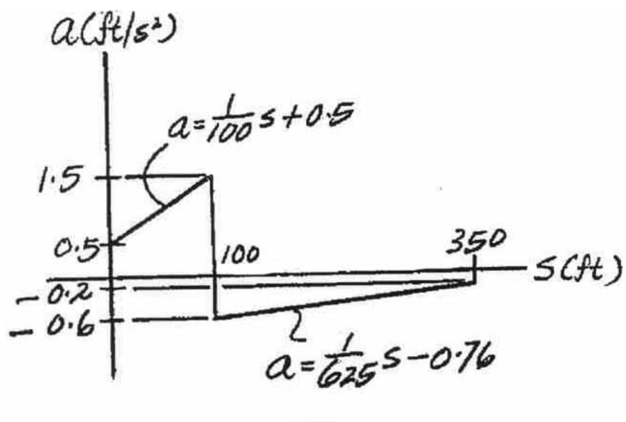
$$\left(\overset{+}{\rightarrow}\right) \qquad a = v\frac{dv}{ds} = \left(-0.04s + 19\right)\left(-0.04\right) = \left(0.0016s - 0.76\right) \text{ ft/s}^2$$

Thus at $s = 100$ ft and 350 ft

$$a\big|_{s=100\text{ ft}} = 0.0016\left(100\right) - 0.76 = -0.6 \text{ ft/s}^2$$

$$a\big|_{s=350\text{ ft}} = 0.0016\left(350\right) - 0.76 = -0.2 \text{ ft/s}^2$$

The $a-s$ graph is shown in Fig. a.



(a)

Thus at $s = 0$ and 100 ft

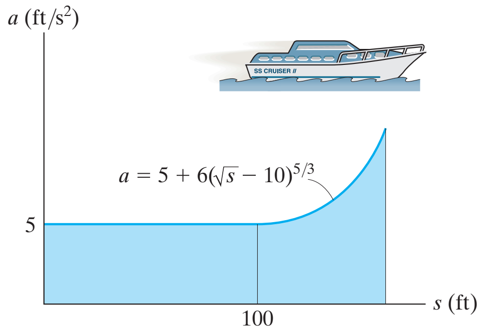
$$a\big|_{s=0} = 0.01\left(0\right) + 0.5 = 0.5 \text{ ft/s}^2$$

$$a\big|_{s=100\text{ ft}} = 0.01\left(100\right) + 0.5 = 1.5 \text{ ft/s}^2$$

At $s = 100$ ft, a changes from $a_{max} = 1.5 \text{ ft/s}^2$ to $a_{min} = -0.6 \text{ ft/s}^2$.

*■ **12–72.**

The a–s graph for a boat moving along a straight path is given. If the boat starts at $s = 0$ when $v = 0$, determine its speed when it is at $s = 75$ ft, and 125 ft, respectively. Use Simpson's rule with $n = 100$ to evaluate v at $s = 125$ ft.



SOLUTION

Velocity: The velocity v in terms of s can be obtained by applying $vdv = ads$. For the interval **0 ft ≤ *s* < 100 ft**,

$$vdv = ads$$

$$\int_0^v vdv = \int_0^s 5ds$$

$$v = \sqrt{10s} = \text{ft/s}$$

At $s = 75$ ft, $\quad v = \sqrt{10(75)} = 27.4$ ft/s **Ans.**

At $s = 100$ ft, $\quad v = \sqrt{10(100)} = 31.62$ ft/s **Ans.**

For the interval **100 ft < *s* ≤ 125 ft**,

$$vdv = ads$$

$$\int_{31.62\text{ ft/s}}^{v} vdv = \int_{100\text{ ft}}^{125\text{ ft}} [5 + 6(\sqrt{s} - 10)^{5/3}]ds$$

Evaluating the integral on the right using Simpson's rule, we have

$$\left.\frac{v^2}{2}\right|_{31.62\text{ ft/s}}^{v} = 201.032$$

At $s = 125$ ft, **Ans.**

$$v = 37.4 \text{ ft/s}$$

12–73.

The position of a particle is defined by $\mathbf{r} = \{5 \cos 2t\, \mathbf{i} + 4 \sin 2t\, \mathbf{j}\}$ m, where t is in seconds and the arguments for the sine and cosine are given in radians. Determine the magnitudes of the velocity and acceleration of the particle when $t = 1$ s. Also, prove that the path of the particle is elliptical.

SOLUTION

Velocity: The velocity expressed in Cartesian vector form can be obtained by applying Eq. 12–7.

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \{-10 \sin 2t\mathbf{i} + 8 \cos 2t\mathbf{j}\} \text{ m/s}$$

When $t = 1$ s, $v = -10 \sin 2(1)\mathbf{i} + 8 \cos 2(1)\mathbf{j} = \{-9.093\mathbf{i} - 3.329\mathbf{j}\}$ m/s. Thus, the magnitude of the velocity is

$$\mathbf{v} = \sqrt{v_x^2 + v_y^2} = \sqrt{(-9.093)^2 + (-3.329)^2} = 9.68 \text{ m/s} \qquad \textbf{Ans.}$$

Acceleration: The acceleration expressed in Cartesian vector from can be obtained by applying Eq. 12–9.

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \{-20 \cos 2t\mathbf{i} - 16 \sin 2t\mathbf{j}\} \text{ m/s}^2$$

When $t = 1$ s, $\mathbf{a} = -20 \cos 2(1)\mathbf{i} - 16 \sin 2(1)\mathbf{j} = \{8.323\mathbf{i} - 14.549\mathbf{j}\}$ m/s^2. Thus, the magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{8.323^2 + (-14.549)^2} = 16.8 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Traveling Path: Here, $x = 5 \cos 2t$ and $y = 4 \sin 2t$. Then,

$$\frac{x^2}{25} = \cos^2 2t \qquad \textbf{(1)}$$

$$\frac{y^2}{16} = \sin^2 2t \qquad \textbf{(2)}$$

Adding Eqs (1) and (2) yields

$$\frac{x^2}{25} + \frac{y^2}{16} = \cos^2 2t + \sin^2 2t$$

However, $\cos^2 2t + \sin^2 2t = 1$. Thus,

$$\frac{x^2}{25} + \frac{y^2}{16} = 1 \qquad \textbf{(Equation of an Ellipse) (Q.E.D.)}$$

12–74.

The velocity of a particle is $\mathbf{v} = \{3\mathbf{i} + (6 - 2t)\mathbf{j}\}$ m/s, where t is in seconds. If $\mathbf{r} = \mathbf{0}$ when $t = 0$, determine the displacement of the particle during the time interval $t = 1$ s to $t = 3$ s.

SOLUTION

Position: The position $\mathbf{r}$ of the particle can be determined by integrating the kinematic equation $d\mathbf{r} = \boldsymbol{v}dt$ using the initial condition $\mathbf{r} = \mathbf{0}$ at $t = 0$ as the integration limit. Thus,

$$d\mathbf{r} = \boldsymbol{v}dt$$

$$\int_0^{\mathbf{r}} d\mathbf{r} = \int_0^t \left[3\mathbf{i} + (6 - 2t)\mathbf{j}\right]dt$$

$$\mathbf{r} = \left[3t\mathbf{i} + \left(6t - t^2\right)\mathbf{j}\right]\text{m}$$

When $t = 1$ s and 3 s,

$$r\big|_{t=1\text{ s}} = 3(1)\mathbf{i} + \left[6(1) - 1^2\right]\mathbf{j} = [3\mathbf{i} + 5\mathbf{j}]\text{ m/s}$$

$$r\big|_{t=3\text{ s}} = 3(3)\mathbf{i} + \left[6(3) - 3^2\right]\mathbf{j} = [9\mathbf{i} + 9\mathbf{j}]\text{ m/s}$$

Thus, the displacement of the particle is

$$\Delta\mathbf{r} = \mathbf{r}\big|_{t=3\text{ s}} - \mathbf{r}\big|_{t=1\text{ s}}$$

$$= (9\mathbf{i} + 9\mathbf{j}) - (3\mathbf{i} + 5\mathbf{j})$$

$$= \{6\mathbf{i} + 4\mathbf{j}\}\text{ m}$$ **Ans.**

12–75.

A particle, originally at rest and located at point (3 ft, 2 ft, 5 ft), is subjected to an acceleration of $\mathbf{a} = \{6t\mathbf{i} + 12t^2\mathbf{k}\}$ ft/s^2. Determine the particle's position (x, y, z) at $t = 1$ s.

SOLUTION

Velocity: The velocity expressed in Cartesian vector form can be obtained by applying Eq. 12–9.

$$dv = \mathrm{a}dt$$

$$\int_0^v dv = \int_0^t (6t\mathbf{i} + 12t^2\mathbf{k})\,dt$$

$$v = \{3t^2\mathbf{i} + 4t^3\mathbf{k}\}\ \text{ft/s}$$

Position: The position expressed in Cartesian vector form can be obtained by applying Eq. 12–7.

$$dr = \mathrm{v}dt$$

$$\int_{\mathrm{r}_1}^{\mathrm{r}} dr = \int_0^t (3t^2\mathbf{i} + 4t^3\mathbf{k})\,dt$$

$$\mathrm{r} - (3\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}) = t^3\mathbf{i} + t^4\mathbf{k}$$

$$\mathrm{r} = \{(t^3 + 3)\,\mathbf{i} + 2\mathbf{j} + (t^4 + 5)\mathbf{k}\}\ \text{ft}$$

When $t = 1\ s$, $\mathrm{r} = (1^3 + 3)\mathbf{i} + 2\mathbf{j} + (1^4 + 5)\mathbf{k} = \{4\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}\}$ ft.

The coordinates of the particle are

(4 ft, 2 ft, 6 ft) **Ans.**

***12–76.**

The velocity of a particle is given by $\boldsymbol{v} = \{16t^2\mathbf{i} + 4t^3\mathbf{j} + (5t + 2)\mathbf{k}\}$ m/s, where t is in seconds. If the particle is at the origin when $t = 0$, determine the magnitude of the particle's acceleration when $t = 2$ s. Also, what is the x, y, z coordinate position of the particle at this instant?

SOLUTION

Acceleration: The acceleration expressed in Cartesian vector form can be obtained by applying Eq. 12–9.

$$\mathbf{a} = \frac{d\boldsymbol{v}}{dt} = \{32t\mathbf{i} + 12t^2\mathbf{j} + 5\mathbf{k}\}\ \text{m/s}^2$$

When $t = 2$ s, $\mathbf{a} = 32(2)\mathbf{i} + 12\left(2^2\right)\mathbf{j} + 5\mathbf{k} = \{64\mathbf{i} + 48\mathbf{j} + 5\mathbf{k}\}\ \text{m/s}^2$. The magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2} = \sqrt{64^2 + 48^2 + 5^2} = 80.2\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Position: The position expressed in Cartesian vector form can be obtained by applying Eq. 12–7.

$$d\mathbf{r} = \boldsymbol{v}\,dt$$

$$\int_0^{\mathbf{r}} d\mathbf{r} = \int_0^t \left(16t^2\mathbf{i} + 4t^3\mathbf{j} + (5t + 2)\mathbf{k}\right) dt$$

$$\mathbf{r} = \left[\frac{16}{3}t^3\mathbf{i} + t^4\mathbf{j} + \left(\frac{5}{2}t^2 + 2t\right)\mathbf{k}\right]\ \text{m}$$

When $t = 2$ s,

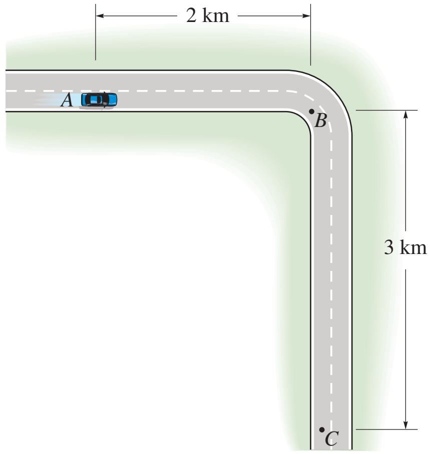
$$\mathbf{r} = \frac{16}{3}\left(2^3\right)\mathbf{i} + \left(2^4\right)\mathbf{j} + \left[\frac{5}{2}\left(2^2\right) + 2(2)\right]\mathbf{k} = \{42.7\mathbf{i} + 16.0\mathbf{j} + 14.0\mathbf{k}\}\ \text{m}.$$

Thus, the coordinate of the particle is

$$(42.7,\ 16.0,\ 14.0)\ \text{m} \qquad \textbf{Ans.}$$

12–77.

The car travels from A to B, and then from B to C, as shown in the figure. Determine the magnitude of the displacement of the car and the distance traveled.



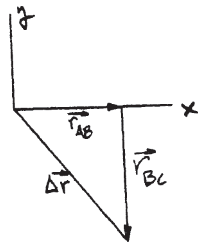
SOLUTION

Displacement: $\Delta\mathbf{r} = \{2\mathbf{i} - 3\mathbf{j}\}$ km

$\Delta r = \sqrt{2^2 + 3^2} = 3.61$ km **Ans.**

Distance Traveled:

$d = 2 + 3 = 5$ km **Ans.**



12–78.

A car travels east 2 km for 5 minutes, then north 3 km for 8 minutes, and then west 4 km for 10 minutes. Determine the total distance traveled and the magnitude of displacement of the car. Also, what is the magnitude of the average velocity and the average speed?

SOLUTION

Total Distance Traveled and Displacement: The total distance traveled is

$$s = 2 + 3 + 4 = 9 \text{ km} \qquad \textbf{Ans.}$$

and the magnitude of the displacement is

$$\Delta r = \sqrt{(2 - 4)^2 + 3^2} = 3.606 \text{ km} = 3.61 \text{ km} \qquad \textbf{Ans.}$$

Average Velocity and Speed: The total time is $\Delta t = 5 + 8 + 10 = 23 \text{ min} = 1380 \text{ s}$ The magnitude of average velocity is

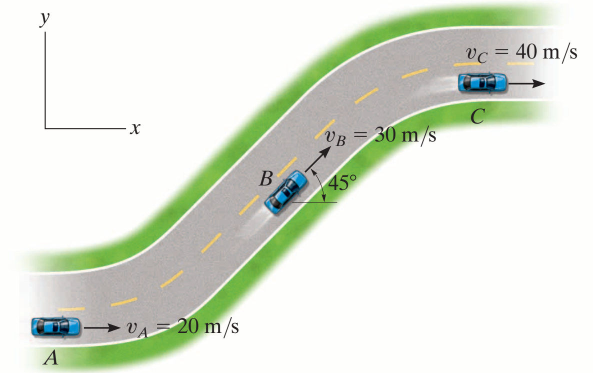
$$v_{\text{avg}} = \frac{\Delta r}{\Delta t} = \frac{3.606\left(10^3\right)}{1380} = 2.61 \text{ m/s} \qquad \textbf{Ans.}$$

and the average speed is

$$\left(v_{sp}\right)_{\text{avg}} = \frac{s}{\Delta t} = \frac{9\left(10^3\right)}{1380} = 6.52 \text{ m/s} \qquad \textbf{Ans.}$$

12–79.

A car traveling along the straight portions of the road has the velocities indicated in the figure when it arrives at points A, B, and C. If it takes 3 s to go from A to B, and then 5 s to go from B to C, determine the average acceleration between points A and B and between points A and C.



SOLUTION

$$\boldsymbol{v}_A = 20\,\mathbf{i}$$

$$\boldsymbol{v}_B = 21.21\,\mathrm{i} + 21.21\mathbf{j}$$

$$\boldsymbol{v}_C = 40\mathbf{i}$$

$$\mathbf{a}_{AB} = \frac{\Delta\,\boldsymbol{v}}{\Delta\,t} = \frac{21.21\,\mathbf{i} + 21.21\,\mathbf{j} - 20\,\mathbf{i}}{3}$$

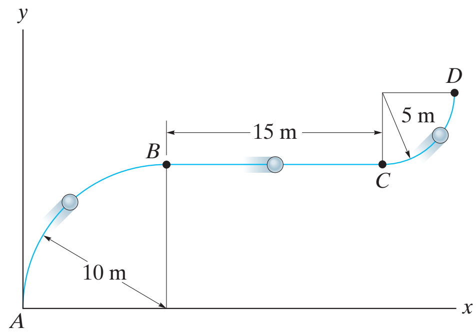
$$\mathbf{a}_{AB} = \{\,0.404\,\mathbf{i} + 7.07\,\mathbf{j}\,\}\ \mathrm{m/s^2}$$ **Ans.**

$$\mathbf{a}_{AC} = \frac{\Delta\,\boldsymbol{v}}{\Delta\,t} = \frac{40\,\mathbf{i} - 20\,\mathbf{i}}{8}$$

$$\mathbf{a}_{AC} = \{\,2.50\,\mathbf{i}\,\}\ \mathrm{m/s^2}$$ **Ans.**

***12–80.**

A particle travels along the curve from A to B in 2 s. It takes 4 s for it to go from B to C and then 3 s to go from C to D. Determine its average speed when it goes from A to D.



SOLUTION

$$s_T = \frac{1}{4}(2\pi)(10)) + 15 + \frac{1}{4}(2\pi(5)) = 38.56$$

$$v_{sP} = \frac{s_T}{t_t} = \frac{38.56}{2 + 4 + 3} = 4.28 \text{ m/s}$$ **Ans.**

12–81.

The position of a crate sliding down a ramp is given by $x = (0.25t^3)$ m, $y = (1.5t^2)$ m, $z = (6 - 0.75t^{5/2})$ m, where t is in seconds. Determine the magnitude of the crate's velocity and acceleration when $t = 2$ s.

SOLUTION

Velocity: By taking the time derivative of x, y, and z, we obtain the x, y, and z components of the crate's velocity.

$$v_x = \dot{x} = \frac{d}{dt}\left(0.25t^3\right) = \left(0.75t^2\right) \text{ m/s}$$

$$v_y = \dot{y} = \frac{d}{dt}\left(1.5t^2\right) = \left(3t\right) \text{ m/s}$$

$$v_z = \dot{z} = \frac{d}{dt}\left(6 - 0.75t^{5/2}\right) = \left(-1.875t^{3/2}\right) \text{ m/s}$$

When $t = 2$ s,

$$v_x = 0.75\left(2^2\right) = 3 \text{ m/s} \qquad v_y = 3(2) = 6 \text{ m/s} \qquad v_z = -1.875\left(2\right)^{3/2} = -5.303 \text{ m/s}$$

Thus, the magnitude of the crate's velocity is

$$v = \sqrt{v_x^{\,2} + v_y^{\,2} + v_z^{\,2}} = \sqrt{3^2 + 6^2 + (-5.303)^2} = 8.551 \text{ ft/s} = 8.55 \text{ ft} \qquad \textbf{Ans.}$$

Acceleration: The x, y, and z components of the crate's acceleration can be obtained by taking the time derivative of the results of v_x, v_y, and v_z, respectively.

$$a_x = \dot{v}_x = \frac{d}{dt}\left(0.75t^2\right) = (1.5t) \text{ m/s}^2$$

$$a_y = \dot{v}_y = \frac{d}{dt}(3t) = 3 \text{ m/s}^2$$

$$a_z = \dot{v}_z = \frac{d}{dt}\left(-1.875t^{3/2}\right) = \left(-2.815t^{1/2}\right) \text{ m/s}^2$$

When $t = 2$ s,

$$a_x = 1.5(2) = 3 \text{ m/s}^2 \qquad a_y = 3 \text{ m/s}^2 \qquad a_z = -2.8125\left(2^{1/2}\right) = -3.977 \text{ m/s}^2$$

Thus, the magnitude of the crate's acceleration is

$$a = \sqrt{a_x^{\,2} + a_y^{\,2} + a_z^{\,2}} = \sqrt{3^2 + 3^2 + (-3.977)^2} = 5.815 \text{ m/s}^2 = 5.82 \text{ m/s} \qquad \textbf{Ans.}$$

12–82.

A rocket is fired from rest at $x = 0$ and travels along a parabolic trajectory described by $y^2 = [120(10^3)x]$ m. If the x component of acceleration is $a_x = \left(\frac{1}{4}t^2\right)$ m/s^2, where t is in seconds, determine the magnitude of the rocket's velocity and acceleration when $t = 10$ s.

SOLUTION

Position: The parameter equation of x can be determined by integrating a_x twice with respect to t.

$$\int dv_x = \int a_x dt$$

$$\int_0^{v_x} dv_x = \int_0^t \frac{1}{4}t^2 dt$$

$$v_x = \left(\frac{1}{12}t^3\right) \text{ m/s}$$

$$\int dx = \int v_x dt$$

$$\int_0^x dx = \int_0^t \frac{1}{12}t^3 dt$$

$$x = \left(\frac{1}{48}t^4\right) \text{ m}$$

Substituting the result of x into the equation of the path,

$$y^2 = 120\left(10^3\right)\left(\frac{1}{48}t^4\right)$$

$$y = \left(50t^2\right) \text{ m}$$

Velocity:

$$v_y = \dot{y} = \frac{d}{dt}\left(50t^2\right) = \left(100t\right) \text{ m/s}$$

When $t = 10$ s,

$$v_x = \frac{1}{12}\left(10^3\right) = 83.33 \text{ m/s} \qquad v_y = 100(10) = 1000 \text{ m/s}$$

Thus, the magnitude of the rocket's velocity is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{83.33^2 + 1000^2} = 1003 \text{ m/s}$$ **Ans.**

Acceleration:

$$a_y = \dot{v}_y = \frac{d}{dt}(100t) = 100 \text{ m/s}^2$$

When $t = 10$ s,

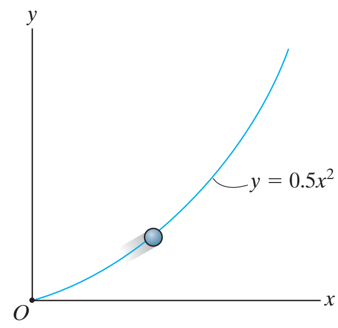
$$a_x = \frac{1}{4}\left(10^2\right) = 25 \text{ m/s}^2$$

Thus, the magnitude of the rocket's acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{25^2 + 100^2} = 103 \text{ m/s}^2$$ **Ans.**

12–83.

The particle travels along the path defined by the parabola $y = 0.5x^2$. If the component of velocity along the x axis is $v_x = (5t)$ ft/s, where t is in seconds, determine the particle's distance from the origin O and the magnitude of its acceleration when $t = 1$ s. When $t = 0$, $x = 0$, $y = 0$.



SOLUTION

Position: The x position of the particle can be obtained by applying the $v_x = \dfrac{dx}{dt}$.

$$dx = v_x dt$$

$$\int_0^x dx = \int_0^t 5t\,dt$$

$$x = \left(2.50t^2\right)\ \text{ft}$$

Thus, $y = 0.5\left(2.50t^2\right)^2 = \left(3.125t^4\right)$ ft. At $t = 1$ s, $x = 2.5\left(1^2\right) = 2.50$ ft and $y = 3.125\left(1^4\right) = 3.125$ ft. The particle's distance from the origin at this moment is

$$d = \sqrt{(2.50 - 0)^2 + (3.125 - 0)^2} = 4.00\ \text{ft} \qquad \textbf{Ans.}$$

Acceleration: Taking the first derivative of the path $y = 0.5x^2$, we have $\dot{y} = x\dot{x}$. The second derivative of the path gives

$$\ddot{y} = \dot{x}^2 + x\ddot{x} \qquad \textbf{(1)}$$

However, $\dot{x} = v_x$, $\ddot{x} = a_x$ and $\ddot{y} = a_y$. Thus, Eq. (1) becomes

$$a_y = v_x^2 + xa_x \qquad \textbf{(2)}$$

When $t = 1$ s, $v_x = 5(1) = 5$ ft/s $a_x = \dfrac{dv_x}{dt} = 5$ ft/s², and $x = 2.50$ ft. Then, from Eq. (2)

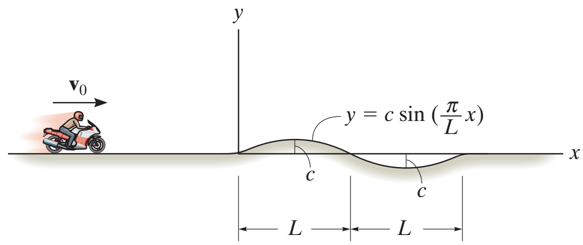
$$a_y = 5^2 + 2.50(5) = 37.5\ \text{ft/s}^2$$

Also,

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{5^2 + 37.5^2} = 37.8\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

***12–84.**

The motorcycle travels with constant speed v_0 along the path that, for a short distance, takes the form of a sine curve. Determine the x and y components of its velocity at any instant on the curve.



SOLUTION

$$y = c \sin\left(\frac{\pi}{L} x\right)$$

$$\dot{y} = \frac{\pi}{L} c\left(\cos \frac{\pi}{L} x\right)\dot{x}$$

$$v_y = \frac{\pi}{L} c\, v_x \left(\cos \frac{\pi}{L} x\right)$$

$$v_0^2 = v_y^2 + v_x^2$$

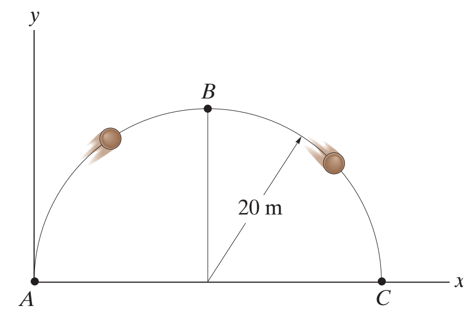
$$v_0^2 = v_x^2\left[1 + \left(\frac{\pi}{L} c\right)^2 \cos^2\left(\frac{\pi}{L} x\right)\right]$$

$$v_x = v_0\left[1 + \left(\frac{\pi}{L} c\right)^2 \cos^2\left(\frac{\pi}{L} x\right)\right]^{-\frac{1}{2}}$$ **Ans.**

$$v_y = \frac{v_0\, \pi c}{L}\left(\cos \frac{\pi}{L} x\right)\left[1 + \left(\frac{\pi}{L} c\right)^2 \cos^2\left(\frac{\pi}{L} x\right)\right]^{-\frac{1}{2}}$$ **Ans.**

12–85.

A particle travels along the curve from A to B in 1 s. If it takes 3 s for it to go from A to C, determine its *average velocity* when it goes from B to C.



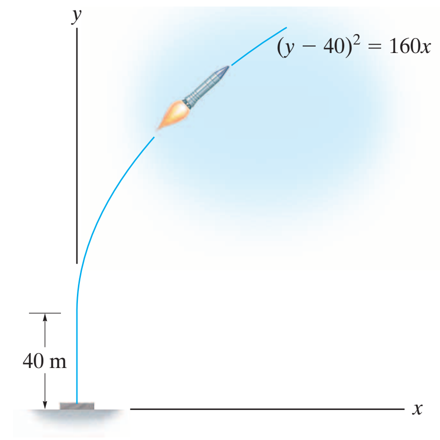
SOLUTION

Time from B to C is $3 - 1 = 2$ s

$$\mathbf{v}_{avg} = \frac{\Delta \mathrm{r}}{\Delta t} = \frac{(\mathbf{r}_{AC} - \mathbf{r}_{AB})}{\Delta t} = \frac{40\mathbf{i} - (20\mathbf{i} + 20\mathbf{j})}{2} = \{10\mathbf{i} - 10\mathbf{j}\}\ \mathrm{m/s} \qquad \textbf{Ans.}$$

12–86.

When a rocket reaches an altitude of 40 m it begins to travel along the parabolic path $(y - 40)^2 = 160x$, where the coordinates are measured in meters. If the component of velocity in the vertical direction is constant at $v_y = 180$ m/s, determine the magnitudes of the rocket's velocity and acceleration when it reaches an altitude of 80 m.



SOLUTION

$v_y = 180 \text{ m/s}$

$(y - 40)^2 = 160\,x$

$2(y - 40)v_y = 160v_x$ **(1)**

$2(80 - 40)(180) = 160v_x$

$v_x = 90 \text{ m/s}$

$v = \sqrt{90^2 + 180^2} = 201 \text{ m/s}$ **Ans.**

$$a_y = \frac{d\,v_y}{dt} = 0$$

From Eq. 1,

$2\,v_y^2 + 2(y - 40)a_y = 160\,a_x$

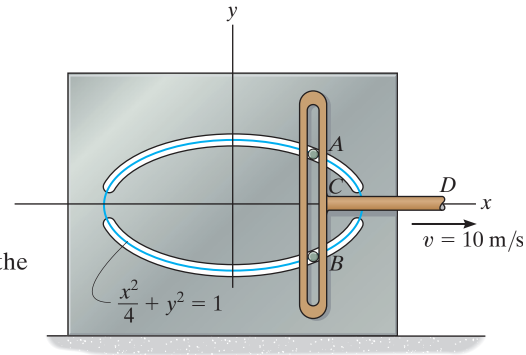
$2(180)^2 + 0 = 160\,a_x$

$a_x = 405 \text{ m/s}^2$

$a = 405 \text{ m/s}^2$ **Ans.**

12–87.

Pegs A and B are restricted to move in the elliptical slots due to the motion of the slotted link. If the link moves with a constant speed of 10 m/s, determine the magnitude of the velocity and acceleration of peg A when $x = 1$ m.



SOLUTION

Velocity: The x and y components of the peg's velocity can be related by taking the first time derivative of the path's equation.

$$\frac{x^2}{4} + y^2 = 1$$

$$\frac{1}{4}(2x\dot{x}) + 2y\dot{y} = 0$$

$$\frac{1}{2}x\dot{x} + 2y\dot{y} = 0$$

or

$$\frac{1}{2}xv_x + 2yv_y = 0 \qquad \textbf{(1)}$$

At $x = 1$ m,

$$\frac{(1)^2}{4} + y^2 = 1 \qquad y = \frac{\sqrt{3}}{2}\text{ m}$$

Here, $v_x = 10$ m/s and $x = 1$. Substituting these values into Eq. (1),

$$\frac{1}{2}(1)(10) + 2\left(\frac{\sqrt{3}}{2}\right)v_y = 0 \qquad v_y = -2.887\text{ m/s} = 2.887\text{ m/s}\downarrow$$

Thus, the magnitude of the peg's velocity is

$$v = \sqrt{v_x{}^2 + v_y{}^2} = \sqrt{10^2 + 2.887^2} = 10.4\text{ m/s} \qquad \textbf{Ans.}$$

Acceleration: The x and y components of the peg's acceleration can be related by taking the second time derivative of the path's equation.

$$\frac{1}{2}(\dot{x}\dot{x} + x\ddot{x}) + 2(\dot{y}\dot{y} + y\ddot{y}) = 0$$

$$\frac{1}{2}\left(\dot{x}^2 + x\ddot{x}\right) + 2\left(\dot{y}^2 + y\ddot{y}\right) = 0$$

or

$$\frac{1}{2}\left(v_x{}^2 + xa_x\right) + 2\left(v_y{}^2 + ya_y\right) = 0 \qquad \textbf{(2)}$$

Since v_x is constant, $a_x = 0$. When $x = 1$ m, $y = \frac{\sqrt{3}}{2}$ m, $v_x = 10$ m/s, and $v_y = -2.887$ m/s. Substituting these values into Eq. (2),

$$\frac{1}{2}\left(10^2 + 0\right) + 2\left[(-2.887)^2 + \frac{\sqrt{3}}{2}a_y\right] = 0$$

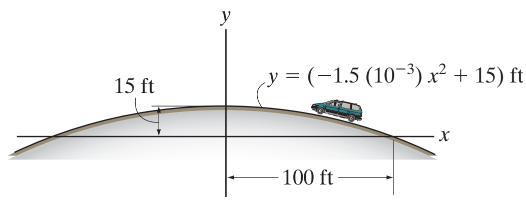
$$a_y = -38.49\text{ m/s}^2 = 38.49\text{ m/s}^2\downarrow$$

Thus, the magnitude of the peg's acceleration is

$$a = \sqrt{a_x{}^2 + a_y{}^2} = \sqrt{0^2 + (-38.49)^2} = 38.5\text{ m/s}^2 \qquad \textbf{Ans.}$$

***12–88.**

The van travels over the hill described by $y = (-1.5(10^{-3})\,x^2 + 15)$ ft. If it has a constant speed of 75 ft/s, determine the x and y components of the van's velocity and acceleration when $x = 50$ ft.



SOLUTION

Velocity: The x and y components of the van's velocity can be related by taking the first time derivative of the path's equation using the chain rule.

$$y = -1.5\left(10^{-3}\right)x^2 + 15$$

$$\dot{y} = -3\left(10^{-3}\right)x\dot{x}$$

or

$$v_y = -3\left(10^{-3}\right)xv_x$$

When $x = 50$ ft,

$$v_y = -3\left(10^{-3}\right)(50)v_x = -0.15v_x \qquad \textbf{(1)}$$

The magnitude of the van's velocity is

$$v = \sqrt{v_x{}^2 + v_y{}^2} \qquad \textbf{(2)}$$

Substituting $v = 75$ ft/s and Eq. (1) into Eq. (2),

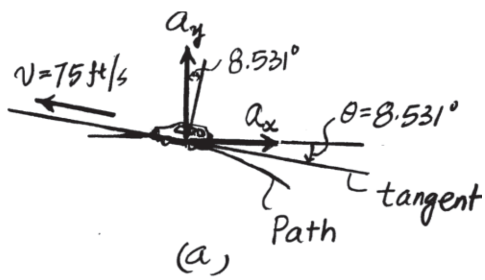
$$75 = \sqrt{v_x{}^2 + (-0.15v_x)^2}$$

$$v_x = 74.2 \text{ ft/s} \leftarrow \qquad \textbf{Ans.}$$

Substituting the result of v_x into Eq. (1), we obtain

$$v_y = -0.15(-74.17) = 11.12 \text{ ft/s} = 11.1 \text{ ft/s} \uparrow \qquad \textbf{Ans.}$$

Acceleration: The x and y components of the van's acceleration can be related by taking the second time derivative of the path's equation using the chain rule.



$$\ddot{y} = -3\left(10^{-3}\right)(\dot{x}\dot{x} + x\ddot{x})$$

or

$$a_y = -3\left(10^{-3}\right)\left(v_x{}^2 + xa_x\right)$$

When $x = 50$ ft, $v_x = -74.17$ ft/s. Thus,

$$a_y = -3\left(10^{-3}\right)\left[(-74.17)^2 + 50a_x\right]$$

$$a_y = -(16.504 + 0.15a_x) \qquad \textbf{(3)}$$

Since the van travels with a constant speed along the path, its acceleration along the tangent of the path is equal to zero. Here, the angle that the tangent makes with the horizontal at $x = 50$ ft is $\theta = \tan^{-1}\left(\dfrac{dy}{dx}\right)\Bigg|_{x=50\text{ ft}} = \tan^{-1}\left[-3\left(10^{-3}\right)x\right]\Bigg|_{x=50\text{ ft}} = \tan^{-1}(-0.15) = -8.531°.$ Thus, from the diagram shown in Fig. a,

$$a_x \cos 8.531° - a_y \sin 8.531° = 0 \qquad \textbf{(4)}$$

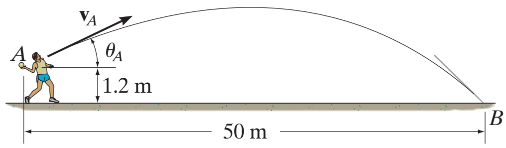
Solving Eqs. (3) and (4) yields

$$a_x = -2.42 \text{ ft/s} = 2.42 \text{ ft/s}^2 \leftarrow \qquad \textbf{Ans.}$$

$$a_y = -16.1 \text{ ft/s} = 16.1 \text{ ft/s}^2 \downarrow \qquad \textbf{Ans.}$$

12–89.

It is observed that the time for the ball to strike the ground at B is 2.5 s. Determine the speed v_A and angle θ_A at which the ball was thrown.



SOLUTION

Coordinate System: The x–y coordinate system will be set so that its origin coincides with point A.

x-Motion: Here, $(v_A)_x = v_A \cos\theta_A$, $x_A = 0$, $x_B = 50$ m, and $t = 2.5$ s. Thus,

$$(\overset{+}{\rightarrow}) \qquad x_B = x_A + (v_A)_x t$$

$$50 = 0 + v_A \cos\theta_A (2.5)$$

$$v_A \cos\theta_A = 20 \qquad \textbf{(1)}$$

y-Motion: Here, $(v_A)_y = v_A \sin\theta_A$, $y_A = 0$, $y_B = -1.2$ m, and $a_y = -g = -9.81 \text{ m/s}^2$. Thus,

$$(+\uparrow) \qquad y_B = y_A + (v_A)_y t + \frac{1}{2} a_y t^2$$

$$-1.2 = 0 + v_A \sin\theta_A (2.5) + \frac{1}{2}(-9.81)(2.5^2)$$

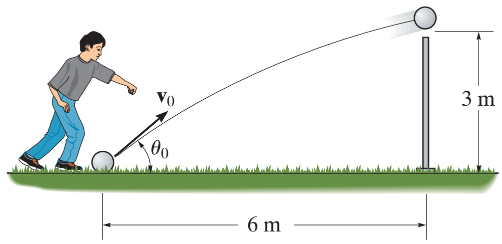
$$v_A \sin\theta_A = 11.7825 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$\theta_A = 30.5^\circ \qquad v_A = 23.2 \text{ m/s} \qquad \textbf{Ans.}$$

12–90.

Determine the minimum initial velocity v_0 and the corresponding angle θ_0 at which the ball must be kicked in order for it to just cross over the 3-m high fence.



SOLUTION

Coordinate System: The $x-y$ coordinate system will be set so that its origin coincides with the ball's initial position.

x-Motion: Here, $(v_0)_x = v_0 \cos\theta$, $x_0 = 0$, and $x = 6$ m. Thus,

$$\left(\overset{+}{\rightarrow}\right) \qquad x = x_0 + (v_0)x^t$$

$$6 = 0 + (v_0 \cos\theta)t$$

$$t = \frac{6}{v_0 \cos\theta} \qquad \textbf{(1)}$$

y-Motion: Here, $(v_0)_x = v_0 \sin\theta$, $a_y = -g = -9.81\,\text{m/s}^2$, and $y_0 = 0$. Thus,

$$(+\uparrow) \qquad y = y_0 + (v_0)_y t + \frac{1}{2}a_y t^2$$

$$3 = 0 + v_0(\sin\theta)t + \frac{1}{2}(-9.81)t^2$$

$$3 = v_0(\sin\theta)t - 4.905t^2 \qquad \textbf{(2)}$$

Substituting Eq. (1) into Eq. (2) yields

$$v_0 = \sqrt{\frac{58.86}{\sin 2\theta - \cos^2\theta}} \qquad \textbf{(3)}$$

From Eq. (3), we notice that v_0 is minimum when $f(\theta) = \sin 2\theta - \cos^2\theta$ is maximum. This requires $\dfrac{df(\theta)}{d\theta} = 0$

$$\frac{df(\theta)}{d\theta} = 2\cos 2\theta + \sin 2\theta = 0$$

$$\tan 2\theta = -2$$

$$2\theta = 116.57°$$

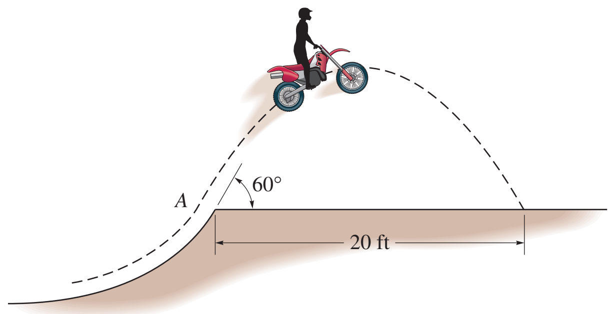
$$\theta = 58.28° = 58.3° \qquad \textbf{Ans.}$$

Substituting the result of θ into Eq. (2), we have

$$(v_0)_{min} = \sqrt{\frac{58.86}{\sin 116.57° - \cos^2 58.28°}} = 9.76\text{ m/s} \qquad \textbf{Ans.}$$

12–91.

During a race the dirt bike was observed to leap up off the small hill at A at an angle of 60° with the horizontal. If the point of landing is 20 ft away, determine the approximate speed at which the bike was traveling just before it left the ground. Neglect the size of the bike for the calculation.



SOLUTION

$(\xrightarrow{+})\ s = s_0 + v_0 t$

$$20 = 0 + v_A \cos 60°\, t$$

$(+\uparrow)\ s = s_0 + v_0 + \frac{1}{2} a_c t^2$

$$0 = 0 + v_A \sin 60°\, t + \frac{1}{2}(-32.2)\, t^2$$

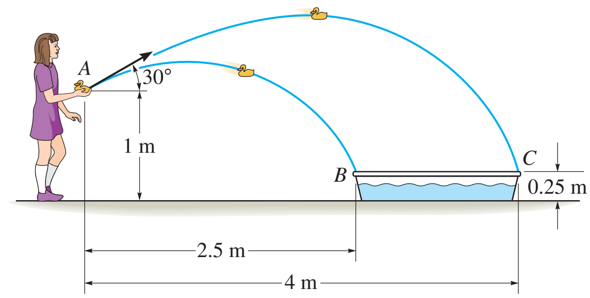
Solving

$t = 1.4668 \text{ s}$

$v_A = 27.3 \text{ ft/s}$ **Ans.**

***12–92.**

The girl always throws the toys at an angle of 30° from point A as shown. Determine the time between throws so that both toys strike the edges of the pool B and C at the same instant. With what speed must she throw each toy?



SOLUTION

To strike B:

$(\overset{+}{\rightarrow})\ s = s_0 + v_0 t$

$2.5 = 0 + v_A \cos 30^\circ\, t$

$(+\uparrow)\ s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$0.25 = 1 + v_A \text{ siv } 30^\circ\, t - \frac{1}{2}(9.81)t^2$

Solving

$t = 0.6687 \text{ s}$

$(v_A)_B = 4.32 \text{ m/s}$ **Ans.**

To strike C:

$(\overset{+}{\rightarrow})\ s = s_0 + v_0 t$

$4 = 0 + v_A \cos 30^\circ\, t$

$(+\uparrow)\ s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$0.25 = 1 + v_A \text{ siv } 30^\circ\, t - \frac{1}{2}(9.81)t^2$

Solving

$t = 0.790 \text{ s}$

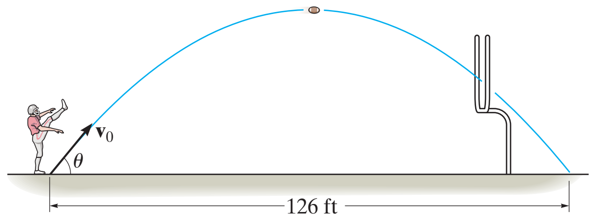
$(v_A)_C = 5.85 \text{ m/s}$ **Ans.**

Time between throws:

$\Delta t = 0.790 \text{ s} - 0.6687 \text{ s} = 0.121 \text{ s}$ **Ans.**

12–93.

The player kicks a football with an initial speed of $v_0 = 90$ ft/s. Determine the time the ball is in the air and the angle θ of the kick.



SOLUTION

Coordinate System: The $x-y$ coordinate system will be set with its origin coinciding with starting point of the football.

x-motion: Here, $x_0 = 0$, $x = 126$ ft, and $(v_0)_x = 90\cos\theta$

$$(\overset{+}{\rightarrow}) \qquad x = x_0 + (v_0)_x t$$

$$126 = 0 + (90\cos\theta)t$$

$$t = \frac{126}{90\cos\theta} \qquad \textbf{(1)}$$

y-motion: Here, $y_0 = y = 0$, $(v_0)_y = 90\sin\theta$, and $a_y = -g = -32.2$ ft. Thus,

$$(+\uparrow) \qquad y = y_0 + (v_0)_y t + \frac{1}{2}a_y t^2$$

$$O = 0 + (90\sin\theta)t + \frac{1}{2}(-32.2)t^2$$

$$O = (90\sin\theta)t - 16.1t^2 \qquad \textbf{(2)}$$

Substitute Eq. (1) into (2) yields

$$O = 90\sin\theta\left(\frac{126}{90\cos\theta}\right) - 16.1\left(\frac{126}{90\cos\theta}\right)^2$$

$$O = \frac{126\sin\theta}{\cos\theta} - \frac{31.556}{\cos^2\theta}$$

$$O = 126\sin\theta\cos\theta - 31.556 \qquad \textbf{(3)}$$

Using the trigonometry identity $\sin 2\theta = 2\sin\theta\cos\theta$, Eq. (3) becomes

$$63\sin 2\theta = 31.556$$

$$\sin 2\theta = 0.5009$$

$$2\theta = 30.06 \text{ or } 149.94$$

$$\theta = 15.03° = 15.0° \text{ or } \theta = 74.97° = 75.0° \qquad \textbf{Ans.}$$

If $\theta = 15.03°$,

$$t = \frac{126}{90\cos 15.03°} = 1.45\text{ s} \qquad \textbf{Ans.}$$

If $\theta = 74.97°$,

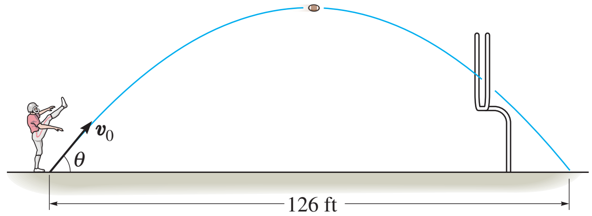
$$t = \frac{126}{90\cos 74.97°} = 5.40\text{ s} \qquad \textbf{Ans.}$$

Thus, $\theta = 15.0°$, $t = 1.45$ s

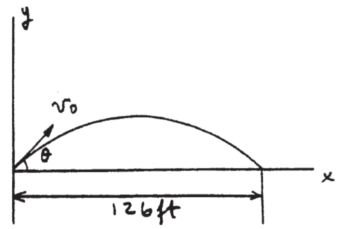
$\theta = 75.0°$, $t = 5.40$ s

12–94.

From a videotape, it was observed that a pro football player kicked a football 126 ft during a measured time of 3.6 seconds. Determine the initial speed of the ball and the angle θ at which it was kicked.



SOLUTION



$(\xrightarrow{+})$ $\quad s = s_0 + v_0 t$

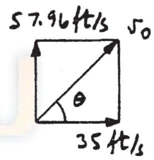
$$126 = 0 + (v_0)_x\,(3.6)$$

$$(v_0)_x = 35 \text{ ft/s}$$

$(+\uparrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$$O = 0 + (v_0)_y\,(3.6) + \frac{1}{2}(-32.2)(3.6)^2$$

$$(v_0)_y = 57.96 \text{ ft/s}$$

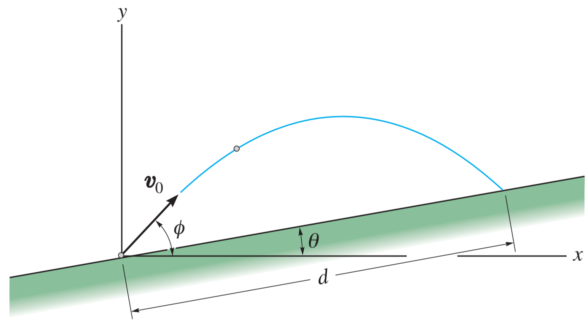


$v_0 = \sqrt{(35)^2 + (57.96)^2} = 67.7 \text{ ft/s}$ **Ans.**

$\theta = \tan^{-1}\left(\frac{57.96}{35}\right) = 58.9°$ **Ans.**

12–95.

A projectile is given a velocity $\mathbf{v}_0$ at an angle ϕ above the horizontal. Determine the distance d to where it strikes the sloped ground. The acceleration due to gravity is g.



SOLUTION

$(\overset{+}{\rightarrow})\qquad s = s_0 + v_0 t$

$$d\cos\theta = 0 + v_0(\cos\phi)t$$

$(+\uparrow)\qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$$d\sin\theta = 0 + v_0(\sin\phi)t + \frac{1}{2}(-g)t^2$$

Thus,

$$d\sin\theta = v_0\sin\phi\left(\frac{d\cos\theta}{v_0\cos\phi}\right) - \frac{1}{2}g\left(\frac{d\cos\theta}{v_0\cos\phi}\right)^2$$

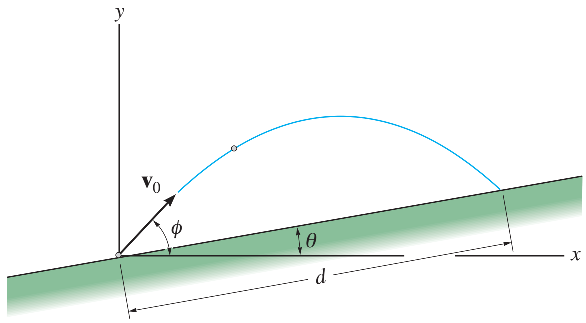
$$\sin\theta = \cos\theta\tan\phi - \frac{gd\cos^2\theta}{2v_0^2\cos^2\phi}$$

$$d = (\cos\theta\tan\phi - \sin\theta)\frac{2v_0^2\cos^2\phi}{g\cos^2\theta}$$

$$d = \frac{v_0^2}{g\cos\theta}\left(\sin 2\phi - 2\tan\theta\cos^2\phi\right)$$ **Ans.**

***12–96.**

A projectile is given a velocity $\mathbf{v}_0$. Determine the angle ϕ at which it should be launched so that d is a maximum. The acceleration due to gravity is g.



SOLUTION

$(\overset{+}{\rightarrow})\qquad s_x = s_0 + v_0 t$

$$d\cos\theta = 0 + v_0(\cos\phi)t$$

$(+\uparrow)\qquad s_y = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$$d\sin\theta = 0 + v_0(\sin\phi)t + \frac{1}{2}(-g)t^2$$

Thus,

$$d\sin\theta = v_0\sin\phi\left(\frac{d\cos\theta}{v_0\cos\phi}\right) - \frac{1}{2}g\left(\frac{d\cos\theta}{v_0\cos\phi}\right)^2$$

$$\sin\theta = \cos\theta\tan\phi - \frac{gd\cos^2\theta}{2v_0^2\cos^2\phi}$$

$$d = (\cos\theta\tan\phi - \sin\theta)\frac{2v_0^2\cos^2\phi}{g\cos^2\theta}$$

$$d = \frac{v_0^2}{g\cos\theta}\left(\sin 2\phi - 2\tan\theta\cos^2\phi\right)$$

Require:

$$\frac{d(d)}{d\phi} = \frac{v_0^2}{g\cos\theta}\left[\cos 2\phi(2) - 2\tan\theta(2\cos\phi)(-\sin\phi)\right] = 0$$

$$\cos 2\phi + \tan\theta\sin 2\phi = 0$$

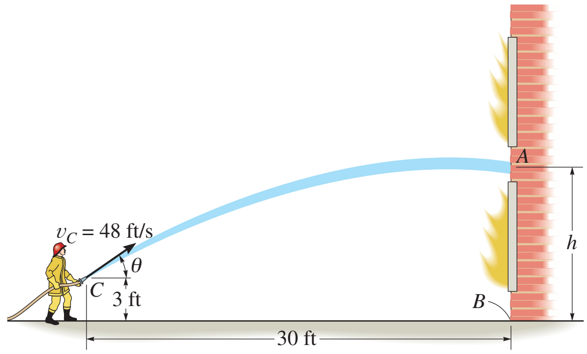
$$\frac{\sin 2\phi}{\cos 2\phi}\tan\theta + 1 = 0$$

$$\tan 2\phi = -\text{ctn}\,\theta$$

$$\phi = \frac{1}{2}\tan^{-1}(-\text{ctn}\,\theta)$$ **Ans.**

12–97.

Determine the maximum height on the wall to which the firefighter can project water from the hose, if the speed of the water at the nozzle is $v_C = 48$ ft/s.



SOLUTION

$(+\uparrow)\; v = v_0 + a_c t$

$$0 = 48 \sin\theta - 32.2\, t$$

$(\xrightarrow{+})\; s = s_0 + v_0 t$

$$30 = 0 + 48\,(\cos\theta)(t)$$

$$48 \sin\theta = 32.2 \frac{30}{48 \cos\theta}$$

$$\sin\theta \cos\theta = 0.41927$$

$$\sin 2\theta = 0.83854$$

$$\theta = 28.5°$$

$$t = 0.7111 \text{ s}$$

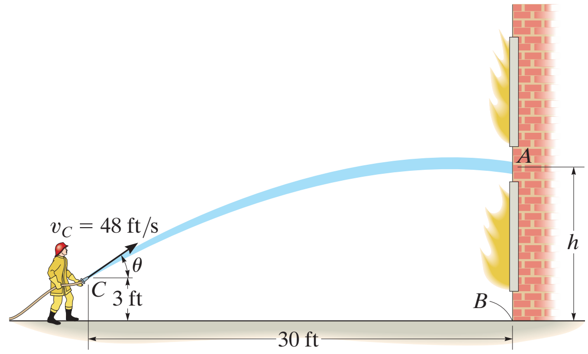
$(+\uparrow)\; s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$h - 3 = 0 + 48 \sin 28.5° \,(0.7111) + \frac{1}{2}(-32.2)(0.7111)^2$$

$$h = 11.1 \text{ ft}$$ **Ans.**

■ **12–98.**

Determine the smallest angle θ, measured above the horizontal, that the hose should be directed so that the water stream strikes the bottom of the wall at B. The speed of the water at the nozzle is $v_C = 48$ ft/s.



SOLUTION

$(\overset{+}{\rightarrow})\qquad s = s_0 + v_0 t$

$$30 = 0 + 48 \cos\theta\, t$$

$$t = \frac{30}{48 \cos\theta}$$

$(+\uparrow)\qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$0 = 3 + 48 \sin\theta\, t + \frac{1}{2}(-32.2)t^2$$

$$0 = 3 + \frac{48 \sin\theta\,(30)}{48 \cos\theta} - 16.1\left(\frac{30}{48 \cos\theta}\right)^2$$

$$0 = 3 \cos^2\theta + 30 \sin\theta \cos\theta - 6.2891$$

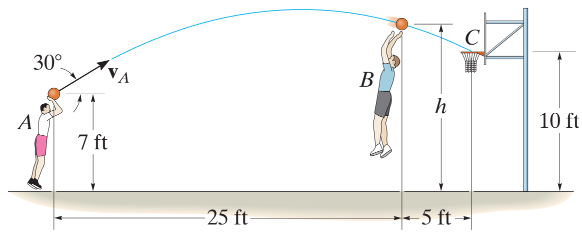
$$3 \cos^2\theta + 15 \sin 2\theta = 6.2891$$

Solving

$\theta = 6.41°$ or $77.9°$ **Ans.**

12–99.

Measurements of a shot recorded on a videotape during a basketball game are shown. The ball passed through the hoop even though it barely cleared the hands of the player B who attempted to block it. Neglecting the size of the ball, determine the magnitude v_A of its initial velocity and the height h of the ball when it passes over player B.



SOLUTION

$(\overset{+}{\rightarrow})$ $$s = s_0 + v_0 t$$

$$30 = 0 + v_A \cos 30°\, t_{AC}$$

$(+\uparrow)$ $$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$10 = 7 + v_A \sin 30°\, t_{AC} - \frac{1}{2}(32.2)(t_{AC}^2)$$

Solving

$v_A = 36.73 = 36.7$ ft/s **Ans.**

$t_{AC} = 0.943$ s

$(\overset{+}{\rightarrow})$ $$s = s_0 + v_0 t$$

$$25 = 0 + 36.73 \cos 30°\, t_{AB}$$

$(+\uparrow)$ $$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$h = 7 + 36.73 \sin 30°\, t_{AB} - \frac{1}{2}(32.2)(t_{AB}^2)$$

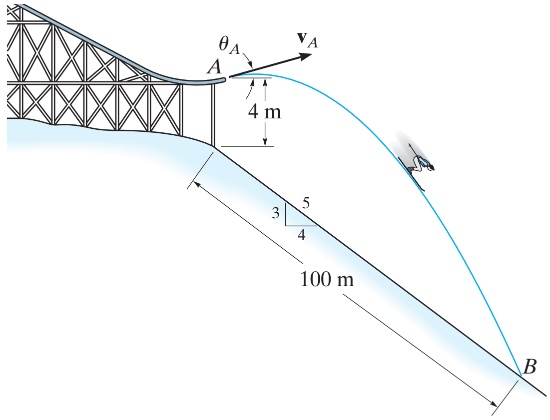
Solving

$t_{AB} = 0.786$ s

$h = 11.5$ ft **Ans.**

***12–100.**

It is observed that the skier leaves the ramp A at an angle $\theta_A = 25°$ with the horizontal. If he strikes the ground at B, determine his initial speed v_A and the time of flight t_{AB}.



SOLUTION

$(\overset{+}{\rightarrow})$ $\quad s = v_0 t$

$$100\left(\frac{4}{5}\right) = v_A \cos 25° t_{AB}$$

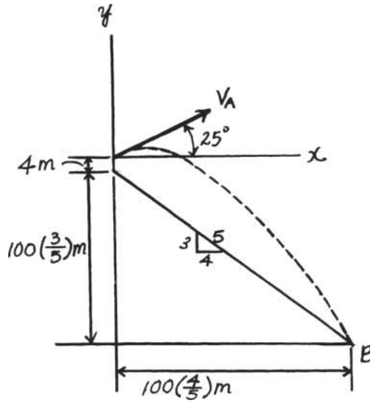
$(+\uparrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$-4 - 100\left(\frac{3}{5}\right) = 0 + v_A \sin 25° t_{AB} + \frac{1}{2}(-9.81)t_{AB}^2$$

Solving,

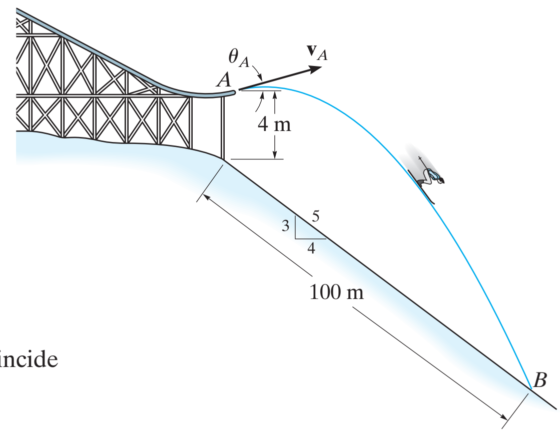
$$v_A = 19.4 \text{ m/s} \qquad \textbf{Ans.}$$

$$t_{AB} = 4.54 \text{ s} \qquad \textbf{Ans.}$$



12–101.

It is observed that the skier leaves the ramp A at an angle $\theta_A = 25°$ with the horizontal. If he strikes the ground at B, determine his initial speed v_A and the speed at which he strikes the ground.



SOLUTION

Coordinate System: $x-y$ coordinate system will be set with its origin to coincide with point A as shown in Fig. a.

x-motion: Here, $x_A = 0$, $x_B = 100\left(\frac{4}{5}\right) = 80$ m and $(v_A)_x = v_A \cos 25°$.

$$\left(\overset{+}{\rightarrow}\right) \qquad x_B = x_A + (v_A)_x t$$

$$80 = 0 + (v_A \cos 25°)t$$

$$t = \frac{80}{v_A \cos 25°} \qquad \textbf{(1)}$$

y-motion: Here, $y_A = 0$, $y_B = -\left[4 + 100\left(\frac{3}{5}\right)\right] = -64$ m and $(v_A)_y = v_A \sin 25°$ and $a_y = -g = -9.81\ \text{m/s}^2$.

$$(+\uparrow) \qquad y_B = y_A + (v_A)_y t + \frac{1}{2} a_y t^2$$

$$-64 = 0 + v_A \sin 25°\, t + \frac{1}{2}(-9.81)t^2$$

$$4.905t^2 - v_A \sin 25°\, t = 64 \qquad \textbf{(2)}$$

Substitute Eq. (1) into (2) yieldS

$$4.905\left(\frac{80}{v_A \cos 25°}\right)^2 = v_A \sin 25°\left(\frac{80}{v_A \cos 25°}\right) = 64$$

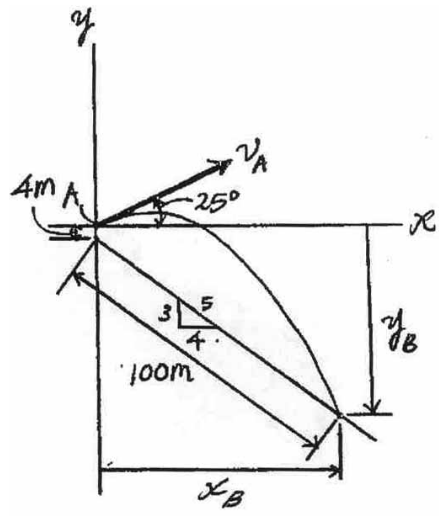
$$\left(\frac{80}{v_A \cos 25°}\right)^2 = 20.65$$

$$\frac{80}{v_A \cos 25°} = 4.545$$

$$v_A = 19.42\ \text{m/s} = 19.4\ \text{m/s} \qquad \textbf{Ans.}$$

Substitute this result into Eq. (1),

$$t = \frac{80}{19.42 \cos 25°} = 4.54465$$



(a)

12–101. continued

Using this result,

$(+\uparrow)$ $$(v_B)_y = (v_A)_y + a_y t$$

$$= 19.42 \sin 25° + (-9.81)(4.5446)$$

$$= -36.37 \text{ m/s} = 36.37 \text{ m/s} \downarrow$$

And

$(\overset{+}{\rightarrow})$ $$(v_B)_x = (v_A)_x = v_A \cos 25° = 19.42 \cos 25° = 17.60 \text{ m/s} \rightarrow$$

Thus,

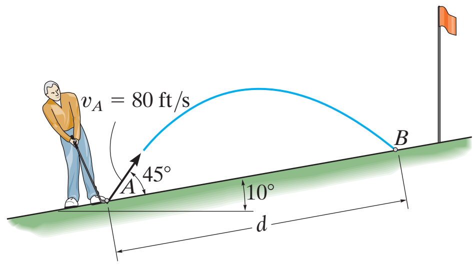
$$v_B = \sqrt{(v_B)_x^2 + (v_B)_y^2}$$

$$= \sqrt{36.37^2 + 17.60^2}$$

$$= 40.4 \text{ m/s}$$ **Ans.**

12–102.

A golf ball is struck with a velocity of 80 ft/s as shown. Determine the distance d to where it will land.



SOLUTION

Horizontal Motion: The horizontal component of velocity is $(v_0)_x = 80\cos 55^\circ = 45.89$ ft/s. The initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = d\cos 10^\circ$, respectively.

$$(\xrightarrow{+})\qquad s_x = (s_0)_x + (v_0)_x\, t$$

$$d\cos 10^\circ = 0 + 45.89t \qquad \textbf{(1)}$$

Vertical Motion: The vertical component of initial velocity is $(v_0)_y = 80\sin 55^\circ = 65.53$ ft/s. The initial and final vertical positions are $(s_0)_y = 0$ and $s_y = d\sin 10^\circ$, respectively.

$$(+\uparrow)\qquad s_y = (s_0)_y + (v_0)_y\, t + \frac{1}{2}(a_c)_y\, t^2$$

$$d\sin 10^\circ = 0 + 65.53t + \frac{1}{2}(-32.2)t^2 \qquad \textbf{(2)}$$

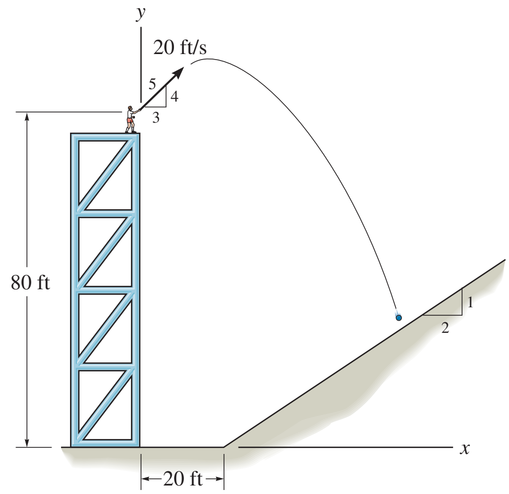
Solving Eqs. (1) and (2) yields

$$d = 166 \text{ ft} \qquad \textbf{Ans.}$$

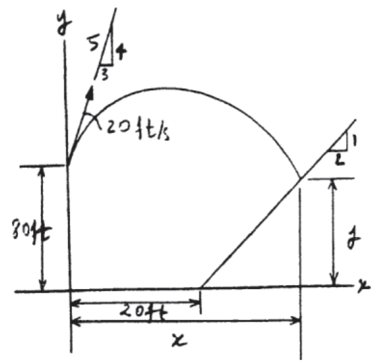
$$t = 3.568 \text{ s}$$

12–103.

The ball is thrown from the tower with a velocity of 20 ft/s as shown. Determine the x and y coordinates to where the ball strikes the slope. Also, determine the speed at which the ball hits the ground.



SOLUTION



Assume ball hits slope.

$(\xrightarrow{+})$ $\quad s = s_0 + v_0 t$

$$x = 0 + \frac{3}{5}(20)t = 12t$$

$(+\uparrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$$y = 80 + \frac{4}{5}(20)t + \frac{1}{2}(-32.2)t^2 = 80 + 16t - 16.1t^2$$

Equation of slope: $y - y_1 = m(x - x_1)$

$$y - 0 = \frac{1}{2}(x - 20)$$

$$y = 0.5x - 10$$

Thus,

$$80 + 16t - 16.1t^2 = 0.5(12t) - 10$$

$$16.1t^2 - 10t - 90 = 0$$

Choosing the positive root:

$t = 2.6952$ s

$x = 12(2.6952) = 32.3$ ft **Ans.**

Since 32.3 ft > 20 ft, assumption is valid.

$y = 80 + 16(2.6952) - 16.1(2.6952)^2 = 6.17$ ft **Ans.**

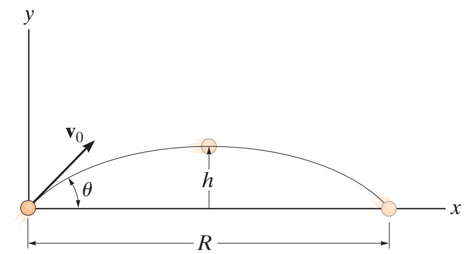
$(\xrightarrow{+})$ $\quad v_x = (v_0)_x = \frac{3}{5}(20) = 12$ ft/s

$(+\uparrow)$ $\quad v_y = (v_0)_y + a_c t = \frac{4}{5}(20) + (-32.2)(2.6952) = -70.785$ ft/s

$v = \sqrt{(12)^2 + (-70.785)^2} = 71.8$ ft/s **Ans.**

***12–104.**

The projectile is launched with a velocity $\mathbf{v}_0$. Determine the range R, the maximum height h attained, and the time of flight. Express the results in terms of the angle θ and v_0. The acceleration due to gravity is g.



SOLUTION

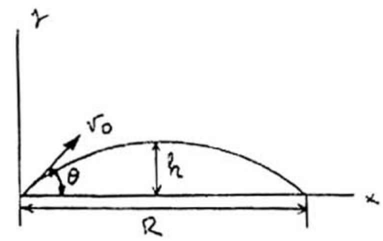
$$(\xrightarrow{+}) \qquad s = s_0 + v_0 t$$

$$R = 0 + (v_0 \cos\theta)t$$

$$(+\uparrow) \qquad s = s_0 + v_0\,t + \frac{1}{2}a_c t^2$$

$$0 = 0 + (v_0 \sin\theta)\,t + \frac{1}{2}(-g)t^2$$

$$0 = v_0 \sin\theta - \frac{1}{2}(g)\left(\frac{R}{v_0\cos\theta}\right)$$



$$R = \frac{v_0^2}{g}\sin 2\theta \qquad \textbf{Ans.}$$

$$t = \frac{R}{v_0\cos\theta} = \frac{v_0^2\,(2\sin\theta\cos\theta)}{v_0\,g\cos\theta}$$

$$= \frac{2v_0}{g}\sin\theta \qquad \textbf{Ans.}$$

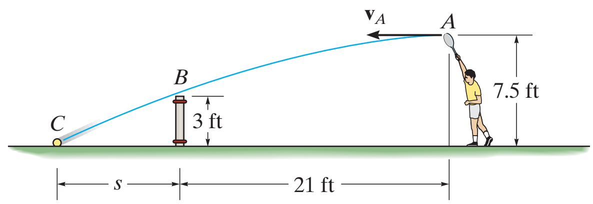
$$(+\uparrow) \qquad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$0 = (v_0\sin\theta)^2 + 2(-g)(h - 0)$$

$$h = \frac{v_0^2}{2g}\sin^2\theta \qquad \textbf{Ans.}$$

12–105.

Determine the horizontal velocity v_A of a tennis ball at A so that it just clears the net at B. Also, find the distance s where the ball strikes the ground.



SOLUTION

Vertical Motion: The vertical component of initial velocity is $(v_0)_y = 0$. For the ball to travel from A to B, the initial and final vertical positions are $(s_0)_y = 7.5$ ft and $s_y = 3$ ft, respectively.

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$3 = 7.5 + 0 + \frac{1}{2}(-32.2)t_1^2$$

$$t_1 = 0.5287 \text{ s}$$

For the ball to travel from A to C, the initial and final vertical positions are $(s_0)_y = 7.5$ ft and $s_y = 0$, respectively.

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$0 = 7.5 + 0 + \frac{1}{2}(-32.2)t_2^2$$

$$t_2 = 0.6825 \text{ s}$$

Horizontal Motion: The horizontal component of velocity is $(v_0)_x = v_A$. For the ball to travel from A to B, the initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = 21$ ft, respectively. The time is $t = t_1 = 0.5287$ s.

$$(\overset{+}{\leftarrow}) \qquad s_x = (s_0)_x + (v_0)_x t$$

$$21 = 0 + v_A(0.5287)$$

$$v_A = 39.72 \text{ ft/s} = 39.7 \text{ ft/s} \qquad \textbf{Ans.}$$

For the ball to travel from A to C, the initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = (21 + s)$ ft, respectively. The time is $t = t_2 = 0.6825$ s.

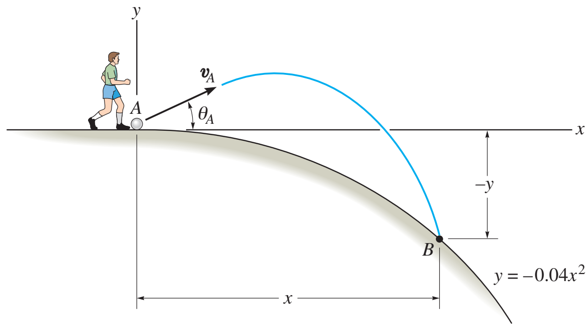
$$(\overset{+}{\leftarrow}) \qquad s_x = (s_0)_x + (v_0)_x t$$

$$21 + s = 0 + 39.72(0.6825)$$

$$s = 6.11 \text{ ft} \qquad \textbf{Ans.}$$

12–106.

The ball at A is kicked with a speed $v_A = 8$ ft/s and at an angle $\theta_A = 30°$. Determine the point $(x, -y)$ where it strikes the ground. Assume the ground has the shape of a parabola as shown.



SOLUTION

$$(v_A)_x = 8 \cos 30° = 6.928 \text{ ft/s}$$

$$(v_A)_y = 8 \sin 30° = 4 \text{ ft/s}$$

$$(\xrightarrow{+})\ s = s_0 + v_0 t$$

$$x = 0 + 6.928\, t \qquad \textbf{(1)}$$

$$(+\uparrow)\ s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$y = 0 + 4t + \frac{1}{2}(-32.2)t^2 \qquad \textbf{(2)}$$

$$y = -0.04\, x^2$$

From Eqs. (1) and (2):

$$y = 0.5774\, x - 0.3354\, x^2$$

$$-0.04\, x^2 = 0.5774x - 0.3354\, x^2$$

$$0.2954\, x^2 = 0.5774x$$

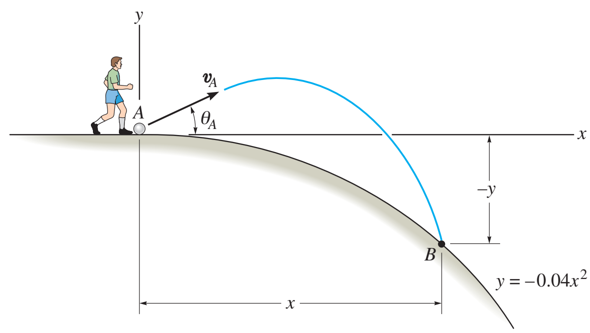
$$x = 1.95 \text{ ft} \qquad \textbf{Ans.}$$

Thus,

$$y = -0.04(1.954)^2 = -0.153 \text{ ft} \qquad \textbf{Ans.}$$

12–107.

The ball at A is kicked such that $\theta_A = 30°$. If it strikes the ground at B having coordinates $x = 15$ ft, $y = -9$ ft, determine the speed at which it is kicked and the speed at which it strikes the ground.



SOLUTION

$(\xrightarrow{+})\ s = s_0 + v_0 t$

$$15 = 0 + v_A \cos 30°\ t$$

$(+\uparrow)\ s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$-9 = 0 + v_A \sin 30°\ t + \frac{1}{2}(-32.2)t^2$$

$v_A = 16.5$ ft/s **Ans.**

$t = 1.047$ s

$(\xrightarrow{+})\ (v_B)_x = 16.54 \cos 30° = 14.32$ ft/s

$(+\uparrow)\ v = v_0 + a_c t$

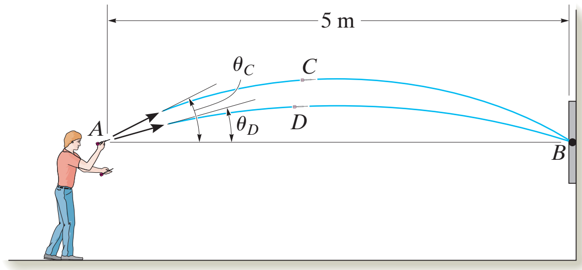
$(v_B)_y = 16.54 \sin 30° + (-32.2)(1.047)$

$= -25.45$ ft/s

$v_B = \sqrt{(14.32)^2 + (-25.45)^2} = 29.2$ ft/s **Ans.**

***12–108.**

The man at A wishes to throw two darts at the target at B so that they arrive at the *same time*. If each dart is thrown with a speed of 10 m/s, determine the angles θ_C and θ_D at which they should be thrown and the time between each throw. Note that the first dart must be thrown at θ_C $(>\theta_D)$, then the second dart is thrown at θ_D.



SOLUTION

$(\xrightarrow{+})\qquad s = s_0 + v_0 t$

$$5 = 0 + (10\cos\theta)\,t \qquad \textbf{(1)}$$

$(+\uparrow)\qquad v = v_0 + a_c t$

$$-10\sin\theta = 10\sin\theta - 9.81\,t$$

$$t = \frac{2(10\sin\theta)}{9.81} = 2.039\sin\theta$$

From Eq. (1),

$$5 = 20.39\sin\theta\cos\theta$$

Since $\quad \sin 2\theta = 2\sin\theta\cos\theta$

$$\sin 2\theta = 0.4905$$

The two roots are $\theta_D = 14.7°$ **Ans.**

$\theta_C = 75.3°$ **Ans.**

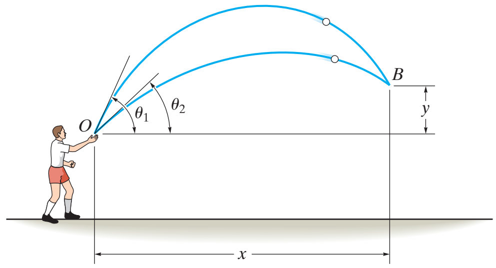
From Eq. (1): $t_D = 0.517$ s

$t_C = 1.97$ s

So that $\Delta t = t_C - t_D = 1.45$ s **Ans.**

12–109.

A boy throws a ball at O in the air with a speed v_0 at an angle θ_1. If he then throws another ball with the same speed v_0 at an angle $\theta_2 < \theta_1$, determine the time between the throws so that the balls collide in mid air at B.



SOLUTION

Vertical Motion: For the first ball, the vertical component of initial velocity is $(v_0)_y = v_0 \sin\theta_1$ and the initial and final vertical positions are $(s_0)_y = 0$ and $s_y = y$, respectively.

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$y = 0 + v_0 \sin\theta_1 t_1 + \frac{1}{2}(-g)t_1^2 \qquad \textbf{(1)}$$

For the second ball, the vertical component of initial velocity is $(v_0)_y = v_0 \sin\theta_2$ and the initial and final vertical positions are $(s_0)_y = 0$ and $s_y = y$, respectively.

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$y = 0 + v_0 \sin\theta_2 t_2 + \frac{1}{2}(-g)t_2^2 \qquad \textbf{(2)}$$

Horizontal Motion: For the first ball, the horizontal component of initial velocity is $(v_0)_x = v_0 \cos\theta_1$ and the initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = x$, respectively.

$$(\overset{+}{\rightarrow}) \qquad s_x = (s_0)_x + (v_0)_x t$$

$$x = 0 + v_0 \cos\theta_1\, t_1 \qquad \textbf{(3)}$$

For the second ball, the horizontal component of initial velocity is $(v_0)_x = v_0 \cos\theta_2$ and the initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = x$, respectively.

$$(\overset{+}{\rightarrow}) \qquad s_x = (s_0)_x + (v_0)_x t$$

$$x = 0 + v_0 \cos\theta_2\, t_2 \qquad \textbf{(4)}$$

Equating Eqs. (3) and (4), we have

$$t_2 = \frac{\cos\theta_1}{\cos\theta_2} t_1 \qquad \textbf{(5)}$$

Equating Eqs. (1) and (2), we have

$$v_0 t_1 \sin\theta_1 - v_0 t_2 \sin\theta_2 = \frac{1}{2} g\left(t_1^2 - t_2^2\right) \qquad \textbf{(6)}$$

Solving Eq. [5] into [6] yields

$$t_1 = \frac{2v_0 \cos\theta_2 \sin(\theta_1 - \theta_2)}{g(\cos^2\theta_2 - \cos^2\theta_1)}$$

$$t_2 = \frac{2v_0 \cos\theta_1 \sin(\theta_1 - \theta_2)}{g(\cos^2\theta_2 - \cos^2\theta_1)}$$

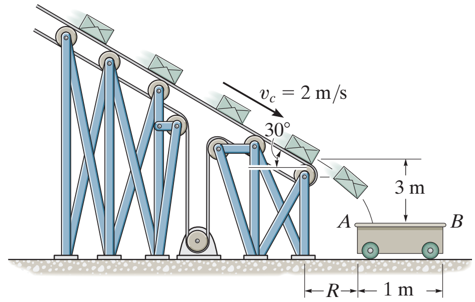
Thus, the time between the throws is

$$\Delta t = t_1 - t_2 = \frac{2v_0 \sin(\theta_1 - \theta_2)(\cos\theta_2 - \cos\theta_1)}{g(\cos^2\theta_2 - \cos^2\theta_1)}$$

$$= \frac{2v_0 \sin(\theta_1 - \theta_2)}{g(\cos\theta_2 + \cos\theta_1)} \qquad \textbf{Ans.}$$

12–110.

Small packages traveling on the conveyor belt fall off into a l-m-long loading car. If the conveyor is running at a constant speed of $v_C = 2$ m/s, determine the smallest and largest distance R at which the end A of the car may be placed from the conveyor so that the packages enter the car.



SOLUTION

Vertical Motion: The vertical component of initial velocity is $(v_0)_y = 2 \sin 30° = 1.00$ m/s. The initial and final vertical positions are $(s_0)_y = 0$ and $s_y = 3$ m, respectively.

$$(+\downarrow) \qquad s_y = (s_0)_y + (v_0)_y\, t + \frac{1}{2}(a_c)_y\, t^2$$

$$3 = 0 + 1.00(t) + \frac{1}{2}(9.81)\left(t^2\right)$$

Choose the positive root $t = 0.6867$ s

Horizontal Motion: The horizontal component of velocity is $(v_0)_x = 2 \cos 30° = 1.732$ m/s and the initial horizontal position is $(s_0)_x = 0$. If $s_x = R$, then

$$(\overset{+}{\rightarrow}) \qquad s_x = (s_0)_x + (v_0)_x\, t$$

$$R = 0 + 1.732(0.6867) = 1.19 \text{ m} \qquad \textbf{Ans.}$$

If $s_x = R + 1$, then

$$(\overset{+}{\rightarrow}) \qquad s_x = (s_0)_x + (v_0)_x\, t$$

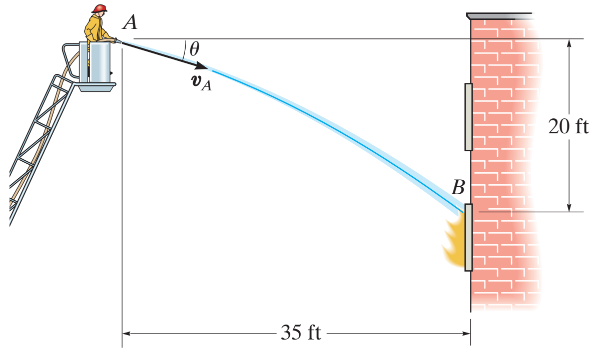
$$R + 1 = 0 + 1.732(0.6867)$$

$$R = 0.189 \text{ m} \qquad \textbf{Ans.}$$

Thus, $R_{\min} = 0.189$ m, $R_{\max} = 1.19$ m **Ans.**

12–111.

The fireman wishes to direct the flow of water from his hose to the fire at B. Determine two possible angles θ_1 and θ_2 at which this can be done. Water flows from the hose at $v_A = 80$ ft/s.



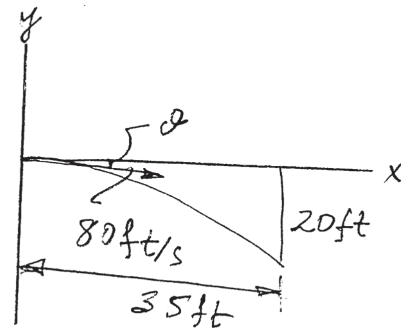
SOLUTION

$(\xrightarrow{+})$ $\quad s = s_0 + v_0 t$

$$35 = 0 + (80)(\cos\theta)t$$

$(+\uparrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$$-20 = 0 - 80(\sin\theta)t + \frac{1}{2}(-32.2)t^2$$



Thus,

$$20 = 80\sin\theta\frac{0.4375}{\cos\theta}t + 16.1\left(\frac{0.1914}{\cos^2\theta}\right)$$

$$20\cos^2\theta = 17.5\sin 2\theta + 3.0816$$

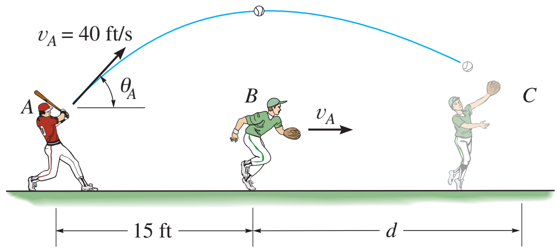
Solving,

$$\theta_1 = 24.9^\circ \quad \text{(below the horizontal)} \qquad \textbf{Ans.}$$

$$\theta_2 = 85.2^\circ \quad \text{(above the horizontal)} \qquad \textbf{Ans.}$$

***12–112.**

The baseball player A hits the baseball at $v_A = 40$ ft/s and $\theta_A = 60°$ from the horizontal. When the ball is directly overhead of player B he begins to run under it. Determine the constant speed at which B must run and the distance d in order to make the catch at the same elevation at which the ball was hit.



SOLUTION

Vertical Motion: The vertical component of initial velocity for the football is $(v_0)_y = 40 \sin 60° = 34.64$ ft/s. The initial and final vertical positions are $(s_0)_y = 0$ and $s_y = 0$, respectively.

$$(+\uparrow) \quad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$0 = 0 + 34.64t + \frac{1}{2}(-32.2)\,t^2$$

$$t = 2.152 \text{ s}$$

Horizontal Motion: The horizontal component of velocity for the baseball is $(v_0)_x = 40 \cos 60° = 20.0$ ft/s. The initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = R$, respectively.

$$(\overset{+}{\rightarrow}) \quad s_x = (s_0)_x + (v_0)_x t$$

$$R = 0 + 20.0(2.152) = 43.03 \text{ ft}$$

The distance for which player B must travel in order to catch the baseball is

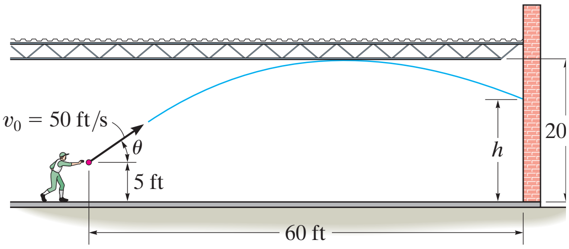
$d = R - 15 = 43.03 - 15 = 28.0$ ft **Ans.**

Player B is required to run at a same speed as the horizontal component of velocity of the baseball in order to catch it.

$v_B = 40 \cos 60° = 20.0$ ft/s **Ans.**

12–113.

The man stands 60 ft from the wall and throws a ball at it with a speed $v_0 = 50$ ft/s. Determine the angle θ at which he should release the ball so that it strikes the wall at the highest point possible. What is this height? The room has a ceiling height of 20 ft.



SOLUTION

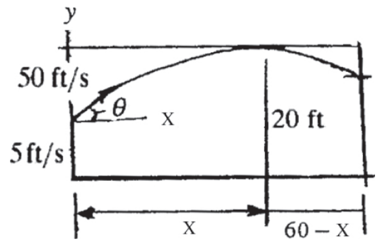
$v_x = 50 \cos \theta$

$(\overset{+}{\rightarrow}) \qquad s = s_0 + v_0 t$

$$x = 0 + 50 \cos \theta \, t \qquad \textbf{(1)}$$

$(+\uparrow) \qquad v = v_0 + a_c t$

$$v_y = 50 \sin \theta - 32.2\, t \qquad \textbf{(2)}$$



$(+\uparrow) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$y = 0 + 50 \sin \theta \, t - 16.1\, t^2 \qquad \textbf{(3)}$$

$(+\uparrow) \qquad v^2 = v_0^2 + 2a_c(s - s_0)$

$$v_y^2 = (50 \sin \theta)^2 + 2(-32.2)(s - 0)$$

$$v_y^2 = 2500 \sin^2 \theta - 64.4\, s \qquad \textbf{(4)}$$

Require $v_y = 0$ at $s = 20 - 5 = 15$ ft

$0 = 2500 \sin^2 \theta - 64.4\,(15)$

$\theta = 38.433° = 38.4°$ **Ans.**

From Eq. (2)

$0 = 50 \sin 38.433° - 32.2\, t$

$t = 0.9652$ s

From Eq. (1)

$x = 50 \cos 38.433°(0.9652) = 37.8$ ft

Time for ball to hit wall

From Eq. (1),

$60 = 50(\cos 38.433°)t$

$t = 1.53193$ s

From Eq. (3)

$y = 50 \sin 38.433°(1.53193) - 16.1(1.53193)^2$

$y = 9.830$ ft

$h = 9.830 + 5 = 14.8$ ft **Ans.**

12–114.

A car is traveling along a circular curve that has a radius of 50 m. If its speed is 16 m/s and is increasing uniformly at 8 m/s^2, determine the magnitude of its acceleration at this instant.

SOLUTION

$v = 16 \text{ m/s}$

$a_t = 8 \text{ m/s}^2$

$r = 50 \text{ m}$

$$a_n = \frac{v^2}{\rho} = \frac{(16)^2}{50} = 5.12 \text{ m/s}^2$$

$$a = \sqrt{(8)^2 + (5.12)^2} = 9.50 \text{ m/s}^2$$ **Ans.**

12–115.

Determine the maximum constant speed a race car can have if the acceleration of the car cannot exceed 7.5 m/s^2 while rounding a track having a radius of curvature of 200 m.

SOLUTION

Acceleration: Since the speed of the race car is constant, its tangential component of acceleration is zero, i.e., $a_t = 0$. Thus,

$$a = a_n = \frac{v^2}{\rho}$$

$$7.5 = \frac{v^2}{200}$$

$$v = 38.7 \text{ m/s}$$ **Ans.**

***12–116.**

A car moves along a circular track of radius 250 ft such that its speed for a short period of time, $0 \leq t \leq 4$ s, is $v = 3(t + t^2)$ ft/s, where t is in seconds. Determine the magnitude of its acceleration when $t = 3$ s. How far has it traveled in $t = 3$ s?

SOLUTION

$$v = 3(t + t^2)$$

$$a_t = \frac{dv}{dt} = 3 + 6\,t$$

When $t = 3$ s, $\quad a_t = 3 + 6(3) = 21 \text{ ft/s}^2$

$$a_n = \frac{[3(3 + 3^2)]^2}{250} = 5.18 \text{ ft/s}^2$$

$$a = \sqrt{(21)^2 + (5.18)^2} = 21.6 \text{ ft/s}^2$$ **Ans.**

$$\int ds = \int_0^3 3(t + t^2)dt$$

$$\Delta s = \frac{3}{2}t^2 + t^3\Big|_0^3$$

$$\Delta s = 40.5 \text{ ft}$$ **Ans.**

12–117.

A car travels along a horizontal circular curved road that has a radius of 600 m. If the speed is uniformly increased at a rate of $2000\ \text{km/h}^2$, determine the magnitude of the acceleration at the instant the speed of the car is 60 km/h.

SOLUTION

$$a_t = \left(\frac{2000\ \text{km}}{\text{h}^2}\right)\left(\frac{1000\ \text{m}}{1\ \text{km}}\right)\left(\frac{1\text{h}}{3600\ \text{s}}\right)^2 = 0.1543\ \text{m/s}^2$$

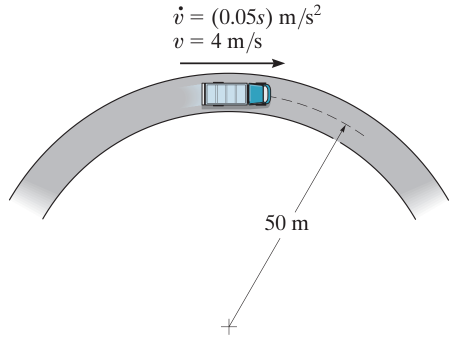
$$v = \left(\frac{60\ \text{km}}{\text{h}}\right)\left(\frac{1000\ \text{m}}{1\ \text{km}}\right)\left(\frac{1\text{h}}{3600\ \text{s}}\right) = 16.67\ \text{m/s}$$

$$a_n = \frac{v^2}{\rho} = \frac{16.67^2}{600} = 0.4630\ \text{m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0.1543^2 + 0.4630^2} = 0.488\ \text{m/s}^2$$ **Ans.**

12–118.

The truck travels in a circular path having a radius of 50 m at a speed of $v = 4$ m/s. For a short distance from $s = 0$, its speed is increased by $\dot{v} = (0.05s)$ m/s^2, where s is in meters. Determine its speed and the magnitude of its acceleration when it has moved $s = 10$ m.



SOLUTION

$$v\,dv = a_t\,ds$$

$$\int_4^v v\,dv = \int_0^{10} 0.05s\,ds$$

$$0.5v^2 - 8 = \frac{0.05}{2}(10)^2$$

$$v = 4.583 = 4.58 \text{ m/s}$$ **Ans.**

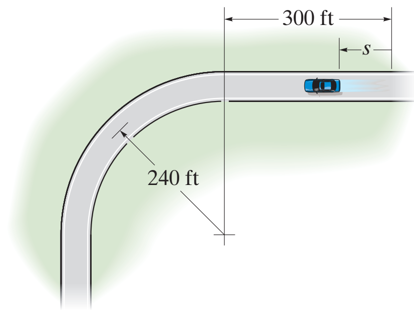
$$a_n = \frac{v^2}{\rho} = \frac{(4.583)^2}{50} = 0.420 \text{ m/s}^2$$

$$a_t = 0.05(10) = 0.5 \text{ m/s}^2$$

$$a = \sqrt{(0.420)^2 + (0.5)^2} = 0.653 \text{ m/s}^2$$ **Ans.**

12–119.

The automobile is originally at rest at $s = 0$. If its speed is increased by $\dot{v} = (0.05t^2)\ \text{ft/s}^2$, where t is in seconds, determine the magnitudes of its velocity and acceleration when $t = 18$ s.



SOLUTION

$a_t = 0.05t^2$

$$\int_0^v dv = \int_0^t 0.05\, t^2\, dt$$

$v = 0.0167\, t^3$

$$\int_0^s ds = \int_0^t 0.0167\, t^3\, dt$$

$s = 4.167(10^{-3})\, t^4$

When $t = 18$ s, $\quad s = 437.4$ ft

Therefore the car is on a curved path.

$v = 0.0167(18^3) = 97.2$ ft/s **Ans.**

$$a_n = \frac{(97.2)^2}{240} = 39.37\ \text{ft/s}^2$$

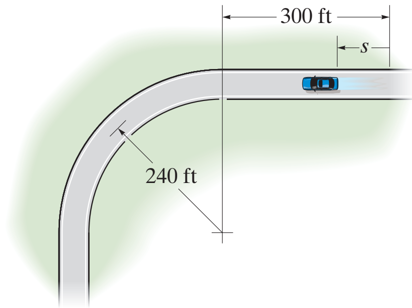
$a_t = 0.05(18^2) = 16.2\ \text{ft/s}^2$

$a = \sqrt{(39.37)^2 + (16.2)^2}$

$a = 42.6\ \text{ft/s}^2$ **Ans.**

***12–120.**

The automobile is originally at rest $s = 0$. If it then starts to increase its speed at $\dot{v} = (0.05t^2)$ ft/s^2, where t is in seconds, determine the magnitudes of its velocity and acceleration at $s = 550$ ft.



SOLUTION

The car is on the curved path.

$$a_t = 0.05\,t^2$$

$$\int_0^v dv = \int_0^t 0.05\,t^2\,dt$$

$$v = 0.0167\,t^3$$

$$\int_0^s ds = \int_0^t 0.0167\,t^3\,dt$$

$$s = 4.167(10^{-3})\,t^4$$

$$550 = 4.167(10^{-3})\,t^4$$

$$t = 19.06 \text{ s}$$

So that

$$v = 0.0167(19.06)^3 = 115.4$$

$$v = 115 \text{ ft/s} \qquad \textbf{Ans.}$$

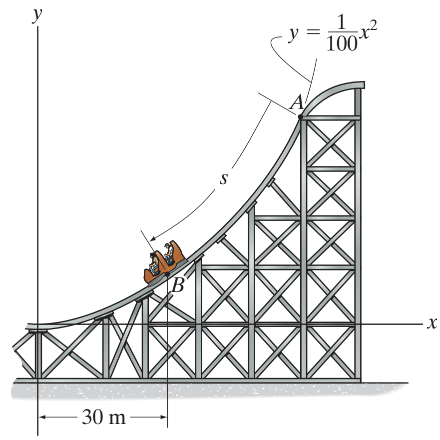
$$a_n = \frac{(115.4)^2}{240} = 55.51 \text{ ft/s}^2$$

$$a_t = 0.05(19.06)^2 = 18.17 \text{ ft/s}^2$$

$$a = \sqrt{(55.51)^2 + (18.17)^2} = 58.4 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

12–121.

When the roller coaster is at B, it has a speed of 25 m/s, which is increasing at $a_t = 3\text{ m/s}^2$. Determine the magnitude of the acceleration of the roller coaster at this instant and the direction angle it makes with the x axis.



SOLUTION

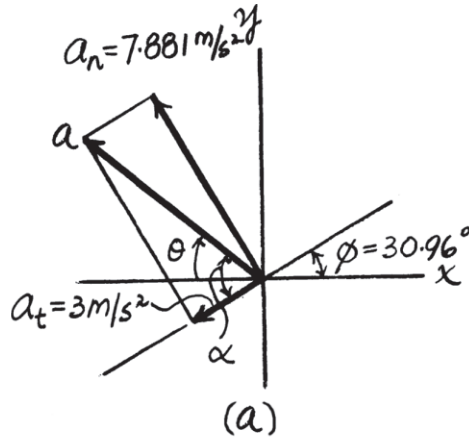
Radius of Curvature:

$$y = \frac{1}{100}x^2$$

$$\frac{dy}{dx} = \frac{1}{50}x$$

$$\frac{d^2y}{dx^2} = \frac{1}{50}$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(\frac{1}{50}x\right)^2\right]^{3/2}}{\left|\frac{1}{50}\right|}\right|_{x=30\text{ m}} = 79.30\text{ m}$$



Acceleration:

$$a_t = \dot{v} = 3\text{ m/s}^2$$

$$a_n = \frac{v_B{}^2}{\rho} = \frac{25^2}{79.30} = 7.881\text{ m/s}^2$$

The magnitude of the roller coaster's acceleration is

$$a = \sqrt{a_t{}^2 + a_n{}^2} = \sqrt{3^2 + 7.881^2} = 8.43\text{ m/s}^2 \qquad \textbf{Ans.}$$

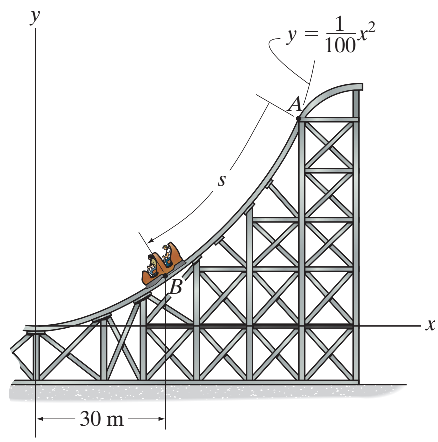
The angle that the tangent at B makes with the x axis is $\phi = \tan^{-1}\left(\left.\frac{dy}{dx}\right|_{x=30\text{ m}}\right) = \tan^{-1}\left[\frac{1}{50}(30)\right] = 30.96°$.

As shown in Fig. a, $\mathbf{a}_n$ is always directed towards the center of curvature of the path. Here, $\alpha = \tan^{-1}\left(\frac{a_n}{a_t}\right) = \tan^{-1}\left(\frac{7.881}{3}\right) = 69.16°$. Thus, the angle θ that the roller coaster's acceleration makes with the x axis is

$$\theta = \alpha - \phi = 38.2° \qquad \textbf{Ans.}$$

12–122.

If the roller coaster starts from rest at A and its speed increases at $a_t = (6 - 0.06s)\ \mathrm{m/s^2}$, determine the magnitude of its acceleration when it reaches B where $s_B = 40$ m.



SOLUTION

Velocity: Using the initial condition $v = 0$ at $s = 0$,

$$v\,dv = a_t\,ds$$

$$\int_0^v v\,dv = \int_0^s (6 - 0.06s)\,ds$$

$$v = \left(\sqrt{12s - 0.06s^2}\right)\ \mathrm{m/s} \qquad \textbf{(1)}$$

Thus,

$$v_B = \sqrt{12(40) - 0.06(40)^2} = 19.60\ \mathrm{m/s}$$

Radius of Curvature:

$$y = \frac{1}{100}x^2$$

$$\frac{dy}{dx} = \frac{1}{50}x$$

$$\frac{d^2y}{dx^2} = \frac{1}{50}$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(\frac{1}{50}x\right)^2\right]^{3/2}}{\left|\frac{1}{50}\right|}\right|_{x=30\ \mathrm{m}} = 79.30\ \mathrm{m}$$

Acceleration:

$$a_t = \dot{v} = 6 - 0.06(40) = 3.600\ \mathrm{m/s^2}$$

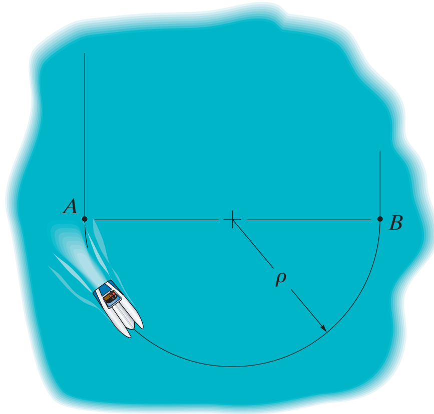
$$a_n = \frac{v^2}{\rho} = \frac{19.60^2}{79.30} = 4.842\ \mathrm{m/s^2}$$

The magnitude of the roller coaster's acceleration at B is

$$a = \sqrt{{a_t}^2 + {a_n}^2} = \sqrt{3.600^2 + 4.842^2} = 6.03\ \mathrm{m/s^2} \qquad \textbf{Ans.}$$

12–123.

The speedboat travels at a constant speed of 15 m/s while making a turn on a circular curve from A to B. If it takes 45 s to make the turn, determine the magnitude of the boat's acceleration during the turn.



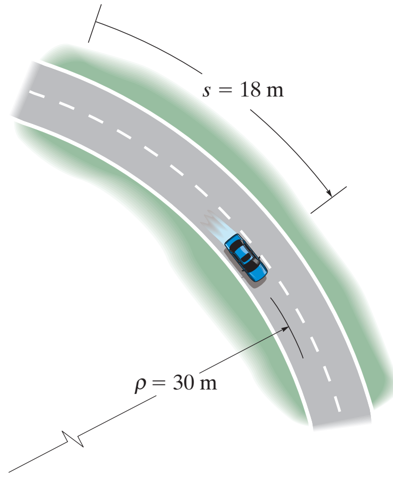
SOLUTION

Acceleration: During the turn, the boat travels $s = vt = 15(45) = 675$ m. Thus, the radius of the circular path is $\rho = \dfrac{s}{\pi} = \dfrac{675}{\pi}$ m. Since the boat has a constant speed, $a_t = 0$. Thus,

$$a = a_n = \frac{v^2}{\rho} = \frac{15^2}{\left(\dfrac{675}{\pi}\right)} = 1.05\ \text{m/s}^2 \qquad \textbf{Ans.}$$

***12–124.**

The car travels along the circular path such that its speed is increased by $a_t = (0.5e^t)\ \text{m/s}^2$, where t is in seconds. Determine the magnitudes of its velocity and acceleration after the car has traveled $s = 18\ \text{m}$ starting from rest. Neglect the size of the car.



SOLUTION

$$\int_0^v dv = \int_0^t 0.5e^t\, dt$$

$$v = 0.5(e^t - 1)$$

$$\int_0^{18} ds = 0.5\int_0^t (e^t - 1)dt$$

$$18 = 0.5(e^t - t - 1)$$

Solving,

$$t = 3.7064\ \text{s}$$

$$v = 0.5(e^{3.7064} - 1) = 19.85\ \text{m/s} = 19.9\ \text{m/s} \qquad \textbf{Ans.}$$

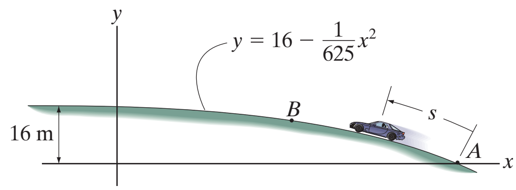
$$a_t = \dot{v} = 0.5e^t\big|_{t=3.7064\ \text{s}} = 20.35\ \text{m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{19.85^2}{30} = 13.14\ \text{m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{20.35^2 + 13.14^2} = 24.2\ \text{m/s}^2 \qquad \textbf{Ans.}$$

12–125.

The car passes point A with a speed of 25 m/s after which its speed is defined by $v = (25 - 0.15s)$ m/s. Determine the magnitude of the car's acceleration when it reaches point B, where $s = 51.5$ m.



SOLUTION

Velocity: The speed of the car at B is

$$v_B = \left[25 - 0.15(51.5)\right] = 17.28 \text{ m/s}$$

Radius of Curvature:

$$y = 16 - \frac{1}{625}x^2$$

$$\frac{dy}{dx} = -3.2\left(10^{-3}\right)x$$

$$\frac{d^2y}{dx^2} = -3.2\left(10^{-3}\right)$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(-3.2\left(10^{-3}\right)x\right)^2\right]^{3/2}}{\left|-3.2\left(10^{-3}\right)\right|}\right|_{x=50\text{ m}} = 324.58 \text{ m}$$

Acceleration:

$$a_n = \frac{v_B{}^2}{\rho} = \frac{17.28^2}{324.58} = 0.9194 \text{ m/s}^2$$

$$a_t = v\frac{dv}{ds} = (25 - 0.15s)(-0.15) = (0.225s - 3.75) \text{ m/s}^2$$

When the car is at B $(s = 51.5 \text{ m})$

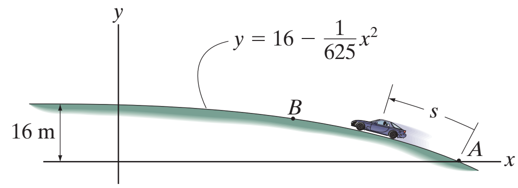
$$a_t = \left[0.225(51.5) - 3.75\right] = -2.591 \text{ m/s}^2$$

Thus, the magnitude of the car's acceleration at B is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(-2.591)^2 + 0.9194^2} = 2.75 \text{ m/s}^2$$ **Ans.**

12–126.

If the car passes point A with a speed of 20 m/s and begins to increase its speed at a constant rate of $a_t = 0.5\ \text{m/s}^2$, determine the magnitude of the car's acceleration when $s = 100$ m.



SOLUTION

Velocity: The speed of the car at C is

$$v_C{}^2 = v_A{}^2 + 2a_t(s_C - s_A)$$

$$v_C{}^2 = 20^2 + 2(0.5)(100 - 0)$$

$$v_C = 22.361\ \text{m/s}$$

Radius of Curvature:

$$y = 16 - \frac{1}{625}x^2$$

$$\frac{dy}{dx} = -3.2\left(10^{-3}\right)x$$

$$\frac{d^2y}{dx^2} = -3.2\left(10^{-3}\right)$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(-3.2\left(10^{-3}\right)x\right)^2\right]^{3/2}}{\left|-3.2\left(10^{-3}\right)\right|}\right|_{x=0} = 312.5\ \text{m}$$

Acceleration:

$$a_t = \dot{v} = 0.5\ \text{m/s}$$

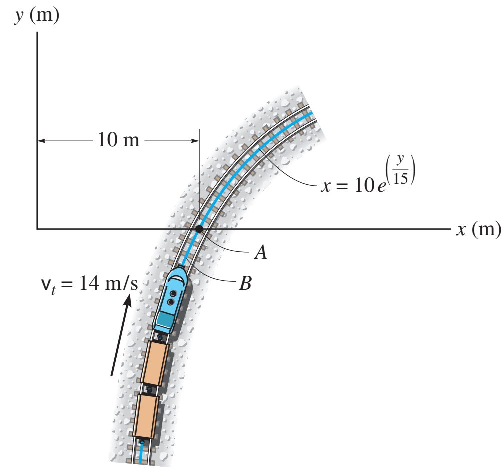
$$a_n = \frac{v_C{}^2}{\rho} = \frac{22.361^2}{312.5} = 1.60\ \text{m/s}^2$$

The magnitude of the car's acceleration at C is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0.5^2 + 1.60^2} = 1.68\ \text{m/s}^2 \qquad \textbf{Ans.}$$

12–127.

A train is traveling with a constant speed of 14 m/s along the curved path. Determine the magnitude of the acceleration of the front of the train, B, at the instant it reaches point A $(y = 0)$.



SOLUTION

$$x = 10e^{\left(\frac{y}{15}\right)}$$

$$y = 15\ln\left(\frac{x}{10}\right)$$

$$\frac{dy}{dx} = 15\left(\frac{10}{x}\right)\left(\frac{1}{10}\right) = \frac{15}{x}$$

$$\frac{d^2y}{dx^2} = -\frac{15}{x^2}$$

At $x = 10$,

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1 + (1.5)^2\right]^{\frac{3}{2}}}{|-0.15|} = 39.06 \text{ m}$$

$$a_t = \frac{dv}{dt} = 0$$

$$a_n = a = \frac{v^2}{\rho} = \frac{(14)^2}{39.06} = 5.02 \text{ m/s}^2$$ **Ans.**

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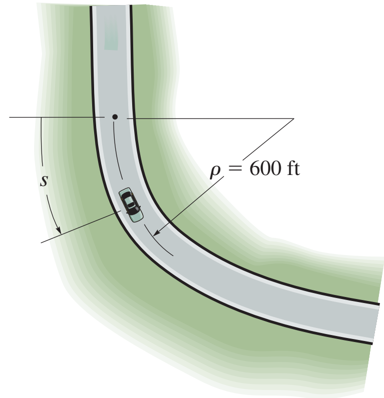
***12–128.**

When a car starts to round a curved road with the radius of curvature of 600 ft, it is traveling at 75 ft/s. If the car's speed begins to decrease at a rate of $\dot{v} = (-0.06t^2)$ ft/s^2, determine the magnitude of the acceleration of the car when it has traveled a distance of $s = 700$ ft.



SOLUTION

Velocity: Using the initial condition $v = 75$ ft/s when $t = 0$ s,

$$\int dt = \int a_t dt$$

$$\int_{v=75\text{ ft/s}}^{v} dv = \int_0^t -0.06t^2 dt$$

$$v = (75 - 0.02t^3)\text{ ft/s}$$

Position: Using the initial condition $s = 0$ at $t = 0$ s,

$$ds = vdt$$

$$\int_0^s ds = \int_0^t (75 - 0.02t^3)dt$$

$$s = [75t - 0.005t^4]\text{ ft}$$

At $s = 700$ ft,

$$700 = 75t - 0.005t^4$$

Solving the above equation by trial and error,

$t = 10$ s and $t = 20$ s. Pick the first solution.

Acceleration: When $t = 10$ s, $a_t = \dot{v} = -0.06(10^2) = -6\text{ ft/s}^2$ and $v = 75 - 0.02(10^3) = 55$ ft/s

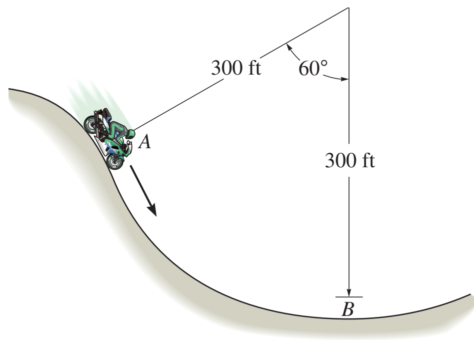
$$a_n = \frac{v^2}{\rho} = \frac{55^2}{600} = 5.042\text{ ft/s}^2$$

Thus, the magnitude of the truck's acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(-6)^2 + 5.042^2} = 7.84\text{ ft/s}^2$$ **Ans.**

12–129.

When the motorcyclist is at A, he increases his speed along the vertical circular path at the rate of $\dot{v} = (0.3t)\ \text{ft/s}^2$, where t is in seconds. If he starts from rest at A, determine the magnitudes of his velocity and acceleration when he reaches B.



SOLUTION

$$\int_0^v dv = \int_0^t 0.3t\,dt$$

$$v = 0.15t^2$$

$$\int_0^s ds = \int_0^t 0.15t^2\,dt$$

$$s = 0.05t^3$$

When $s = \frac{\pi}{3}(300)$ ft, $\quad \frac{\pi}{3}(300) = 0.05t^3 \quad t = 18.453\text{ s}$

$v = 0.15(18.453)^2 = 51.08\text{ ft/s} = 51.1\text{ ft/s}$ **Ans.**

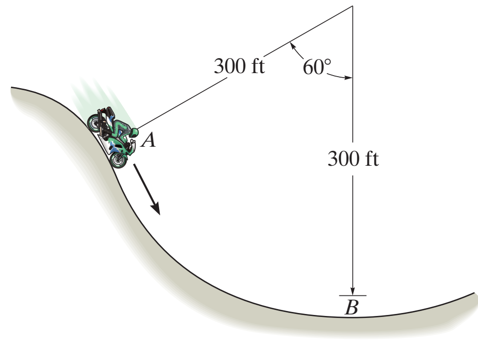
$a_t = \dot{v} = 0.3t|_{t=18.453\text{ s}} = 5.536\ \text{ft/s}^2$

$$a_n = \frac{v^2}{\rho} = \frac{51.08^2}{300} = 8.696\ \text{ft/s}^2$$

$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(5.536)^2 + (8.696)^2} = 10.3\ \text{ft/s}^2$ **Ans.**

12–130.

When the motorcyclist is at A, he increases his speed along the vertical circular path at the rate of $\dot{v} = (0.04s)$ ft/s^2 where s is in ft. If he starts at $v_A = 2$ ft/s where $s = 0$ at A, determine the magnitude of his velocity when he reaches B. Also, what is his initial acceleration?



SOLUTION

Velocity: At $s = 0, v = 2$. Here, $a_c = \dot{v} = 0.045$. Then

$$\int v\,dv = \int a_t\,ds$$

$$\int_2^v v\,dv = \int_0^s 0.04s\,ds$$

$$\frac{v^2}{2}\bigg|_2^v = 0.025^2\bigg|_0^s$$

$$\frac{v^2}{2} - 2 = 0.025^2$$

$$v^2 = 0.045^2 + 4 = 0.04(s^2 + 100)$$

$$v = 0.2\sqrt{s^2 + 100}$$

At B, $s = r\theta = 300\left(\frac{\pi}{3}\right) = 100\pi$ ft. Thus

$$v\bigg|_{s = 100\pi\text{ ft}} = 0.2\sqrt{(100\pi)^2 + 100} = 62.9\text{ ft/s}$$ **Ans.**

Acceleration: At $t = 0, s = 0$, and $v = 2$.

$$a_t = \dot{v} = 0.04\text{ s}$$

$$a_t\bigg|_{s=0} = 0$$

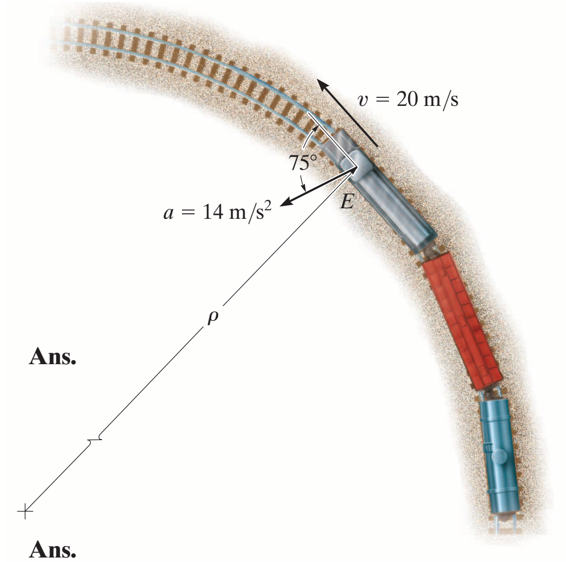
$$a_n = \frac{v^2}{\rho}$$

$$a_n\bigg|_{s=0} = \frac{(2)^2}{300} = 0.01333\text{ ft/s}^2$$

$$a = \sqrt{(0)^2 + (0.01333)^2} = 0.0133\text{ ft/s}^2$$

12–131.

At a given instant the train engine at E has a speed of 20 m/s and an acceleration of 14 m/s^2 acting in the direction shown. Determine the rate of increase in the train's speed and the radius of curvature ρ of the path.



SOLUTION

$a_t = 14 \cos 75° = 3.62 \text{ m/s}^2$ **Ans.**

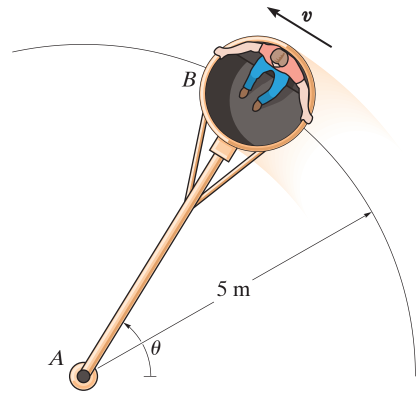
$a_n = 14 \sin 75°$

$$a_n = \frac{(20)^2}{\rho}$$

$\rho = 29.6 \text{ m}$ **Ans.**

***12–132.**

Car B turns such that its speed is increased by $(a_t)_B = (0.5e^t)\ \text{m/s}^2$, where t is in seconds. If the car starts from rest when $\theta = 0°$, determine the magnitudes of its velocity and acceleration when the arm AB rotates $\theta = 30°$. Neglect the size of the car.



SOLUTION

Velocity: The speed v in terms of time t can be obtained by applying $a = \dfrac{dv}{dt}$.

$$dv = a\,dt$$

$$\int_0^v dv = \int_0^t 0.5e^t\,dt$$

$$v = 0.5\left(e^t - 1\right) \qquad \textbf{(1)}$$

When $\theta = 30°$, the car has traveled a distance of $s = r\theta = 5\left(\dfrac{30°}{180°}\pi\right) = 2.618\ \text{m}$. The time required for the car to travel this distance can be obtained by applying $v = \dfrac{ds}{dt}$.

$$ds = v\,dt$$

$$\int_0^{2.618\ \text{m}} ds = \int_0^t 0.5\left(e^t - 1\right)dt$$

$$2.618 = 0.5\left(e^t - t - 1\right)$$

Solving by trial and error $\quad t = 2.1234\ \text{s}$

Substituting $t = 2.1234\ \text{s}$ into Eq. (1) yields

$$v = 0.5\left(e^{2.1234} - 1\right) = 3.680\ \text{m/s} = 3.68\ \text{m/s} \qquad \textbf{Ans.}$$

Acceleration: The tangential acceleration for the car at $t = 2.1234\ \text{s}$ is $a_t = 0.5e^{2.1234} = 4.180\ \text{m/s}^2$. To determine the normal acceleration, apply Eq. 12–20.

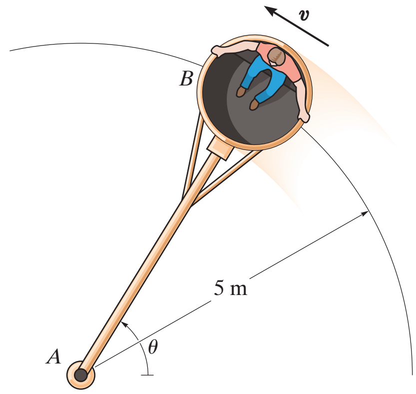
$$a_n = \frac{v^2}{\rho} = \frac{3.680^2}{5} = 2.708\ \text{m/s}^2$$

The magnitude of the acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{4.180^2 + 2.708^2} = 4.98\ \text{m/s}^2 \qquad \textbf{Ans.}$$

12–133.

Car B turns such that its speed is increased by $(a_t)_B = (0.5e^t)\ \text{m/s}^2$, where t is in seconds. If the car starts from rest when $\theta = 0°$, determine the magnitudes of its velocity and acceleration when $t = 2$ s. Neglect the size of the car.



SOLUTION

Velocity: The speed v in terms of time t can be obtained by applying $a = \dfrac{dv}{dt}$.

$$dv = adt$$

$$\int_0^v dv = \int_0^t 0.5e^t\, dt$$

$$v = 0.5\left(e^t - 1\right)$$

When $t = 2$ s, $\quad v = 0.5\left(e^2 - 1\right) = 3.195\ \text{m/s} = 3.19\ \text{m/s}$ **Ans.**

Acceleration: The tangential acceleration of the car at $t = 2\ \text{s}$ is $a_t = 0.5e^2 = 3.695\ \text{m/s}^2$. To determine the normal acceleration, apply Eq. 12–20.

$$a_n = \frac{v^2}{\rho} = \frac{3.195^2}{5} = 2.041\ \text{m/s}^2$$

The magnitude of the acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{3.695^2 + 2.041^2} = 4.22\ \text{m/s}^2$$ **Ans.**

12–134.

A boat is traveling along a circular curve having a radius of 100 ft. If its speed at $t = 0$ is 15 ft/s and is increasing at $\dot{v} = (0.8t)$ ft/s^2, determine the magnitude of its acceleration at the instant $t = 5$ s.

SOLUTION

$$\int_{15}^{v} dv = \int_{0}^{5} 0.8t\,dt$$

$$v = 25 \text{ ft/s}$$

$$a_n = \frac{v^2}{\rho} = \frac{25^2}{100} = 6.25 \text{ ft/s}^2$$

At $t = 5$ s, $\quad a_t = \dot{v} = 0.8(5) = 4 \text{ ft/s}^2$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{4^2 + 6.25^2} = 7.42 \text{ ft/s}^2$$ **Ans.**

12–135.

A boat is traveling along a circular path having a radius of 20 m. Determine the magnitude of the boat's acceleration when the speed is $v = 5$ m/s and the rate of increase in the speed is $\dot{v} = 2$ m/s^2.

SOLUTION

$$a_t = 2 \text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{5^2}{20} = 1.25 \text{ m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{2^2 + 1.25^2} = 2.36 \text{ m/s}^2$$ **Ans.**

***■12–136.**

Starting from rest, a bicyclist travels around a horizontal circular path, $\rho = 10$ m, at a speed of $v = (0.09t^2 + 0.1t)$ m/s, where t is in seconds. Determine the magnitudes of his velocity and acceleration when he has traveled $s = 3$ m.

SOLUTION

$$\int_0^s ds = \int_0^t (0.09t^2 + 0.1t)dt$$

$$s = 0.03t^3 + 0.05t^2$$

When $s = 3$ m, $\quad 3 = 0.03t^3 + 0.05t^2$

Solving,

$t = 4.147$ s

$$v = \frac{ds}{dt} = 0.09t^2 + 0.1t$$

$v = 0.09(4.147)^2 + 0.1(4.147) = 1.96$ m/s **Ans.**

$$a_t = \frac{dv}{dt} = 0.18t + 0.1\Big|_{t=4.147\text{ s}} = 0.8465\text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{1.96^2}{10} = 0.3852\text{ m/s}^2$$

$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(0.8465)^2 + (0.3852)^2} = 0.930$ m/s^2 **Ans.**

12–137.

A particle travels around a circular path having a radius of 50 m. If it is initially traveling with a speed of 10 m/s and its speed then increases at a rate of $\dot{v} = (0.05\,v)\ \text{m/s}^2$, determine the magnitude of the particle's acceleraton four seconds later.

SOLUTION

Velocity: Using the initial condition $v = 10$ m/s at $t = 0$ s,

$$dt = \frac{dv}{a}$$

$$\int_0^t dt = \int_{10\text{ m/s}}^{v} \frac{dv}{0.05v}$$

$$t = 20\ln\frac{v}{10}$$

$$v = (10e^{t/20})\ \text{m/s}$$

When $t = 4$ s,

$$v = 10e^{4/20} = 12.214\ \text{m/s}$$

Acceleration: When $v = 12.214$ m/s ($t = 4$ s),

$$a_t = 0.05(12.214) = 0.6107\ \text{m/s}^2$$

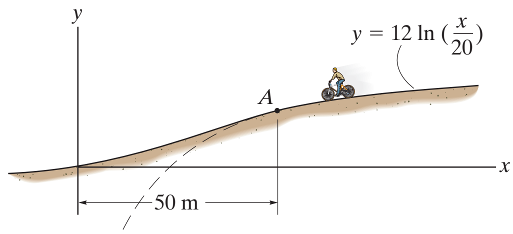
$$a_n = \frac{v^2}{\rho} = \frac{(12.214)^2}{50} = 2.984\ \text{m/s}^2$$

Thus, the magnitude of the particle's acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0.6107^2 + 2.984^2} = 3.05\ \text{m/s}^2 \qquad \textbf{Ans.}$$

12–138.

When the bicycle passes point A, it has a speed of 6 m/s, which is increasing at the rate of $\dot{v} = (0.5)\ \text{m/s}^2$. Determine the magnitude of its acceleration when it is at point A.



SOLUTION

Radius of Curvature:

$$y = 12 \ln\left(\frac{x}{20}\right)$$

$$\frac{dy}{dx} = 12\left(\frac{1}{x/20}\right)\left(\frac{1}{20}\right) = \frac{12}{x}$$

$$\frac{d^2y}{dx^2} = -\frac{12}{x^2}$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(\frac{12}{x}\right)^2\right]^{3/2}}{\left|-\frac{12}{x^2}\right|}\right|_{x=50\text{ m}} = 226.59\text{ m}$$

Acceleration:

$$a_t = \dot{v} = 0.5\ \text{m/s}^2$$

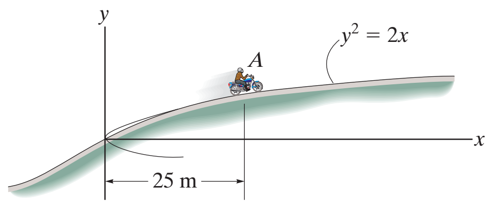
$$a_n = \frac{v^2}{\rho} = \frac{6^2}{226.59} = 0.1589\ \text{m/s}^2$$

The magnitude of the bicycle's acceleration at A is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0.5^2 + 0.1589^2} = 0.525\ \text{m/s}^2 \qquad \textbf{Ans.}$$

12–139.

The motorcycle is traveling at a constant speed of 60 km/h. Determine the magnitude of its acceleration when it is at point A.



SOLUTION

Radius of Curvature:

$$y = \sqrt{2}x^{1/2}$$

$$\frac{dy}{dx} = \frac{1}{2}\sqrt{2}x^{-1/2}$$

$$\frac{d^2y}{dx^2} = -\frac{1}{4}\sqrt{2}x^{-3/2}$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(\frac{1}{2}\sqrt{2}x^{-1/2}\right)^2\right]^{3/2}}{\left|-\frac{1}{4}\sqrt{2}x^{-3/2}\right|}\right|_{x=25\text{ m}} = 364.21\text{ m}$$

Acceleration: The speed of the motorcycle at a is

$$v = \left(60\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 16.67\text{ m/s}$$

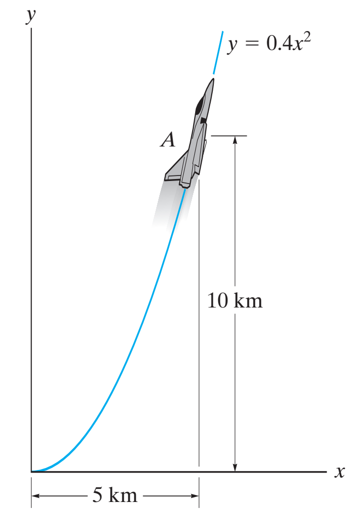
$$a_n = \frac{v^2}{\rho} = \frac{16.67^2}{364.21} = 0.7627\text{ m/s}^2$$

Since the motorcycle travels with a constant speed, $a_t = 0$. Thus, the magnitude of the motorcycle's acceleration at A is

$$a = \sqrt{a_t^{\,2} + a_n^{\,2}} = \sqrt{0^2 + 0.7627^2} = 0.763\text{ m/s}^2$$ **Ans.**

***12–140.**

The jet plane travels along the vertical parabolic path. When it is at point A it has a speed of 200 m/s, which is increasing at the rate of $0.8\ \text{m/s}^2$. Determine the magnitude of acceleration of the plane when it is at point A.



SOLUTION

$$y = 0.4x^2$$

$$\frac{dy}{dx} = 0.8x\Big|_{x=5\ \text{km}} = 4$$

$$\frac{d^2y}{dx^2} = 0.8$$

$$\rho = \frac{[1 + (4)^2]^{3/2}}{0.8} = 87.62\ \text{km}$$

$$a_t = 0.8\ \text{m/s}^2$$

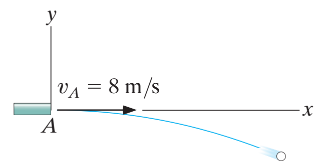
$$a_n = \frac{(0.200)^2}{87.62} = 0.457(10^{-3})\ \text{km/s}^2$$

$$a_n = 0.457\ \text{km/s}^2$$

$$a = \sqrt{(0.8)^2 + (0.457)^2} = 0.921\ \text{m/s}^2$$ **Ans.**

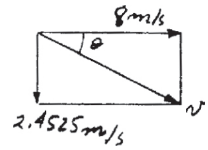
12–141.

The ball is ejected horizontally from the tube with a speed of 8 m/s. Find the equation of the path, $y = f(x)$, and then find the ball's velocity and the normal and tangential components of acceleration when $t = 0.25$ s.



SOLUTION

$v_x = 8$ m/s

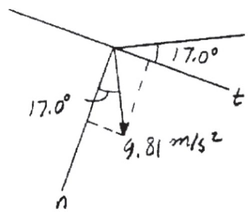


$(\overset{+}{\rightarrow})$ $\quad s = v_0 t$

$x = 8t$

$(+\uparrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$y = 0 + 0 + \frac{1}{2}(-9.81)t^2$



$y = -4.905t^2$

$y = -4.905\left(\frac{x}{8}\right)^2$

$y = -0.0766x^2$ (Parabola) **Ans.**

$v = v_0 + a_c t$

$v_y = 0 - 9.81t$

When $t = 0.25$ s,

$v_y = -2.4525$ m/s

$v = \sqrt{(8)^2 + (2.4525)^2} = 8.37$ m/s **Ans.**

$\theta = \tan^{-1}\left(\frac{2.4525}{8}\right) = 17.04°$

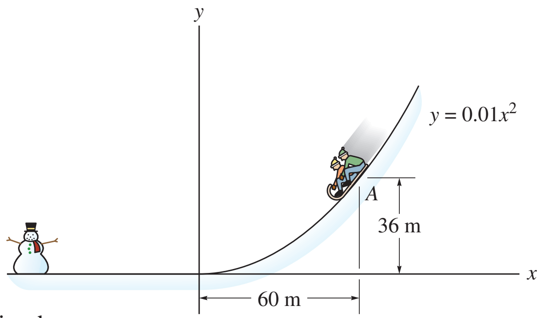
$a_x = 0 \quad a_y = 9.81\ \text{m/s}^2$

$a_n = 9.81 \cos 17.04° = 9.38\ \text{m/s}^2$ **Ans.**

$a_t = 9.81 \sin 17.04° = 2.88\ \text{m/s}^2$ **Ans.**

12–142.

A toboggan is traveling down along a curve which can be approximated by the parabola $y = 0.01x^2$. Determine the magnitude of its acceleration when it reaches point A, where its speed is $v_A = 10$ m/s, and it is increasing at the rate of $\dot{v}_A = 3\ \text{m/s}^2$.



SOLUTION

Acceleration: The radius of curvature of the path at point A must be determined first. Here, $\dfrac{dy}{dx} = 0.02x$ and $\dfrac{d^2y}{dx^2} = 0.02$, then

$$\rho = \frac{[1 + (dy/dx)^2]^{3/2}}{|d^2y/dx^2|} = \left.\frac{[1 + (0.02x)^2]^{3/2}}{|0.02|}\right|_{x=60\text{ m}} = 190.57\text{ m}$$

To determine the normal acceleration, apply Eq. 12–20.

$$a_n = \frac{v^2}{\rho} = \frac{10^2}{190.57} = 0.5247\ \text{m/s}^2$$

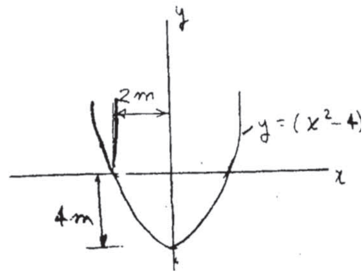
Here, $a_t = \dot{v}_A = 3$ m/s. Thus, the magnitude of acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{3^2 + 0.5247^2} = 3.05\ \text{m/s}^2 \qquad \textbf{Ans.}$$

12–143.

A particle P moves along the curve $y = (x^2 - 4)$ m with a constant speed of 5 m/s. Determine the point on the curve where the maximum magnitude of acceleration occurs and compute its value.

SOLUTION



$$y = (x^2 - 4)$$

$$a_t = \frac{dv}{dt} = 0,$$

To obtain maximum $a = a_n$, ρ must be a minimum.

This occurs at:

$$x = 0, \quad y = -4 \text{ m}$$ **Ans.**

Hence,

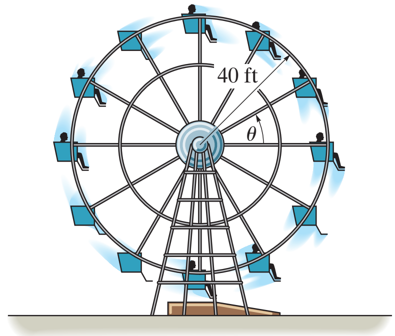
$$\left.\frac{dy}{dx}\right|_{x=0} = 2x = 0; \quad \frac{d^2y}{dx^2} = 2$$

$$\rho_{min} = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{[1 + 0]^{\frac{3}{2}}}{|2|} = \frac{1}{2}$$

$$(a)_{max} = (a_n)_{max} = \frac{v^2}{\rho_{min}} = \frac{5^2}{\frac{1}{2}} = 50 \text{ m/s}^2$$ **Ans.**

***12–144.**

The Ferris wheel turns such that the speed of the passengers is increased by $\dot{v} = (4t)\ \text{ft/s}^2$, where t is in seconds. If the wheel starts from rest when $\theta = 0°$, determine the magnitudes of the velocity and acceleration of the passengers when the wheel turns $\theta = 30°$.



SOLUTION

$$\int_0^v dv = \int_0^t 4t\,dt$$

$$v = 2t^2$$

$$\int_0^s ds = \int_0^t 2t^2\,dt$$

$$s = \frac{2}{3}t^3$$

When $s = \frac{\pi}{6}(40)$ ft, $\quad \frac{\pi}{6}(40) = \frac{2}{3}t^3 \quad t = 3.1554$ s

$v = 2(3.1554)^2 = 19.91$ ft/s $= 19.9$ ft/s **Ans.**

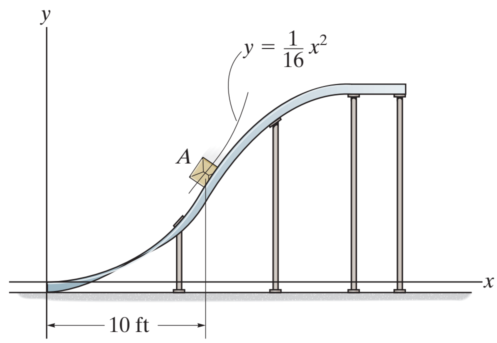
$$a_t = \dot{v} = 4t\,|_{t=3.1554\text{ s}} = 12.62\ \text{ft/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{19.91^2}{40} = 9.91\ \text{ft/s}^2$$

$a = \sqrt{a_t^2 + a_n^2} = \sqrt{12.62^2 + 9.91^2} = 16.0\ \text{ft/s}^2$ **Ans.**

12–145.

If the speed of the crate at A is 15 ft/s, which is increasing at a rate $\dot{v} = 3\text{ ft/s}^2$, determine the magnitude of the acceleration of the crate at this instant.



SOLUTION

Radius of Curvature:

$$y = \frac{1}{16}x^2$$

$$\frac{dy}{dx} = \frac{1}{8}x$$

$$\frac{d^2y}{dx^2} = \frac{1}{8}$$

Thus,

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(\frac{1}{8}x\right)^2\right]^{3/2}}{\left|\frac{1}{8}\right|}\right|_{x=10\text{ ft}} = 32.82\text{ ft}$$

Acceleration:

$$a_t = \dot{v} = 3\text{ft/s}^2$$

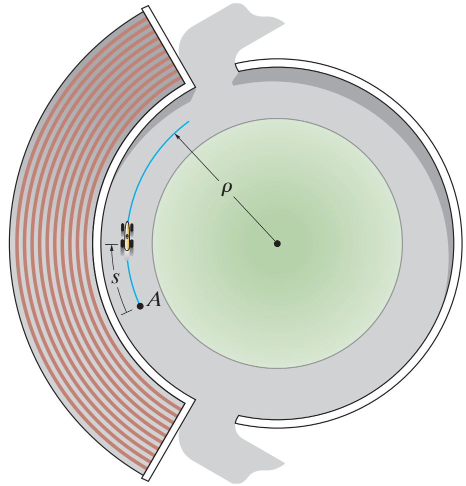
$$a_n = \frac{v^2}{\rho} = \frac{15^2}{32.82} = 6.856\text{ ft/s}^2$$

The magnitude of the crate's acceleration at A is

$$a = \sqrt{a_t^{\,2} + a_n^{\,2}} = \sqrt{3^2 + 6.856^2} = 7.48\text{ ft/s}^2$$ **Ans.**

12–146.

The race car has an initial speed $v_A = 15$ m/s at A. If it increases its speed along the circular track at the rate $a_t = (0.4s)$ m/s^2, where s is in meters, determine the time needed for the car to travel 20 m. Take $\rho = 150$ m.



SOLUTION

$$a_t = 0.4s = \frac{v\,dv}{ds}$$

$$a\,ds = v\,dv$$

$$\int_0^s 0.4s\,ds = \int_{15}^{v} v\,dv$$

$$\frac{0.4s^2}{2}\bigg|_0^s = \frac{v^2}{2}\bigg|_{15}^{v}$$

$$\frac{0.4s^2}{2} = \frac{v^2}{2} - \frac{225}{2}$$

$$v^2 = 0.4s^2 + 225$$

$$v = \frac{ds}{dt} = \sqrt{0.4s^2 + 225}$$

$$\int_0^s \frac{ds}{\sqrt{0.4s^2 + 225}} = \int_0^t dt$$

$$\int_0^s \frac{ds}{\sqrt{s^2 + 562.5}} = 0.632\,456t$$

$$1\text{n}\,(s + \sqrt{s^2 + 562.5})\bigg|_0^s = 0.632\,456t$$

$$1\text{n}\,(s + \sqrt{s^2 + 562.5}) - 3.166\,196 = 0.632\,456t$$

At $s = 20$ m,

$t = 1.21$ s **Ans.**

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12–147.

A boy sits on a merry-go-round so that he is always located at $r = 8$ ft from the center of rotation. The merry-go-round is originally at rest, and then due to rotation the boy's speed is increased at $2\ \text{ft/s}^2$. Determine the time needed for his acceleration to become $4\ \text{ft/s}^2$.

SOLUTION

$$a = \sqrt{a_n^2 + a_t^2}$$

$$a_t = 2$$

$$v = v_0 + a_c t$$

$$v = 0 + 2t$$

$$a_n = \frac{v^2}{\rho} = \frac{(2t)^2}{8}$$

$$4 = \sqrt{(2)^2 + \left(\frac{(2t)^2}{8}\right)^2}$$

$$16 = 4 + \frac{16\,t^4}{64}$$

$t = 2.63$ s **Ans.**

***12–148.**

A particle travels along the path $y = a + bx + cx^2$, where a, b, c are constants. If the speed of the particle is constant, $v = v_0$, determine the x and y components of velocity and the normal component of acceleration when $x = 0$.

SOLUTION

$y = a + bx + cx^2$

$\dot{y} = b\dot{x} + 2\,c\,x\,\dot{x}$

$\ddot{y} = b\ddot{x} + 2c\,(\dot{x})^2 + 2c\,x\ddot{x}$

When $x = 0$, $\quad \dot{y} = b\,\dot{x}$

$v_0^2 + \dot{x}^2 + b^2\,\dot{x}^2$

$$v_x = \dot{x} = \frac{v_0}{\sqrt{1 + b^2}} \qquad \textbf{Ans.}$$

$$v_y = \frac{v_0\,b}{\sqrt{1 + b^2}} \qquad \textbf{Ans.}$$

$$a_n = \frac{v_0^2}{\rho}$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|}$$

$$\frac{dy}{dx} = b + 2c\,x$$

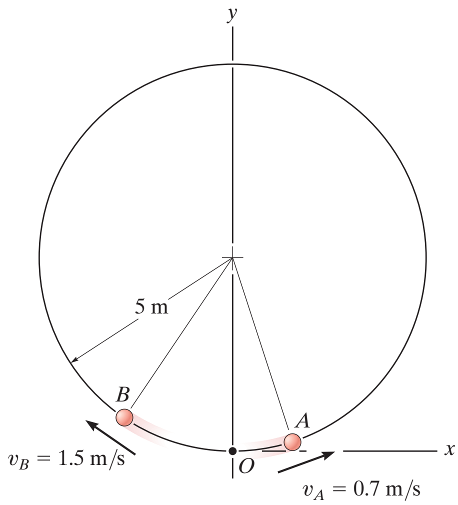
$$\frac{d^2y}{dx^2} = 2\,c$$

At $x = 0$, $\quad \rho = \dfrac{(1 + b^2)^{3/2}}{2\,c}$

$$a_n = \frac{2\,c\,v_0^2}{(1 + b^2)^{3/2}} \qquad \textbf{Ans.}$$

12–149.

The two particles A and B start at the origin O and travel in opposite directions along the circular path at constant speeds $v_A = 0.7\ \text{m/s}$ and $v_B = 1.5\ \text{m/s}$, respectively. Determine in $t = 2\ \text{s}$, (a) the displacement along the path of each particle, (b) the position vector to each particle, and (c) the shortest distance between the particles.



SOLUTION

(a) $s_A = 0.7(2) = 1.40\ \text{m}$ **Ans.**

$s_B = 1.5(2) = 3\ \text{m}$ **Ans.**

(b) $\theta_A = \dfrac{1.40}{5} = 0.280\ \text{rad.} = 16.04^\circ$

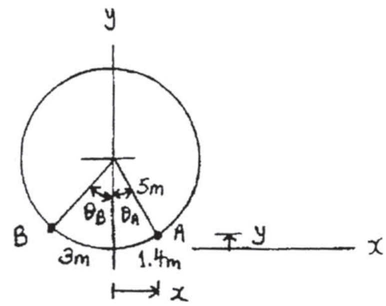
$\theta_B = \dfrac{3}{5} = 0.600\ \text{rad.} = 34.38^\circ$

For A

$x = 5 \sin 16.04^\circ = 1.382 = 1.38\ \text{m}$

$y = 5(1 - \cos 16.04^\circ) = 0.1947 = 0.195\ \text{m}$

$\mathbf{r}_A = \{1.38\mathbf{i} + 0.195\mathbf{j}\}\ \text{m}$ **Ans.**



For B

$x = -5 \sin 34.38^\circ = -2.823 = -2.82\ \text{m}$

$y = 5(1 - \cos 34.38^\circ) = 0.8734 = 0.873\ \text{m}$

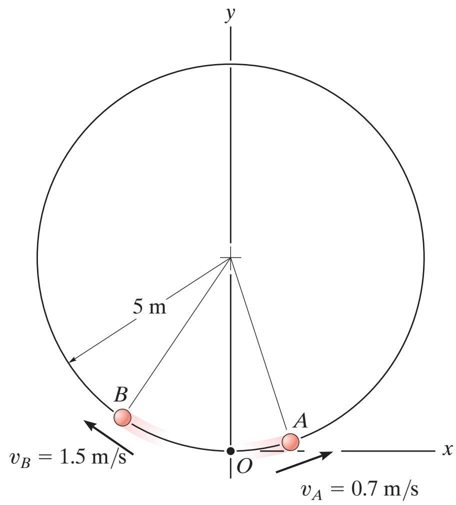
$\mathbf{r}_B = \{-2.82\mathbf{i} + 0.873\mathbf{j}\}\ \text{m}$ **Ans.**

(c) $\Delta\mathbf{r} = \mathbf{r}_B - \mathbf{r}_A = \{-4.20\mathbf{i} + 0.678\mathbf{j}\}\ \text{m}$

$\Delta r = \sqrt{(-4.20)^2 + (0.678)^2} = 4.26\ \text{m}$ **Ans.**

12–150.

The two particles A and B start at the origin O and travel in opposite directions along the circular path at constant speeds $v_A = 0.7\ \text{m/s}$ and $v_B = 1.5\ \text{m/s}$, respectively. Determine the time when they collide and the magnitude of the acceleration of B just before this happens.



SOLUTION

$s_t = 2\pi(5) = 31.4159\ \text{m}$

$s_A = 0.7\,t$

$s_B = 1.5\,t$

Require

$s_A + s_B = 31.4159$

$0.7\,t + 1.5\,t = 31.4159$

$t = 14.28\ \text{s} = 14.3\ \text{s}$ **Ans.**

$a_B = \dfrac{v_B^2}{\rho} = \dfrac{(1.5)^2}{5} = 0.45\ \text{m/s}^2$ **Ans.**

12–151.

The position of a particle traveling along a curved path is $s = (3t^3 - 4t^2 + 4)$ m, where t is in seconds. When $t = 2$ s, the particle is at a position on the path where the radius of curvature is 25 m. Determine the magnitude of the particle's acceleration at this instant.

SOLUTION

Velocity:

$$v = \frac{d}{dt}\left(3t^3 - 4t^2 + 4\right) = \left(9t^2 - 8t\right)\text{m/s}$$

When $t = 2$ s,

$$v|_{t=2\text{ s}} = 9\left(2^2\right) - 8(2) = 20\text{ m/s}$$

Acceleration:

$$a_t = \frac{dv}{ds} = \frac{d}{dt}\left(9t^2 - 8t\right) = \left(18t - 8\right)\text{m/s}^2$$

$$a_t|_{t=2\text{ s}} = 18(2) - 8 = 28\text{ m/s}^2$$

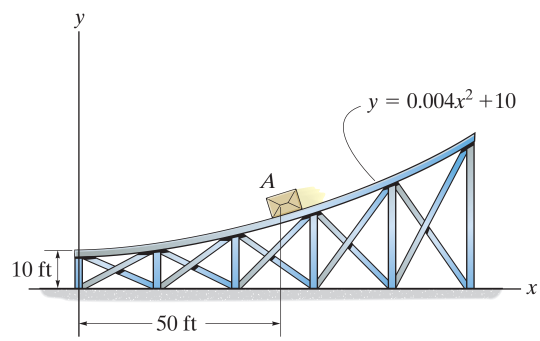
$$a_n = \frac{\left(v|_{t=2\text{s}}\right)^2}{\rho} = \frac{20^2}{25} = 16\text{ m/s}^2$$

Thus,

$$a = \sqrt{a_t^{\,2} + a_n^{\,2}} = \sqrt{28^2 + 16^2} = 32.2\text{ m/s}^2 \qquad \textbf{Ans.}$$

***12–152.**

If the speed of the box at point A on the track is 30 ft/s which is increasing at the rate of $\dot{v} = 5\ \text{ft/s}^2$, determine the magnitude of the acceleration of the box at this instant.



SOLUTION

Radius of Curvature:

$$y = 0.004x^2 + 10$$

$$\frac{dy}{dx} = 0.008x$$

$$\frac{d^2y}{dx^2} = 0.008$$

Thus,

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + (0.008x)^2\right]^{3/2}}{|0.008|}\right|_{x=50\text{ ft}} = 156.17\text{ ft}$$

Acceleration:

$$a_n = \frac{v^2}{\rho} = \frac{30^2}{156.17} = 5.763\ \text{ft/s}^2$$

$$a_t = \dot{v} = 5\ \text{ft/s}^2$$

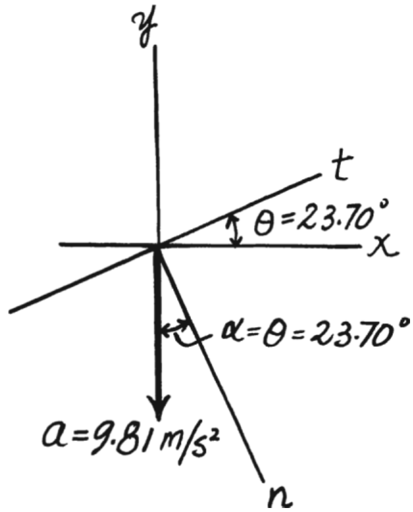
The magnitude of the box's acceleration at A is therefore

$$a = \sqrt{a_t^{\,2} + a_n^{\,2}} = \sqrt{5^2 + 5.763^2} = 7.63\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

■ **12–153.**

A go-cart moves along a circular track of radius 100 ft such that its speed for a short period of time, $0 \le t \le 4$ s, is $v = 60(1 - e^{-t^2})$ ft/s. Determine the magnitude of its acceleration when $t = 2$ s. How far has it traveled in $t = 2$ s? Use Simpson's rule with $n = 50$ to evaluate the integral.

SOLUTION



$$v = 60(1 - e^{-t^2})$$

$$a_t = \frac{dv}{dt} = 60(-e^{-t^2})(-2t) = 120\,t\,e^{-t^2}$$

$$a_t|_{t=2} = 120(2)e^{-4} = 4.3958$$

$$v|_{t=2} = 60(1 - e^{-4}) = 58.9011$$

$$a_n = \frac{(58.9011)^2}{100} = 34.693$$

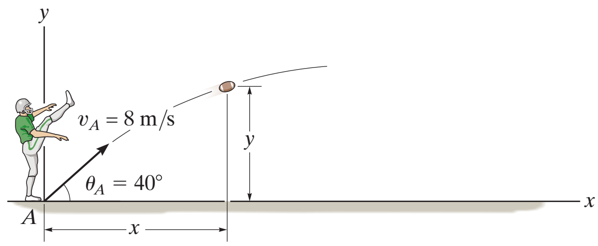
$$a = \sqrt{(4.3958)^2 + (34.693)^2} = 35.0 \text{ m/s}^2$$ **Ans.**

$$\int_0^s ds = \int_0^2 60(1 - e^{-t^2})\,dt$$

$$s = 67.1 \text{ ft}$$ **Ans.**

■ **12–154.**

The ball is kicked with an initial speed $v_A = 8$ m/s at an angle $\theta_A = 40°$ with the horizontal. Find the equation of the path, $y = f(x)$, and then determine the normal and tangential components of its acceleration when $t = 0.25$ s.



SOLUTION

Horizontal Motion: The horizontal component of velocity is $(v_0)_x = 8 \cos 40° = 6.128$ m/s and the initial horizontal and final positions are $(s_0)_x = 0$ and $s_x = x$, respectively.

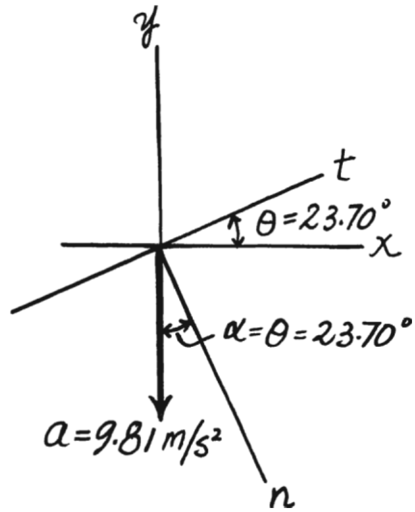
$(\overset{+}{\rightarrow})$ $$s_x = (s_0)_x + (v_0)_x t$$

$$x = 0 + 6.128t \qquad \textbf{(1)}$$

Vertical Motion: The vertical component of initial velocity is $(v_0)_y = 8 \sin 40° = 5.143$ m/s. The initial and final vertical positions are $(s_0)_y = 0$ and $s_y = y$, respectively.

$(+\uparrow)$ $$s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$y = 0 + 5.143t + \frac{1}{2}(-9.81)\left(t^2\right) \qquad \textbf{(2)}$$



Eliminate t from Eqs (1) and (2),we have

$$y = \{0.8391x - 0.1306x^2\}\text{ m} = \{0.839x - 0.131x^2\}\text{ m} \qquad \textbf{Ans.}$$

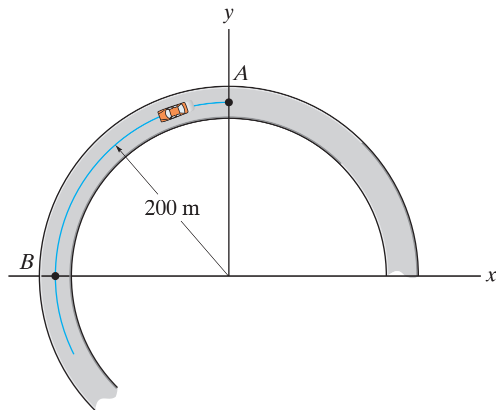
Acceleration: When $t = 0.25$ s, from Eq. (1), $x = 0 + 6.128(0.25) = 1.532$ m. Here, $\frac{dy}{dx} = 0.8391 - 0.2612x$. At $x = 1.532$ m, $\frac{dy}{dx} = 0.8391 - 0.2612(1.532) = 0.4389$ and the tangent of the path makes an angle $\theta = \tan^{-1} 0.4389 = 23.70°$ with the x axis. The magnitude of the acceleration is $a = 9.81$ m/s^2 and is directed downward. From the figure, $\alpha = 23.70°$. Therefore,

$$a_t = -a \sin \alpha = -9.81 \sin 23.70° = -3.94 \text{ m/s}^2 \qquad \textbf{Ans.}$$

$$a_n = a \cos \alpha = 9.81 \cos 23.70° = 8.98 \text{ m/s}^2 \qquad \textbf{Ans.}$$

12–155.

The race car travels around the circular track with a speed of 16 m/s. When it reaches point A it increases its speed at $a_t = (\frac{4}{3}v^{1/4})\ \text{m/s}^2$, where v is in m/s. Determine the magnitudes of the velocity and acceleration of the car when it reaches point B. Also, how much time is required for it to travel from A to B?



SOLUTION

$$a_t = \frac{4}{3}v^{\frac{1}{4}}$$

$$dv = a_t\ dt$$

$$dv = \frac{4}{3}v^{\frac{1}{4}}\ dt$$

$$\int_{16}^{v} 0.75\frac{dv}{v^{\frac{1}{4}}} = \int_0^t dt$$

$$v^{\frac{3}{4}}\Big|_{16}^{v} = t$$

$$v^{\frac{3}{4}} - 8 = t$$

$$v = (t + 8)^{\frac{4}{3}}$$

$$ds = v\,dt$$

$$\int_0^s ds = \int_0^t (t + 8)^{\frac{4}{3}}\,dt$$

$$s = \frac{3}{7}(t + 8)^{\frac{7}{3}}\Big|_0^t$$

$$s = \frac{3}{7}(t + 8)^{\frac{7}{3}} - 54.86$$

For $s = \dfrac{\pi}{2}(200) = 100\pi = \dfrac{3}{7}(t + 8)^{\frac{7}{3}} - 54.86$

$t = 10.108\ \text{s} = 10.1\ \text{s}$ **Ans.**

$v = (10.108 + 8)^{\frac{4}{3}} = 47.551 = 47.6\ \text{m/s}$ **Ans.**

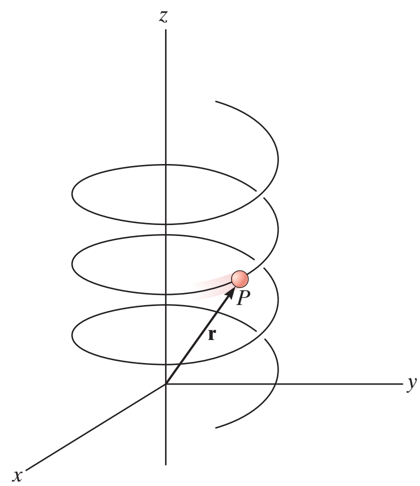
$$a_t = \frac{4}{3}(47.551)^{\frac{1}{4}} = 3.501\ \text{m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{(47.551)^2}{200} = 11.305\ \text{m/s}^2$$

$a = \sqrt{(3.501)^2 + (11.305)^2} = 11.8\ \text{m/s}^2$ **Ans.**

***12–156.**

A particle P travels along an elliptical spiral path such that its position vector $\mathbf{r}$ is defined by $\mathbf{r} = \{2\cos(0.1t)\mathbf{i} + 1.5\sin(0.1t)\mathbf{j} + (2t)\mathbf{k}\}$ m, where t is in seconds and the arguments for the sine and cosine are given in radians. When $t = 8$ s, determine the coordinate direction angles α, β, and γ, which the binormal axis to the osculating plane makes with the x, y, and z axes. *Hint:* Solve for the velocity $\mathbf{v}_P$ and acceleration $\mathbf{a}_P$ of the particle in terms of their $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ components. The binormal is parallel to $\mathbf{v}_P \times \mathbf{a}_P$. Why?



SOLUTION

$$\mathbf{r}_P = 2\cos(0.1t)\mathbf{i} + 1.5\sin(0.1t)\mathbf{j} + 2t\mathbf{k}$$

$$\mathbf{v}_P = \dot{\mathbf{r}} = -0.2\sin(0.1t)\mathbf{i} + 0.15\cos(0.1t)\mathbf{j} + 2\mathbf{k}$$

$$\mathbf{a}_P = \ddot{\mathbf{r}} = -0.02\cos(0.1t)\mathbf{i} - 0.015\sin(0.1t)\mathbf{j}$$

When $t = 8$ s,

$$\mathbf{v}_P = -0.2\sin(0.8\text{ rad})\mathbf{i} + 0.15\cos(0.8\text{ rad})\mathbf{j} + 2\mathbf{k} = -0.143\,47\mathbf{i} + 0.104\,51\mathbf{j} + 2\mathbf{k}$$

$$\mathbf{a}_P = -0.02\cos(0.8\text{ rad})\mathbf{i} - 0.015\sin(0.8\text{ rad})\mathbf{j} = -0.013\,934\mathbf{i} - 0.010\,76\mathbf{j}$$

Since the binormal vector is perpendicular to the plane containing the n–t axis, and $\mathbf{a}_p$ and $\mathbf{v}_p$ are in this plane, then by the definition of the cross product,

$$\mathbf{b} = \mathbf{v}_P \times \mathbf{a}_P = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.14\,347 & 0.104\,51 & 2 \\ -0.013\,934 & -0.010\,76 & 0 \end{vmatrix} = 0.021\,52\mathbf{i} - 0.027\,868\mathbf{j} + 0.003\mathbf{k}$$

$$b = \sqrt{(0.02152)^2 + (-0.027868)^2 + (0.003)^2} = 0.035\,338$$

$$\mathbf{u}_b = 0.608\,99\mathbf{i} - 0.788\,62\mathbf{j} + 0.085\mathbf{k}$$

$\alpha = \cos^{-1}(0.608\,99) = 52.5°$ **Ans.**

$\beta = \cos^{-1}(-0.788\,62) = 142°$ **Ans.**

$\gamma = \cos^{-1}(0.085) = 85.1°$ **Ans.**

Note: The direction of the binormal axis may also be specified by the unit vector $\mathbf{u}_{b'} = -\mathbf{u}_b$, which is obtained from $\mathbf{b}' = \mathbf{a}_p \times \mathbf{v}_p$.

For this case, $\alpha = 128°$, $\beta = 37.9°$, $\gamma = 94.9°$ **Ans.**

12–157.

The motion of a particle is defined by the equations $x = (2t + t^2)$ m and $y = (t^2)$ m, where t is in seconds. Determine the normal and tangential components of the particle's velocity and acceleration when $t = 2$ s.

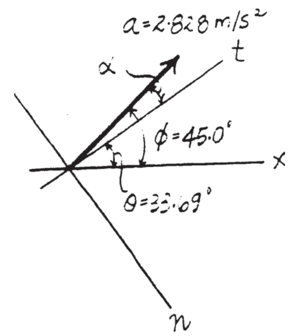
SOLUTION

Velocity: Here, $\mathbf{r} = \left\{\left(2t + t^2\right)\mathbf{i} + t^2\mathbf{j}\right\}$ m. To determine the velocity $\mathbf{v}$, apply Eq. 12–7.

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \{(2 + 2t)\,\mathbf{i} + 2t\mathbf{j}\}\ \text{m/s}$$

When $t = 2$ s, $\mathbf{v} = [2 + 2(2)]\mathbf{i} + 2(2)\mathbf{j} = \{6\mathbf{i} + 4\mathbf{j}\}$ m/s. Then $v = \sqrt{6^2 + 4^2}$ $= 7.21$ m/s. Since the velocity is always directed tangent to the path,

$$v_n = 0 \qquad \text{and} \qquad v_t = 7.21\ \text{m/s} \qquad \textbf{Ans.}$$



The velocity $\mathbf{v}$ makes an angle $\theta = \tan^{-1}\dfrac{4}{6} = 33.69°$ with the x axis.

Acceleration: To determine the acceleration $\mathbf{a}$, apply Eq. 12–9.

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \{2\mathbf{i} + 2\mathbf{j}\}\ \text{m/s}^2$$

Then

$$a = \sqrt{2^2 + 2^2} = 2.828\ \text{m/s}^2$$

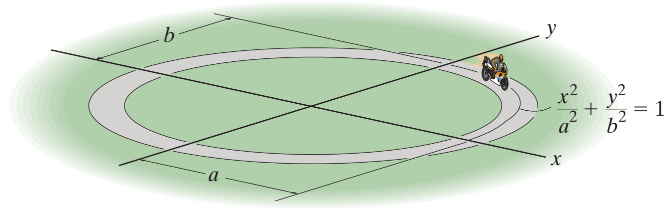
The acceleration $\mathbf{a}$ makes an angle $\phi = \tan^{-1}\dfrac{2}{2} = 45.0°$ with the x axis. From the figure, $\alpha = 45° - 33.69 = 11.31°$. Therefore,

$$a_n = a \sin\alpha = 2.828 \sin 11.31° = 0.555\ \text{m/s}^2 \qquad \textbf{Ans.}$$

$$a_t = a \cos\alpha = 2.828 \cos 11.31° = 2.77\ \text{m/s}^2 \qquad \textbf{Ans.}$$

12–158.

The motorcycle travels along the elliptical track at a constant speed v. Determine the greatest magnitude of the acceleration if $a > b$.



SOLUTION

Acceleration: Differentiating twice the expression $y = \frac{b}{a}\sqrt{a^2 - x^2}$, we have

$$\frac{dy}{dx} = -\frac{bx}{a\sqrt{a^2 - x^2}}$$

$$\frac{d^2y}{dx^2} = -\frac{ab}{(a^2 - x^2)^{3/2}}$$

The radius of curvature of the path is

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1 + \left(-\frac{bx}{a\sqrt{a^2 - x^2}}\right)^2\right]^{3/2}}{\left|-\frac{ab}{(a^2 - x^2)^{3/2}}\right|} = \frac{\left[1 + \frac{b^2x^2}{a^2(a^2 - x^2)}\right]^{3/2}}{\frac{ab}{(a^2 - x^2)^{3/2}}} \qquad \textbf{(1)}$$

To have the maximum normal acceleration, the radius of curvature of the path must be a minimum. By observation, this happens when $y = 0$ and $x = a$. When $x \rightarrow a$, $\frac{b^2x^2}{a^2(a^2 - x^2)} >> 1$. Then, $\left[1 + \frac{b^2x^2}{a^2(a^2 - x^2)}\right]^{3/2} \rightarrow \left[\frac{b^2x^2}{a^2(a^2 - x^2)}\right]^{3/2} = \frac{b^3x^3}{a^3(a^2 - x^2)^{3/2}}$.

Substituting this value into Eq. [1] yields $\rho = \frac{b^2}{a^4}x^3$. At $x = a$,

$$\rho = \frac{b^2}{a^4}\left(a^3\right) = \frac{b^2}{a}$$

To determine the normal acceleration, apply Eq. 12–20.

$$(a_n)_{\max} = \frac{v^2}{\rho} = \frac{v^2}{b^2/a} = \frac{a}{b^2}v^2$$

Since the motorcycle is traveling at a constant speed, $a_t = 0$. Thus,

$$a_{\max} = (a_n)_{\max} = \frac{a}{b^2}v^2 \qquad \textbf{Ans.}$$

12–159.

A particle is moving along a circular path having a radius of 4 in. such that its position as a function of time is given by $\theta = \cos 2t$, where θ is in radians and t is in seconds. Determine the magnitude of the acceleration of the particle when $\theta = 30°$.

SOLUTION

When $\theta = \frac{\pi}{6}$ rad, $\quad \frac{\pi}{6} = \cos 2t \quad t = 0.5099\text{ s}$

$$\dot{\theta} = \frac{d\theta}{dt} = -2\sin 2t\bigg|_{t=0.5099\text{ s}} = -1.7039\text{ rad/s}$$

$$\ddot{\theta} = \frac{d^2\theta}{dt^2} = -4\cos 2t\bigg|_{t=0.5099\text{ s}} = -2.0944\text{ rad/s}^2$$

$$r = 4 \qquad \dot{r} = 0 \qquad \ddot{r} = 0$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 4(-1.7039)^2 = -11.6135\text{ in./s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 4(-2.0944) + 0 = -8.3776\text{ in./s}^2$$

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-11.6135)^2 + (-8.3776)^2} = 14.3\text{ in./s}^2$$ **Ans.**

***12–160.**

A particle travels around a limaçon, defined by the equation $r = b - a\cos\theta$, where a and b are constants. Determine the particle's radial and transverse components of velocity and acceleration as a function of θ and its time derivatives.

SOLUTION

$$r = b - a\cos\theta$$

$$\dot{r} = a\sin\theta\dot{\theta}$$

$$\dot{r} = a\cos\theta\dot{\theta}^2 + a\sin\theta\ddot{\theta}$$

$$v_r = \dot{r} = a\sin\theta\dot{\theta}$$ **Ans.**

$$v_\theta = r\theta = (b - a\cos\theta)\dot{\theta}$$ **Ans.**

$$a_r = \ddot{r} - r\dot{\theta}^2 = a\cos\theta\dot{\theta}^2 + a\sin\theta\ddot{\theta} - (b - a\cos\theta)\dot{\theta}^2$$

$$= (2a\cos\theta - b)\,\dot{\theta}^2 + a\sin\theta\ddot{\theta}$$ **Ans.**

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (b - a\cos\theta)\ddot{\theta} + 2\left(a\sin\theta\dot{\theta}\right)\dot{\theta}$$

$$= (b - a\cos\theta)\ddot{\theta} + 2a\dot{\theta}^2\sin\theta$$ **Ans.**

12–161.

If a particle's position is described by the polar coordinates $r = 4(1 + \sin t)$ m and $\theta = (2e^{-t})$ rad, where t is in seconds and the argument for the sine is in radians, determine the radial and transverse components of the particle's velocity and acceleration when $t = 2$ s.

SOLUTION

When $t = 2$ s,

$r = 4(1+\sin t) = 7.637$

$\dot{r} = 4\cos t = -1.66459$

$\ddot{r} = -4\sin t = -3.6372$

$\theta = 2\,e^{-t}$

$\dot{\theta} = -2\,e^{-t} = -0.27067$

$\ddot{\theta} = 2\,e^{-t} = 0.270665$

$v_r = \dot{r} = -1.66$ m/s **Ans.**

$v_\theta = r\dot{\theta} = 7.637(-0.27067) = -2.07$ m/s **Ans.**

$a_r = \ddot{r} - r(\dot{\theta})^2 = -3.6372 - 7.637(-0.27067)^2 = -4.20\text{ m/s}^2$ **Ans.**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 7.637(0.270665) + 2(-1.66459)(-0.27067) = 2.97\text{ m/s}^2$ **Ans.**

12–162.

An airplane is flying in a straight line with a velocity of 200 mi/h and an acceleration of 3 mi/h^2. If the propeller has a diameter of 6 ft and is rotating at an angular rate of 120 rad/s, determine the magnitudes of velocity and acceleration of a particle located on the tip of the propeller.

SOLUTION

$$v_{Pl} = \left(\frac{200 \text{ mi}}{\text{h}}\right)\left(\frac{5280 \text{ ft}}{1 \text{ mi}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 293.3 \text{ ft/s}$$

$$a_{Pl} = \left(\frac{3 \text{ mi}}{\text{h}^2}\right)\left(\frac{5280 \text{ ft}}{1 \text{ mi}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right)^2 = 0.001\,22 \text{ ft/s}^2$$

$$v_{Pr} = 120(3) = 360 \text{ ft/s}$$

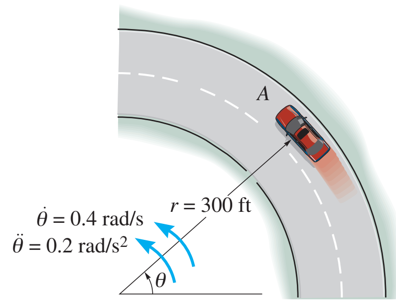
$$v = \sqrt{v_{Pl}^2 + v_{Pr}^2} = \sqrt{(293.3)^2 + (360)^2} = 464 \text{ ft/s}$$ **Ans.**

$$a_{Pr} = \frac{v_{Pr}^2}{\rho} = \frac{(360)^2}{3} = 43\,200 \text{ ft/s}^2$$

$$a = \sqrt{a_{Pl}^2 + a_{Pr}^2} = \sqrt{(0.001\,22)^2 + (43\,200)^2} = 43.2(10^3) \text{ ft/s}^2$$ **Ans.**

12–163.

A car is traveling along the circular curve of radius $r = 300$ ft. At the instant shown, its angular rate of rotation is $\dot{\theta} = 0.4$ rad/s, which is increasing at the rate of $\ddot{\theta} = 0.2$ rad/s^2. Determine the magnitudes of the car's velocity and acceleration at this instant.



SOLUTION

Velocity: Applying Eq. 12–25, we have

$$v_r = \dot{r} = 0 \qquad v_\theta = r\dot{\theta} = 300(0.4) = 120 \text{ ft/s}$$

Thus, the magnitude of the velocity of the car is

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{0^2 + 120^2} = 120 \text{ ft/s} \qquad \textbf{Ans.}$$

Acceleration: Applying Eq. 12–29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 300\left(0.4^2\right) = -48.0 \text{ ft/s}^2$$

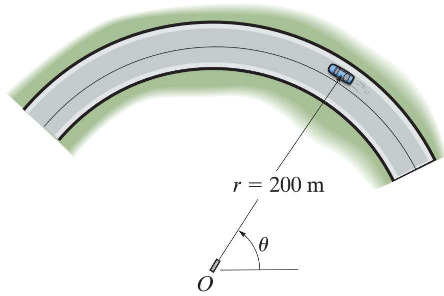
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 300(0.2) + 2(0)(0.4) = 60.0 \text{ ft/s}^2$$

Thus, the magnitude of the acceleration of the car is

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-48.0)^2 + 60.0^2} = 76.8 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

***12–164.**

A radar gun at O rotates with the angular velocity of $\dot{\theta} = 0.1$ rad/s and angular acceleration of $\ddot{\theta} = 0.025$ rad/s^2, at the instant $\theta = 45°$, as it follows the motion of the car traveling along the circular road having a radius of $r = 200$ m. Determine the magnitudes of velocity and acceleration of the car at this instant.



SOLUTION

Time Derivatives: Since r is constant,

$$\dot{r} = \ddot{r} = 0$$

Velocity:

$$v_r = \dot{r} = 0$$

$$v_\theta = r\dot{\theta} = 200(0.1) = 20 \text{ m/s}$$

Thus, the magnitude of the car's velocity is

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{0^2 + 20^2} = 20 \text{ m/s} \qquad \textbf{Ans.}$$

Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 200(0.1^2) = -2 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 200(0.025) + 0 = 5 \text{ m/s}^2$$

Thus, the magnitude of the car's acceleration is

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-2)^2 + 5^2} = 5.39 \text{ m/s}^2 \qquad \textbf{Ans.}$$

12–165.

If a particle moves along a path such that $r = (2\cos t)$ ft and $\theta = (t/2)$ rad, where t is in seconds, plot the path $r = f(\theta)$ and determine the particle's radial and transverse components of velocity and acceleration.

SOLUTION

$$r = 2\cos t \quad \dot{r} = -2\sin t \qquad \ddot{r} = -2\cos t$$

$$\theta = \frac{t}{2} \qquad \dot{\theta} = \frac{1}{2} \qquad \ddot{\theta} = 0$$

$$v_r = \dot{r} = -2\sin t \qquad \textbf{Ans.}$$

$$v_\theta = r\dot{\theta} = (2\cos t)\left(\frac{1}{2}\right) = \cos t \qquad \textbf{Ans.}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -2\cos t - (2\cos t)\left(\frac{1}{2}\right)^2 = -\frac{5}{2}\cos t \qquad \textbf{Ans.}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2\cos t(0) + 2(-2\sin t)\left(\frac{1}{2}\right) = -2\sin t \qquad \textbf{Ans.}$$

12–166.

If a particle's position is described by the polar coordinates $r = (2 \sin 2\theta)$ m and $\theta = (4t)$ rad, where t is in seconds, determine the radial and transverse components of its velocity and acceleration when $t = 1$ s.

SOLUTION

When $t = 1$ s,

$\theta = 4t = 4$

$\dot{\theta} = 4$

$\ddot{\theta} = 0$

$r = 2 \sin 2\theta = 1.9787$

$\dot{r} = 4 \cos 2\theta \, \dot{\theta} = -2.3280$

$\ddot{r} = -8 \sin 2\theta (\dot{\theta})^2 + 8 \cos 2\theta \, \ddot{\theta} = -126.638$

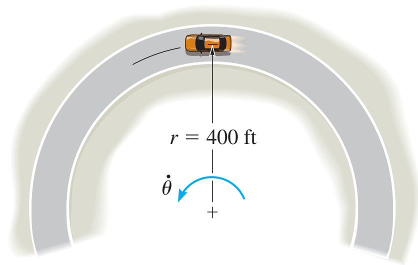
$v_r = \dot{r} = -2.33 \text{ m/s}$ **Ans.**

$v_\theta = r\dot{\theta} = 1.9787(4) = 7.91 \text{ m/s}$ **Ans.**

$a_r = \ddot{r} - r(\dot{\theta})^2 = -126.638 - (1.9787)(4)^2 = -158 \text{ m/s}^2$ **Ans.**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 1.9787(0) + 2(-2.3280)(4) = -18.6 \text{ m/s}^2$ **Ans.**

12–167.

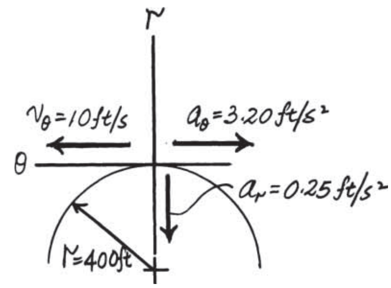


The car travels along the circular curve having a radius $r = 400$ ft. At the instant shown, its angular rate of rotation is $\dot{\theta} = 0.025$ rad/s, which is decreasing at the rate $\ddot{\theta} = -0.008$ rad/s^2. Determine the radial and transverse components of the car's velocity and acceleration at this instant and sketch these components on the curve.

SOLUTION

$r = 400 \qquad \dot{r} = 0 \qquad \ddot{r} = 0$

$\dot{\theta} = 0.025 \qquad \theta = -0.008$



$v_r = \dot{r} = 0$ **Ans.**

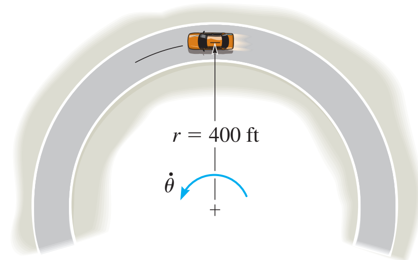
$v_\theta = r\dot{\theta} = 400(0.025) = 10$ ft/s **Ans.**

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 400(0.025)^2 = -0.25$ ft/s^2 **Ans.**

$a_\theta = r\,\ddot{\theta} + 2\dot{r}\dot{\theta} = 400(-0.008) + 0 = -3.20$ ft/s^2 **Ans.**

***12–168.**

The car travels along the circular curve of radius $r = 400$ ft with a constant speed of $v = 30$ ft/s. Determine the angular rate of rotation $\dot{\theta}$ of the radial line r and the magnitude of the car's acceleration.



SOLUTION

$r = 400 \text{ ft} \qquad \dot{r} = 0 \qquad \ddot{r} = 0$

$v_r = \dot{r} = 0 \qquad v_\theta = r\dot{\theta} = 400\left(\dot{\theta}\right)$

$$v = \sqrt{(0)^2 + \left(400\,\dot{\theta}\right)^2} = 30$$

$\dot{\theta} = 0.075 \text{ rad/s}$ **Ans.**

$\ddot{\theta} = 0$

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 400(0.075)^2 = -2.25 \text{ ft/s}^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 400(0) + 2(0)(0.075) = 0$

$a = \sqrt{(-2.25)^2 + (0)^2} = 2.25 \text{ ft/s}^2$ **Ans.**

12–169.

The time rate of change of acceleration is referred to as the *jerk*, which is often used as a means of measuring passenger discomfort. Calculate this vector, $\dot{\mathbf{a}}$, in terms of its cylindrical components, using Eq. 12–32.

SOLUTION

$$\mathbf{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\mathbf{u}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\mathbf{u}_\theta + \ddot{z}\,\mathbf{u}_z$$

$$\ddot{\mathbf{a}} = \left(\dddot{r} - \dot{r}\dot{\theta}^2 - 2r\dot{\theta}\ddot{\theta}\right)\mathbf{u}_r + \left(\ddot{r} - r\dot{\theta}^2\right)\dot{\mathbf{u}}_r + \left(\dot{r}\ddot{\theta} + r\dddot{\theta} + 2\ddot{r}\dot{\theta} + 2\dot{r}\ddot{\theta}\right)\mathbf{u}_\theta + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\dot{\mathbf{u}}_\theta + \dddot{z}\,\mathbf{u}_z + \ddot{z}\dot{\mathbf{u}}_z$$

But, $\mathbf{u}_r = \dot{\theta}\mathbf{u}_\theta \quad \dot{\mathbf{u}}_\theta = -\dot{\theta}\mathbf{u}_r \quad \dot{\mathbf{u}}_z = 0$

Substituting and combining terms yields

$$\dot{\mathbf{a}} = \left(\dddot{r} - 3r\dot{\theta}^2 - 3r\dot{\theta}\ddot{\theta}\right)\mathbf{u}_r + \left(3\dot{r}\ddot{\theta} + r\dddot{\theta} + 3\ddot{r}\dot{\theta} - r\dot{\theta}^3\right)\mathbf{u}_\theta + \left(\dddot{z}\right)\mathbf{u}_z \qquad \textbf{Ans.}$$

12–170.

A particle is moving along a circular path having a radius of 6 in. such that its position as a function of time is given by $\theta = \sin 3t$, where θ is in radians, the argument for the sine are in radians, and t is in seconds. Determine the acceleration of the particle at $\theta = 30°$. The particle starts from rest at $\theta = 0°$.

SOLUTION

$r = 6$ in., $\qquad \dot{r} = 0, \qquad \ddot{r} = 0$

$\theta = \sin 3t$

$\dot{\theta} = 3 \cos 3t$

$\ddot{\theta} = -9 \sin 3t$

At $\theta = 30°$,

$\dfrac{30°}{180°}\pi = \sin 3t$

$t = 10.525$ s

Thus,

$\dot{\theta} = 2.5559$ rad/s

$\ddot{\theta} = -4.7124$ rad/s^2

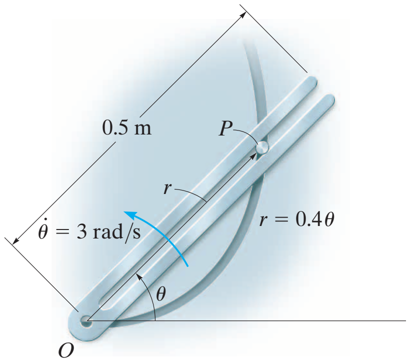
$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 6(2.5559)^2 = -39.196$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 6(-4.7124) + 0 = -28.274$

$a = \sqrt{(-39.196)^2 + (-28.274)^2} = 48.3$ in./s^2 **Ans.**

12–171.

The slotted link is pinned at O, and as a result of the constant angular velocity $\dot{\theta} = 3$ rad/s it drives the peg P for a short distance along the spiral guide $r = (0.4\theta)$ m, where θ is in radians. Determine the radial and transverse components of the velocity and acceleration of P at the instant $\theta = \pi/3$ rad.



SOLUTION

$\dot{\theta} = 3$ rad/s $\quad r = 0.4\,\theta$

$$\dot{r} = 0.4\,\dot{\theta}$$

$$\ddot{r} = 0.4\,\ddot{\theta}$$

At $\theta = \dfrac{\pi}{3}$, $\quad r = 0.4189$

$$\dot{r} = 0.4(3) = 1.20$$

$$\ddot{r} = 0.4(0) = 0$$

$v = \dot{r} = 1.20$ m/s **Ans.**

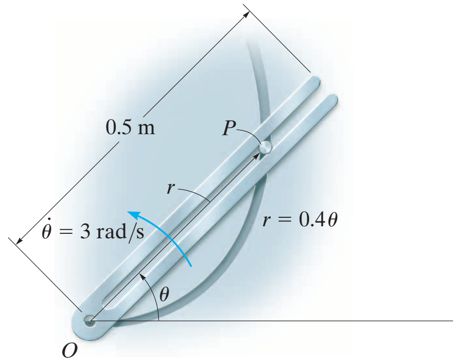
$v_\theta = r\dot{\theta} = 0.4189(3) = 1.26$ m/s **Ans.**

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 0.4189(3)^2 = -3.77$ m/s^2 **Ans.**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(1.20)(3) = 7.20$ m/s^2 **Ans.**

12–172.

Solve Prob. 12–171 if the slotted link has an angular acceleration $\ddot{\theta} = 8 \text{ rad/s}^2$ when $\dot{\theta} = 3 \text{ rad/s}$ at $\dot{\theta} = \pi/3$ rad.



SOLUTION

$\dot{\theta} = 3 \text{ rad/s} \qquad r = 0.4\,\theta$

$\dot{r} = 0.4\,\dot{\theta}$

$\ddot{r} = 0.4\,\ddot{\theta}$

$\theta = \dfrac{\pi}{3}$

$\dot{\theta} = 3$

$\ddot{\theta} = 8$

$r = 0.4189$

$\dot{r} = 1.20$

$\ddot{r} = 0.4(8) = 3.20$

$v_r = \dot{r} = 1.20 \text{ m/s}$ **Ans.**

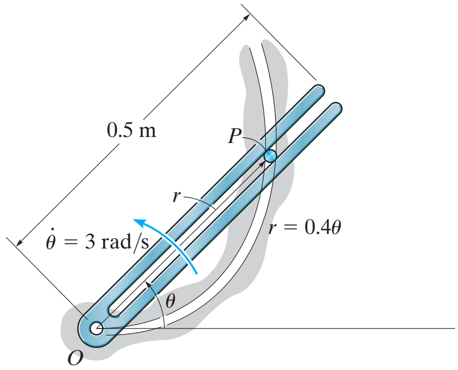
$v_\theta = r\,\dot{\theta} = 0.4189(3) = 1.26 \text{ m/s}$ **Ans.**

$a_r = \ddot{r} - r\dot{\theta}^2 = 3.20 - 0.4189(3)^2 = -0.570 \text{ m/s}^2$ **Ans.**

$a_\theta = r\,\ddot{\theta} + 2\,\dot{r}\dot{\theta} = 0.4189(8) + 2(1.20)(3) = 10.6 \text{ m/s}^2$ **Ans.**

12–173.

The slotted link is pinned at O, and as a result of the constant angular velocity $\dot{\theta} = 3$ rad/s it drives the peg P for a short distance along the spiral guide $r = (0.4\theta)$ m, where θ is in radians. Determine the velocity and acceleration of the particle at the instant it leaves the slot in the link, i.e., when $r = 0.5$ m.



SOLUTION

$r = 0.4\,\theta$

$\dot{r} = 0.4\,\dot{\theta}$

$\ddot{r} = 0.4\,\ddot{\theta}$

$\dot{\theta} = 3$

$\ddot{\theta} = 0$

At $r = 0.5$ m,

$$\theta = \frac{0.5}{0.4} = 1.25 \text{ rad}$$

$\dot{r} = 1.20$

$\ddot{r} = 0$

$v_r = \dot{r} = 1.20$ m/s **Ans.**

$v_\theta = r\,\dot{\theta} = 0.5(3) = 1.50$ m/s **Ans.**

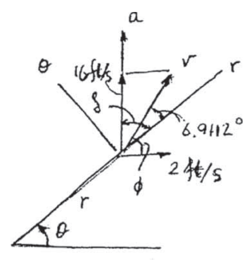
$a_r = \ddot{r} - r(\dot{\theta})^2 = 0 - 0.5(3)^2 = -4.50 \text{ m/s}^2$ **Ans.**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(1.20)(3) = 7.20 \text{ m/s}^2$ **Ans.**

12–174.

A particle moves in the x–y plane such that its position is defined by $\mathbf{r} = \{2t\mathbf{i} + 4t^2\mathbf{j}\}$ ft, where t is in seconds. Determine the radial and transverse components of the particle's velocity and acceleration when $t = 2$ s.

SOLUTION



$\mathbf{r} = 2t\mathbf{i} + 4\,t^2\mathbf{j}|_{t=2} = 4\mathbf{i} + 16\mathbf{j}$

$\mathbf{v} = 2\mathbf{i} + 8\,t\mathbf{j}|_{t=2} = 2\mathbf{i} + 16\mathbf{j}$

$\mathbf{a} = 8\mathbf{j}$

$$\theta = \tan^{-1}\left(\frac{16}{4}\right) = 75.964^\circ$$

$$v = \sqrt{(2)^2 + (16)^2} = 16.1245 \text{ ft/s}$$

$$\phi = \tan^{-1}\left(\frac{16}{2}\right) = 82.875^\circ$$

$a = 8 \text{ ft/s}^2$

$\phi - \theta = 6.9112^\circ$

$v_r = 16.1245 \cos 6.9112^\circ = 16.0 \text{ ft/s}$ **Ans.**

$v_\theta = 16.1245 \sin 6.9112^\circ = 1.94 \text{ ft/s}$ **Ans.**

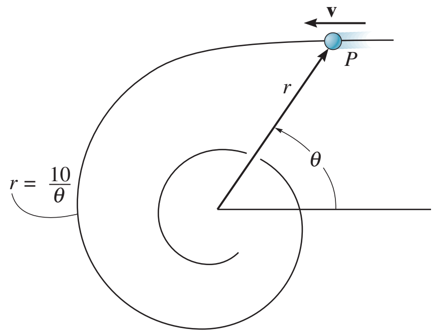
$\delta = 90^\circ - \theta = 14.036^\circ$

$a_r = 8 \cos 14.036^\circ = 7.76 \text{ ft/s}^2$ **Ans.**

$a_\theta = 8 \sin 14.036^\circ = 1.94 \text{ ft/s}^2$ **Ans.**

12–175.

A particle P moves along the spiral path $r = (10/\theta)$ ft, where θ is in radians. If it maintains a constant speed of $v = 20$ ft/s, determine the magnitudes v_r and v_θ as functions of θ and evaluate each at $\theta = 1$ rad.



SOLUTION

$$r = \frac{10}{\theta}$$

$$\dot{r} = -\left(\frac{10}{\theta^2}\right)\dot{\theta}$$

Since $v^2 = \dot{r}^2 + \left(r\dot{\theta}\right)^2$

$$(20)^2 = \left(\frac{10^2}{\theta^4}\right)\dot{\theta}^2 + \left(\frac{10^2}{\theta^2}\right)\dot{\theta}^2$$

$$(20)^2 = \left(\frac{10^2}{\theta^4}\right)(1 + \theta^2)\dot{\theta}^2$$

Thus, $\dot{\theta} = \dfrac{2\theta^2}{\sqrt{1 + \theta^2}}$

$$v_r = \dot{r} = -\left(\frac{10}{\theta^2}\right)\left(\frac{2\theta^2}{\sqrt{1 + \theta^2}}\right) = -\frac{20}{\sqrt{1 + \theta^2}}$$ **Ans.**

$$v_\theta = r\dot{\theta} = \left(\frac{10}{\theta}\right)\left(\frac{2\theta^2}{\sqrt{1 + \theta^2}}\right) = \frac{20\theta}{\sqrt{1 + \theta^2}}$$ **Ans.**

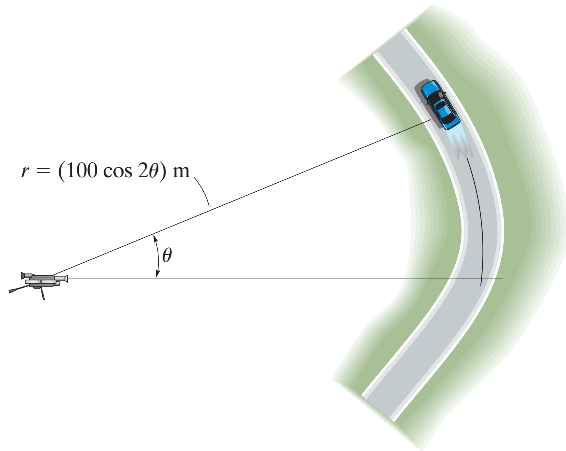
When $\theta = 1$ rad

$$v_r = \left(-\frac{20}{\sqrt{2}}\right) = -14.1 \text{ ft/s}$$ **Ans.**

$$v_\theta = \left(\frac{20}{\sqrt{2}}\right) = 14.1 \text{ ft/s}$$ **Ans.**

***12–176.**

The driver of the car maintains a constant speed of 40 m/s. Determine the angular velocity of the camera tracking the car when $\theta = 15°$.



SOLUTION

Time Derivatives:

$$r = 100 \cos 2\theta$$

$$\dot{r} = (-200 (\sin 2\theta)\dot{\theta}) \text{ m/s}$$

At $\theta = 15°$,

$$r\big|_{\theta=15°} = 100 \cos 30° = 86.60 \text{ m}$$

$$\dot{r}\big|_{\theta=15°} = -200 \sin 30°\dot{\theta} = -100\dot{\theta} \text{ m/s}$$

Velocity: Referring to Fig. *a*, $v_r = -40 \cos \phi$ and $v_\theta = 40 \sin \phi$.

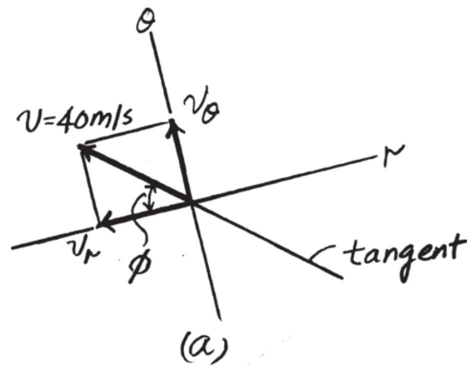
$$v_r = \dot{r}$$

$$-40 \cos \phi = -100\dot{\theta} \qquad \textbf{(1)}$$

and

$$v_\theta = r\dot{\theta}$$

$$40 \sin \phi = 86.60\dot{\theta} \qquad \textbf{(2)}$$



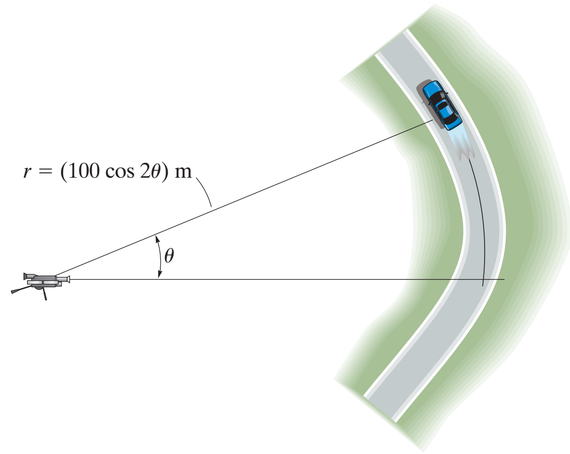
Solving Eqs. (1) and (2) yields

$$\phi = 40.89°$$

$$\dot{\theta} = 0.3024 \text{ rad/s} = 0.302 \text{ rad/s} \qquad \textbf{Ans.}$$

12–177.

When $\theta = 15°$, the car has a speed of 50 m/s which is increasing at 6 m/s^2. Determine the angular velocity of the camera tracking the car at this instant.



SOLUTION

Time Derivatives:

$$r = 100 \cos 2\theta$$

$$\dot{r} = (-200 (\sin 2\theta)\, \dot{\theta})\ \text{m/s}$$

$$\ddot{r} = -200\left[(\sin\ 2\theta)\, \ddot{\theta} + 2 (\cos 2\theta)\, \dot{\theta}^2\right]\ \text{m/s}^2$$

At $\theta = 15°$,

$$r\big|_{\theta=15°} = 100 \cos 30° = 86.60\ \text{m}$$

$$\dot{r}\big|_{\theta=15°} = -200 \sin 30° \dot{\theta} = -100\dot{\theta}\ \text{m/s}$$

$$\dot{r}\big|_{\theta=15°} = -200\left[\sin 30° \ddot{\theta} + 2 \cos 30° \dot{\theta}^2\right] = \left(-100\ddot{\theta} - 346.41\dot{\theta}^2\right)\ \text{m/s}^2$$

Velocity: Referring to Fig. a, $v_r = -50 \cos \phi$ and $v_\theta = 50 \sin \phi$. Thus,

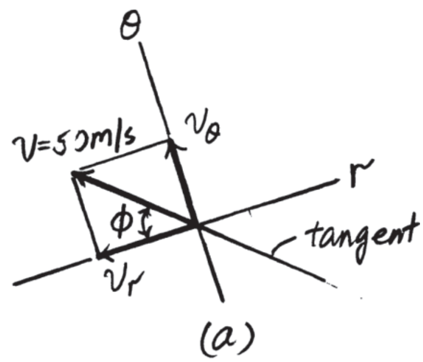
$$v_r = \dot{r}$$

$$-50 \cos \phi = -100\dot{\theta} \qquad \textbf{(1)}$$

and

$$v_\theta = r\dot{\theta}$$

$$50 \sin \phi = 86.60\dot{\theta} \qquad \textbf{(2)}$$



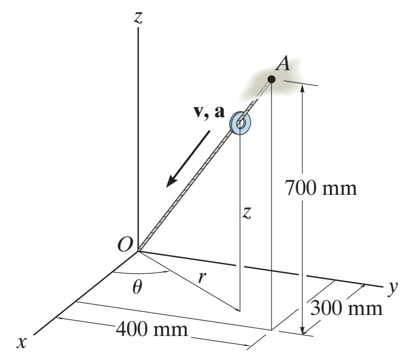
Solving Eqs. (1) and (2) yields

$$\phi = 40.89°$$

$$\dot{\theta} = 0.378\ \text{rad/s} \qquad \textbf{Ans.}$$

12–178.

The small washer slides down the cord OA. When it is at the midpoint, its speed is 200 mm/s and its acceleration is 10 mm/s^2. Express the velocity and acceleration of the washer at this point in terms of its cylindrical components.



SOLUTION

$$OA = \sqrt{(400)^2 + (300)^2 + (700)^2} = 860.23 \text{ mm}$$

$$OB = \sqrt{(400)^2 + (300)^2} = 500 \text{ mm}$$

$$v_r = (200)\left(\frac{500}{860.23}\right) = 116 \text{ mm/s}$$

$$v_\theta = 0$$

$$v_z = (200)\left(\frac{700}{860.23}\right) = 163 \text{ mm/s}$$

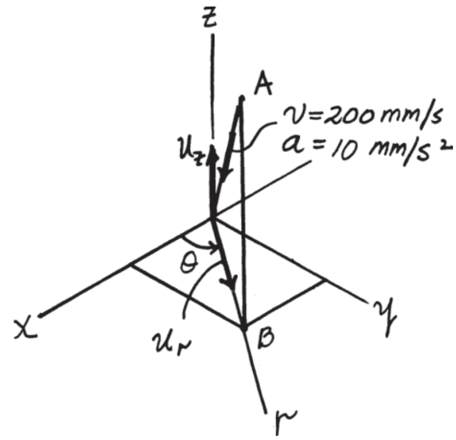
Thus, $\mathbf{v} = \{-116\mathbf{u}_r - 163\mathbf{u}_z\}$ mm/s **Ans.**

$$a_r = 10\left(\frac{500}{860.23}\right) = 5.81$$

$$a_\theta = 0$$

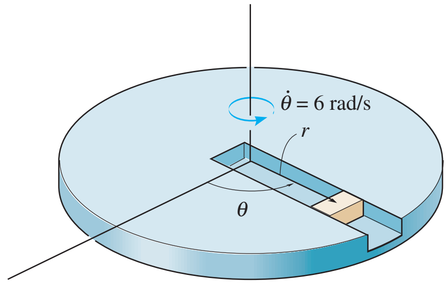
$$a_z = 10\left(\frac{700}{860.23}\right) = 8.14$$

Thus, $\mathbf{a} = \{-5.81\mathbf{u}_r - 8.14\mathbf{u}_z\}$ mm/s^2 **Ans.**



12–179.

A block moves outward along the slot in the platform with a speed of $\dot{r} = (4t)$ m/s, where t is in seconds. The platform rotates at a constant rate of 6 rad/s. If the block starts from rest at the center, determine the magnitudes of its velocity and acceleration when $t = 1$ s.



SOLUTION

$\dot{r} = 4t|_{t=1} = 4 \qquad \ddot{r} = 4$

$\dot{\theta} = 6 \qquad \ddot{\theta} = 0$

$$\int_0^1 dr = \int_0^1 4t\,dt$$

$r = 2t^2\big]_0^1 = 2$ m

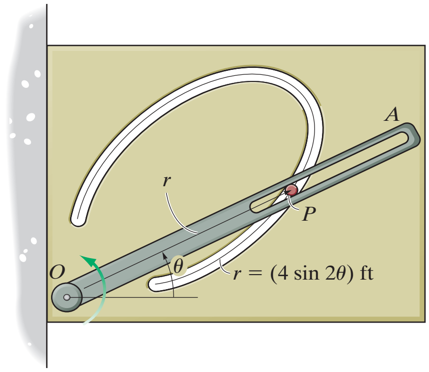
$v = \sqrt{(\dot{r})^2 + (r\dot{\theta})^2} = \sqrt{(4)^2 + [2(6)]^2} = 12.6$ m/s **Ans.**

$a = \sqrt{(\ddot{r} - r\dot{\theta}^2)^2 + (r\ddot{\theta} + 2\dot{r}\dot{\theta})^2} = \sqrt{[4 - 2(6)^2]^2 + [0 + 2(4)(6)]^2}$ **Ans.**

$= 83.2$ m/s^2

***12–180.**

Pin P is constrained to move along the curve defined by the lemniscate $r = (4 \sin 2\theta)$ ft. If the slotted arm OA rotates counterclockwise with a constant angular velocity of $\dot{\theta} = 1.5$ rad/s, determine the magnitudes of the velocity and acceleration of peg P when $\theta = 60°$.



SOLUTION

Time Derivatives:

$$r = 4 \sin 2\theta$$

$$\dot{r} = (8(\cos 2\theta)\dot{\theta})\ \text{ft/s} \qquad \dot{\theta} = 1.5\ \text{rad/s}$$

$$\ddot{r} = 8[(\cos 2\theta)\ddot{\theta} - 2\sin 2\theta(\dot{\theta})^2]\ \text{ft/s}^2 \qquad \ddot{\theta} = 0$$

When $\theta = 60°$,

$$r|_{\theta=60°} = 4 \sin 120° = 3.464\ \text{ft}$$

$$\dot{r}|_{\theta=60°} = 8 \cos 120°(1.5) = -6\ \text{ft/s}$$

$$\ddot{r}|_{\theta=60°} = 8[0 - 2 \sin 120°(1.5^2)] = -31.18\ \text{ft/s}^2$$

Velocity:

$$v_r = \dot{r} = -6\ \text{ft/s} \qquad v_\theta = r\dot{\theta} = 3.464(1.5) = 5.196\ \text{ft/s}$$

Thus, the magnitude of the peg's velocity is

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{(-6)^2 + 5.196^2} = 7.94\ \text{ft/s} \qquad \textbf{Ans.}$$

Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 = -31.18 - 3.464(1.5^2) = -38.97\ \text{ft/s}^2$$

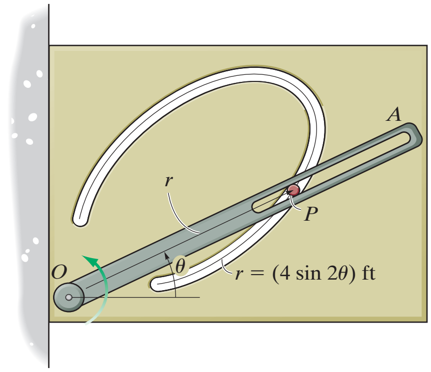
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(-6)(1.5) = -18\ \text{ft/s}^2$$

Thus, the magnitude of the peg's acceleration is

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-38.97)^2 + (-18)^2} = 42.9\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

12–181.

Pin P is constrained to move along the curve defined by the lemniscate $r = (4\sin 2\theta)$ ft. If the angular position of the slotted arm OA is defined by $\theta = (3t^{3/2})$ rad, determine the magnitudes of the velocity and acceleration of the pin P when $\theta = 60°$.



SOLUTION

Time Derivatives:

$$r = 4\sin 2\theta$$

$$\dot{r} = (8(\cos 2\theta)\dot{\theta})\ \text{ft/s}$$

$$\ddot{r} = 8[(\cos 2\theta)\ddot{\theta} - 2(\sin 2\theta)\dot{\theta}^2]\ \text{ft/s}^2$$

When $\theta = 60° = \dfrac{\pi}{3}$ rad,

$$\frac{\pi}{3} = 3t^{3/2} \qquad t = 0.4958\,\text{s}$$

Thus, the angular velocity and angular acceleration of arm OA when $\theta = \dfrac{\pi}{3}$ rad $(t = 0.4958\,\text{s})$ are

$$\dot{\theta} = \frac{9}{2}t^{1/2}\bigg|_{t=0.4958\,\text{s}} = 3.168\ \text{rad/s}$$

$$\ddot{\theta} = \frac{9}{4}t^{1/2}\bigg|_{t=0.4958\,\text{s}} = 3.196\ \text{rad/s}^2$$

Thus,

$$r|_{\theta=60°} = 4\sin 120° = 3.464\ \text{ft}$$

$$\dot{r}|_{\theta=60°} = 8\cos 120°(3.168) = -12.67\ \text{ft/s}$$

$$\ddot{r}|_{\theta=60°} = 8[\cos 120°(3.196) - 2\sin 120°(3.168^2)] = -151.89\ \text{ft/s}^2$$

Velocity:

$$v_r = \dot{r} = -12.67\ \text{ft/s} \qquad v_\theta = r\dot{\theta} = 3.464(3.168) = 10.98\ \text{ft/s}$$

Thus, the magnitude of the peg's velocity is

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{(-12.67)^2 + 10.98^2} = 16.8\ \text{ft/s} \qquad \textbf{Ans.}$$

Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 = -151.89 - 3.464(3.168^2) = -186.67\ \text{ft/s}^2$$

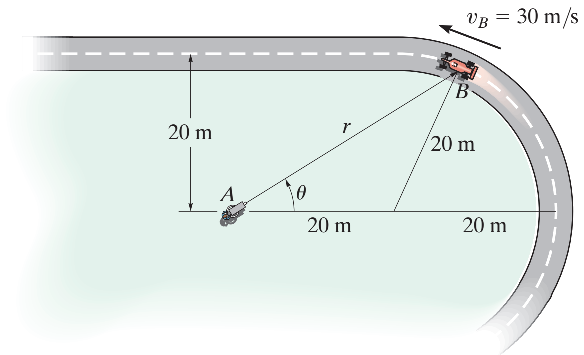
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 3.464(3.196) + 2(-12.67)(3.168) = -69.24\ \text{ft/s}^2$$

Thus, the magnitude of the peg's acceleration is

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-186.67)^2 + (-69.24)^2} = 199\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

12–182.

A cameraman standing at A is following the movement of a race car, B, which is traveling around a curved track at a constant speed of 30 m/s. Determine the angular rate $\dot{\theta}$ at which the man must turn in order to keep the camera directed on the car at the instant $\theta = 30°$.



SOLUTION

$$r = 2(20)\cos\theta = 40\cos\theta$$

$$\dot{r} = -(40\sin\theta)\dot{\theta}$$

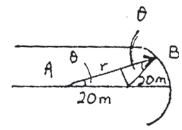
$$\mathbf{v} = \dot{r}\,\mathbf{u}_r + r\,\dot{\theta}\,\mathbf{u}_\theta$$

$$v = \sqrt{(\dot{r})^2 + (r\,\dot{\theta})^2}$$

$$(30)^2 = (-40\sin\theta)^2(\dot{\theta})^2 + (40\cos\theta)^2(\dot{\theta})^2$$

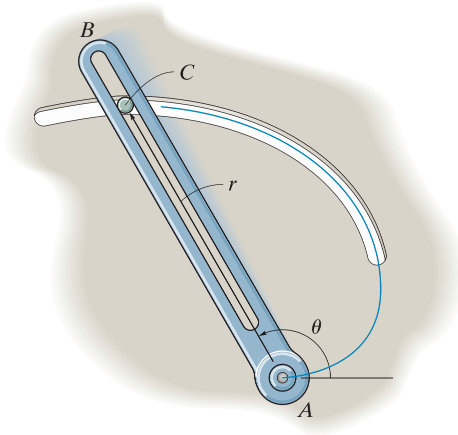
$$(30)^2 = (40)^2[\sin^2\theta + \cos^2\theta](\dot{\theta})^2$$

$$\dot{\theta} = \frac{30}{40} = 0.75 \text{ rad/s} \qquad \textbf{Ans.}$$



12–183.

The slotted arm AB drives pin C through the spiral groove described by the equation $r = a\theta$. If the angular velocity is constant at $\dot{\theta}$, determine the radial and transverse components of velocity and acceleration of the pin.



SOLUTION

Time Derivatives: Since $\dot{\theta}$ is constant, then $\ddot{\theta} = 0$.

$$r = a\theta \qquad \dot{r} = a\dot{\theta} \qquad \ddot{r} = a\ddot{\theta} = 0$$

Velocity: Applying Eq. 12–25, we have

$$v_r = \dot{r} = a\dot{\theta}$$ **Ans.**

$$v_\theta = r\dot{\theta} = a\theta\dot{\theta}$$ **Ans.**

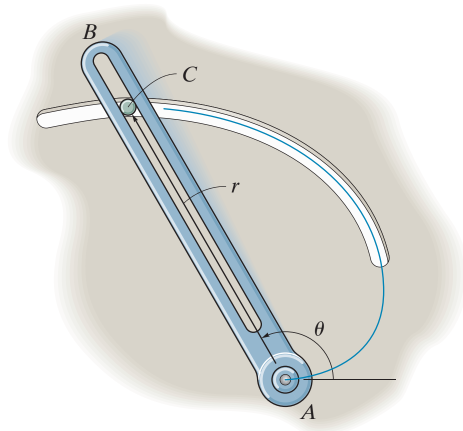
Acceleration: Applying Eq. 12–29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - a\theta\dot{\theta}^2 = -a\theta\dot{\theta}^2$$ **Ans.**

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(a\dot{\theta})(\dot{\theta}) = 2a\dot{\theta}^2$$ **Ans.**

***12–184.**

The slotted arm AB drives pin C through the spiral groove described by the equation $r = (1.5\,\theta)$ ft, where θ is in radians. If the arm starts from rest when $\theta = 60°$ and is driven at an angular velocity of $\dot{\theta} = (4t)$ rad/s, where t is in seconds, determine the radial and transverse components of velocity and acceleration of the pin C when $t = 1$ s.



SOLUTION

Time Derivatives: Here, $\dot{\theta} = 4t$ and $\ddot{\theta} = 4$ rad/s^2.

$$r = 1.5\theta \qquad \dot{r} = 1.5\dot{\theta} = 1.5(4t) = 6t \qquad \ddot{r} = 1.5\ddot{\theta} = 1.5(4) = 6 \text{ ft/s}^2$$

Velocity: Integrate the angular rate, $\int_{\frac{\pi}{3}}^{\theta} d\theta = \int_0^t 4t\,dt$, we have $\theta = \frac{1}{3}(6t^2 + \pi)$ rad.

Then, $r = \left\{\frac{1}{2}(6t^2 + \pi)\right\}$ ft. At $t = 1$ s, $r = \frac{1}{2}\left[6(1^2) + \pi\right] = 4.571$ ft, $\dot{r} = 6(1) = 6.00$ ft/s.

and $\dot{\theta} = 4(1) = 4$ rad/s. Applying Eq. 12–25, we have

$$v_r = \dot{r} = 6.00 \text{ ft/s} \qquad \textbf{Ans.}$$

$$v_\theta = r\dot{\theta} = 4.571\,(4) = 18.3 \text{ ft/s} \qquad \textbf{Ans.}$$

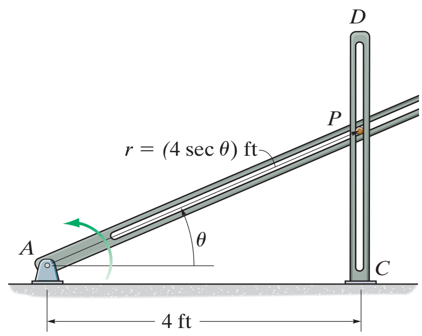
Acceleration: Applying Eq. 12–29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = 6 - 4.571\left(4^2\right) = -67.1 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 4.571(4) + 2(6)\,(4) = 66.3 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

12–185.

If the slotted arm AB rotates counterclockwise with a constant angular velocity of $\dot{\theta} = 2$ rad/s, determine the magnitudes of the velocity and acceleration of peg P at $\theta = 30°$. The peg is constrained to move in the slots of the fixed bar CD and rotating bar AB.



SOLUTION

Time Derivatives:

$$r = 4\sec\theta$$

$$\dot{r} = (4\sec\theta(\tan\theta)\dot{\theta})\text{ ft/s} \qquad \dot{\theta} = 2\text{ rad/s}$$

$$\ddot{r} = 4[\sec\theta(\tan\theta)\ddot{\theta} + \dot{\theta}(\sec\theta(\sec^2\theta)\dot{\theta} + \tan\theta\sec\theta(\tan\theta)\dot{\theta})] \qquad \ddot{\theta} = 0$$

$$= 4[\sec\theta(\tan\theta)\ddot{\theta} + \dot{\theta}^2(\sec 3\theta + \tan^2\theta\sec\theta)]\text{ ft/s}^2$$

When $\theta = 30°$,

$$r|_{\theta=30°} = 4\sec 30° = 4.619\text{ ft}$$

$$\dot{r}|_{\theta=30°} = (4\sec 30°\tan 30°)(2) = 5.333\text{ ft/s}$$

$$\ddot{r}|_{\theta=30°} = 4[0 + 2^2(\sec^3 30° + \tan^2 30°\sec 30°)] = 30.79\text{ ft/s}^2$$

Velocity:

$$v_r = \dot{r} = 5.333\text{ ft/s} \qquad v_\theta = r\dot{\theta} = 4.619(2) = 9.238\text{ ft/s}$$

Thus, the magnitude of the peg's velocity is

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{5.333^2 + 9.238^2} = 10.7\text{ ft/s} \qquad \textbf{Ans.}$$

Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 = 30.79 - 4.619(2^2) = 12.32\text{ ft/s}^2$$

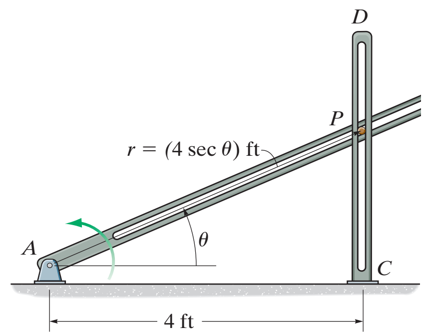
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(5.333)(2) = 21.23\text{ ft/s}^2$$

Thus, the magnitude of the peg's acceleration is

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{12.32^2 + 21.23^2} = 24.6\text{ ft/s}^2 \qquad \textbf{Ans.}$$

12–186.

The peg is constrained to move in the slots of the fixed bar CD and rotating bar AB. When $\theta = 30°$, the angular velocity and angular acceleration of arm AB are $\dot{\theta} = 2$ rad/s and $\ddot{\theta} = 3$ rad/s^2, respectively. Determine the magnitudes of the velocity and acceleration of the peg P at this instant.



SOLUTION

Time Derivatives:

$$r = 4\sec\theta$$

$$\dot{r} = (4\sec\theta(\tan\theta)\dot{\theta})\text{ ft/s} \qquad \dot{\theta} = 2\text{ rad/s}$$

$$\ddot{r} = 4[\sec\theta(\tan\theta)\ddot{\theta} + \dot{\theta}(\sec\theta\sec^2\theta\dot{\theta} + \tan\theta\sec\theta(\tan\theta)\dot{\theta})] \qquad \ddot{\theta} = 3\text{ rad/s}^2$$

$$= 4[\sec\theta(\tan\theta)\ddot{\theta} + \dot{\theta}^2(\sec^3\theta° + \tan^2\theta°\sec\theta°)]\text{ ft/s}^2$$

When $\theta = 30°$,

$$r|_{\theta=30°} = 4\sec 30° = 4.619\text{ ft}$$

$$\dot{r}|_{\theta=30°} = (4\sec 30°\tan 30°)(2) = 5.333\text{ ft/s}$$

$$\ddot{r}|_{\theta=30°} = 4[(\sec 30°\tan 30°)(3) + 2^2(\sec^3 30° + \tan^2 30°\sec 30°)] = 38.79\text{ ft/s}^2$$

Velocity:

$$v_r = \dot{r} = 5.333\text{ ft/s} \qquad v_\theta = r\dot{\theta} = 4.619(2) = 9.238\text{ ft/s}$$

Thus, the magnitude of the peg's velocity is

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{5.333^2 + 9.238^2} = 10.7\text{ ft/s} \qquad \textbf{Ans.}$$

Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 = 38.79 - 4.619(2^2) = 20.32\text{ ft/s}^2$$

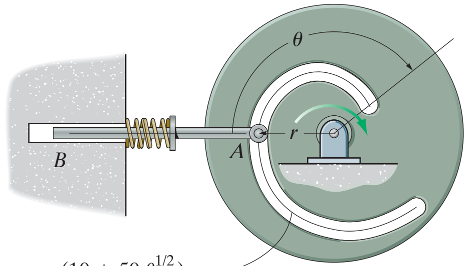
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 4.619(3) + 2(5.333)(2) = 35.19\text{ ft/s}^2$$

Thus, the magnitude of the peg's acceleration is

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{20.32^2 + 35.19^2} = 40.6\text{ ft/s}^2 \qquad \textbf{Ans.}$$

12–187.

If the circular plate rotates clockwise with a constant angular velocity of $\dot{\theta} = 1.5$ rad/s, determine the magnitudes of the velocity and acceleration of the follower rod AB when $\theta = 2/3\pi$ rad.



SOLUTION

Time Derivaties:

$$r = \left(10 + 50\theta^{1/2}\right)\text{ mm}$$

$$\dot{r} = 25\theta^{-1/2}\dot{\theta}\text{ mm/s}$$

$$\ddot{r} = 25\left[\theta^{-1/2}\ddot{\theta} - \frac{1}{2}\theta^{-3/2}\dot{\theta}^2\right]\text{mm/s}^2$$

When $\theta = \dfrac{2\pi}{3}$rad,

$$r|_{\theta=\frac{2\pi}{3}} = \left[10 + 50\left(\frac{2\pi}{3}\right)^{1/2}\right] = 82.36\text{ mm}$$

$$\dot{r}|_{\theta=\frac{2\pi}{3}} = 25\left(\frac{2\pi}{3}\right)^{-1/2}(1.5) = 25.91\text{ mm/s}$$

$$\ddot{r}|_{\theta=\frac{2\pi}{3}} = 25\left[0 - \frac{1}{2}\left(\frac{2\pi}{3}\right)^{-3/2}(1.5^2)\right] = -9.279\text{ mm/s}^2$$

Velocity: The radial component gives the rod's velocity.

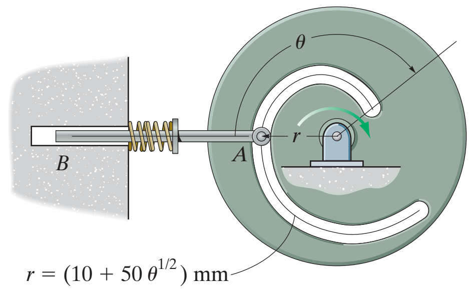
$$v_r = \dot{r} = 25.9\text{ mm/s}$$ **Ans.**

Acceleration: The radial component gives the rod's acceleration.

$$a_r = \ddot{r} - r\dot{\theta}^2 = -9.279 - 82.36\left(1.5^2\right) = -195\text{ mm/s}^2$$ **Ans.**

***12–188.**

When $\theta = 2/3\pi$ rad, the angular velocity and angular acceleration of the circular plate are $\dot{\theta} = 1.5$ rad/s and $\ddot{\theta} = 3$ rad/s^2, respectively. Determine the magnitudes of the velocity and acceleration of the rod AB at this instant.



SOLUTION

Time Derivatives:

$$r = (10 + 50\theta^{1/2})\text{ mm}$$

$$\dot{r} = 25\theta^{-1/2}\dot{\theta}\text{ mm/s}$$

$$\ddot{r} = 25\left[\theta^{-1/2}\ddot{\theta} - \frac{1}{2}\theta^{-3/2}\dot{\theta}^2\right]\text{mm/s}^2$$

When $\theta = \dfrac{2\pi}{3}$ rad,

$$r|_{\theta=\frac{2\pi}{3}} = \left[10 + 50\left(\frac{2\pi}{3}\right)^{1/2}\right] = 82.36\text{ mm}$$

$$\dot{r}|_{\theta=\frac{2\pi}{3}} = 25\left(\frac{2\pi}{3}\right)^{-1/2}(1.5) = 25.91\text{ mm/s}$$

$$\ddot{r}|_{\theta=\frac{2\pi}{3}} = 25\left[\left(\frac{2\pi}{3}\right)^{-1/2}(3) - \frac{1}{2}\left(\frac{2\pi}{3}\right)^{-3/2}(1.5^2)\right] = 42.55\text{ mm/s}^2$$

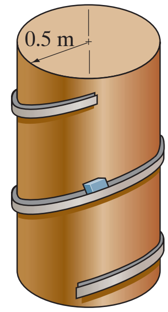
For the rod,

$$v = \dot{r} = 25.9\text{ mm/s}$$ **Ans.**

$$a = \ddot{r} = 42.5\text{ mm/s}^2$$ **Ans.**

12–189.

The box slides down the helical ramp with a constant speed of $v = 2$ m/s. Determine the magnitude of its acceleration. The ramp descends a vertical distance of 1 m for every full revolution. The mean radius of the ramp is $r = 0.5$ m.



SOLUTION

Velocity: The inclination angle of the ramp is $\phi = \tan^{-1}\dfrac{L}{2\pi r} = \tan^{-1}\left[\dfrac{1}{2\pi(0.5)}\right] = 17.66°$.

Thus, from Fig. *a*, $v_\theta = 2\cos 17.66° = 1.906$ m/s and $v_z = 2\sin 17.66° = 0.6066$ m/s. Thus,

$$v_\theta = r\dot{\theta}$$

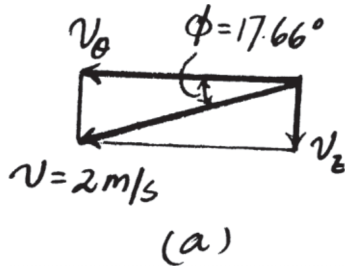
$$1.906 = 0.5\dot{\theta}$$

$$\dot{\theta} = 3.812 \text{ rad/s}$$

Acceleration: Since $r = 0.5$ m is constant, $\dot{r} = \ddot{r} = 0$. Also, $\dot{\theta}$ is constant, then $\ddot{\theta} = 0$. Using the above results,

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 0.5(3.812)^2 = -7.264 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.5(0) + 2(0)(3.812) = 0$$

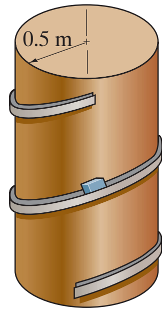


Since $\mathbf{v}_z$ is constant $a_z = 0$. Thus, the magnitude of the box's acceleration is

$$a = \sqrt{{a_r}^2 + {a_\theta}^2 + {a_z}^2} = \sqrt{(-7.264)^2 + 0^2 + 0^2} = 7.26 \text{ m/s}^2 \qquad \textbf{Ans.}$$

12–190.

The box slides down the helical ramp which is defined by $r = 0.5$ m, $\theta = (0.5t^3)$ rad, and $z = (2 - 0.2t^2)$ m, where t is in seconds. Determine the magnitudes of the velocity and acceleration of the box at the instant $\theta = 2\pi$ rad.



SOLUTION

Time Derivatives:

$$r = 0.5 \text{ m}$$

$$\dot{r} = \ddot{r} = 0$$

$$\dot{\theta} = \left(1.5t^2\right) \text{ rad/s} \qquad \ddot{\theta} = (3t) \text{ rad/s}^2$$

$$z = 2 - 0.2t^2$$

$$\dot{z} = (-0.4t) \text{ m/s} \qquad \ddot{z} = -0.4 \text{ m/s}^2$$

When $\theta = 2\pi$ rad,

$$2\pi = 0.5t^3 \qquad t = 2.325 \text{ s}$$

Thus,

$$\dot{\theta}\big|_{t=2.325\text{ s}} = 1.5(2.325)^2 = 8.108 \text{ rad/s}$$

$$\ddot{\theta}\big|_{t=2.325\text{ s}} = 3(2.325) = 6.975 \text{ rad/s}^2$$

$$\dot{z}\big|_{t=2.325\text{ s}} = -0.4(2.325) = -0.92996 \text{ m/s}$$

$$\ddot{z}\big|_{t=2.325\text{ s}} = -0.4 \text{ m/s}^2$$

Velocity:

$$v_r = \dot{r} = 0$$

$$v_\theta = r\dot{\theta} = 0.5(8.108) = 4.05385 \text{ m/s}$$

$$v_z = \dot{z} = -0.92996 \text{ m/s}$$

Thus, the magnitude of the box's velocity is

$$v = \sqrt{{v_r}^2 + {v_\theta}^2 + {v_z}^2} = \sqrt{0^2 + 4.05385^2 + (-0.92996)^2} = 4.16 \text{ m/s} \qquad \textbf{Ans.}$$

Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 0.5(8.108)^2 = -32.867 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.5(6.975) + 2(0)(8.108)^2 = 3.487 \text{ m/s}^2$$

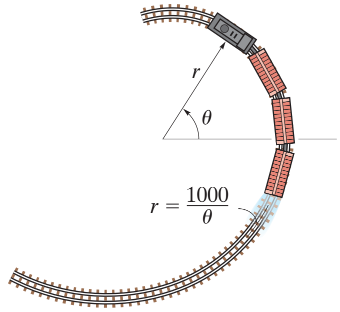
$$a_z = \ddot{z} = -0.4 \text{ m/s}^2$$

Thus, the magnitude of the box's acceleration is

$$a = \sqrt{{a_r}^2 + {a_\theta}^2 + {a_z}^2} = \sqrt{(-32.867)^2 + 3.487^2 + (-0.4)^2} = 33.1 \text{ m/s}^2 \qquad \textbf{Ans.}$$

12–191.

For a short distance the train travels along a track having the shape of a spiral, $r = (1000/\theta)$ m, where θ is in radians. If it maintains a constant speed $v = 20$ m/s, determine the radial and transverse components of its velocity when $\theta = (9\pi/4)$ rad.



SOLUTION

$$r = \frac{1000}{\theta}$$

$$\dot{r} = -\frac{1000}{\theta^2}\dot{\theta}$$

Since

$$v^2 = (\dot{r})^2 + (r\,\dot{\theta})^2$$

$$(20)^2 = \frac{(1000)^2}{\theta^4}(\dot{\theta})^2 + \frac{(1000)^2}{\theta^2}(\dot{\theta})^2$$

$$(20)^2 = \frac{(1000)^2}{\theta^4}(1 + \theta^2)(\dot{\theta})^2$$

Thus,

$$\dot{\theta} = \frac{0.02\theta^2}{\sqrt{1 + \theta^2}}$$

At $\theta = \dfrac{9\pi}{4}$

$$\dot{\theta} = 0.140$$

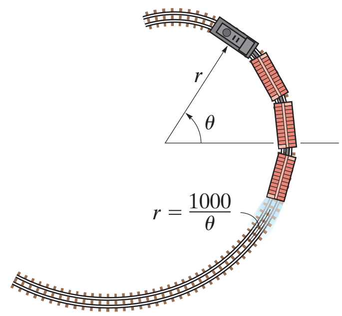
$$\dot{r} = \frac{-1000}{(9\pi/4)^2}(0.140) = -2.80$$

$v_r = \dot{r} = -2.80$ m/s **Ans.**

$v_\theta = r\dot{\theta} = \dfrac{1000}{(9\pi/4)}(0.140) = 19.8$ m/s **Ans.**

12–192.

For a *short distance* the train travels along a track having the shape of a spiral, $r = (1000/\theta)$ m, where θ is in radians. If the angular rate is constant, $\dot{\theta} = 0.2$ rad/s, determine the radial and transverse components of its velocity and acceleration when $\theta = (9\pi/4)$ rad.



SOLUTION

$\dot{\theta} = 0.2$

$\ddot{\theta} = 0$

$r = \dfrac{1000}{\theta}$

$\dot{r} = -1000(\theta^{-2})\dot{\theta}$

$\ddot{r} = 2000(\theta^{-3})(\dot{\theta})^2 - 1000(\theta^{-2})\ddot{\theta}$

When $\theta = \dfrac{9\pi}{4}$

$r = 141.477$

$\dot{r} = -4.002812$

$\ddot{r} = 0.226513$

$v_r = \dot{r} = -4.00$ m/s **Ans.**

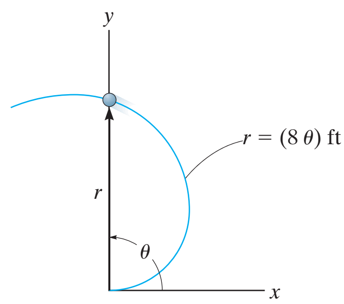
$v_\theta = r\dot{\theta} = 141.477(0.2) = 28.3$ m/s **Ans.**

$a_r = \ddot{r} - r(\dot{\theta})^2 = 0.226513 - 141.477(0.2)^2 = -5.43$ m/s^2 **Ans.**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(-4.002812)(0.2) = -1.60$ m/s^2 **Ans.**

12–193.

A particle moves along an Archimedean spiral $r = (8\theta)$ ft, where θ is given in radians. If $\dot{\theta} = 4$ rad/s (constant), determine the radial and transverse components of the particle's velocity and acceleration at the instant $\theta = \pi/2$ rad. Sketch the curve and show the components on the curve.



SOLUTION

Time Derivatives: Since $\dot{\theta}$ is constant, $\ddot{\theta} = 0$.

$$r = 8\theta = 8\left(\frac{\pi}{2}\right) = 4\pi \text{ ft} \qquad \dot{r} = 8\dot{\theta} = 8(4) = 32.0 \text{ ft/s} \qquad \ddot{r} = 8\ddot{\theta} = 0$$

Velocity: Applying Eq. 12–25, we have

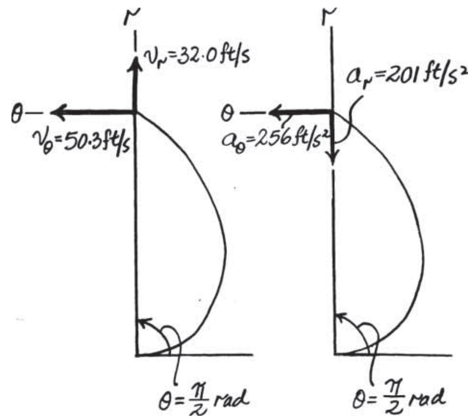
$$v_r = \dot{r} = 32.0 \text{ ft/s}$$ **Ans.**

$$v_\theta = r\dot{\theta} = 4\pi(4) = 50.3 \text{ ft/s}$$ **Ans.**

Acceleration: Applying Eq. 12–29, we have

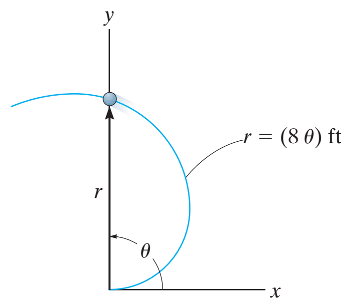
$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 4\pi\left(4^2\right) = -201 \text{ ft/s}^2$$ **Ans.**

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(32.0)(4) = 256 \text{ ft/s}^2$$ **Ans.**



12–194.

Solve Prob. 12–193 if the particle has an angular acceleration $\ddot{\theta} = 5 \text{ rad/s}^2$ when $\dot{\theta} = 4$ rad/s at $\theta = \pi\ 2$ rad.



SOLUTION

Time Derivatives: Here,

$$r = 8\theta = 8\left(\frac{\pi}{2}\right) = 4\pi \text{ ft} \qquad \dot{r} = 8\dot{\theta} = 8(4) = 32.0 \text{ ft/s}$$

$$\ddot{r} = 8\ddot{\theta} = 8(5) = 40 \text{ ft/s}^2$$

Velocity: Applying Eq. 12–25, we have

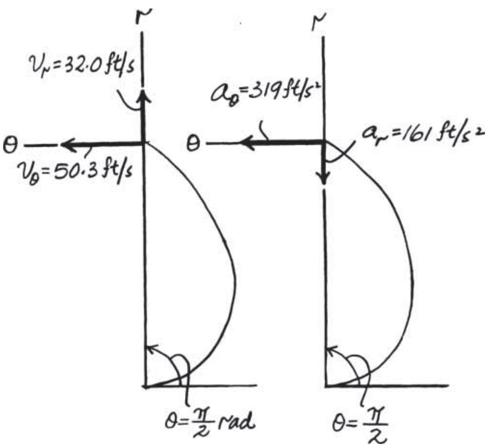
$$v_r = \dot{r} = 32.0 \text{ ft/s} \qquad \textbf{Ans.}$$

$$v_\theta = r\dot{\theta} = 4\pi(4) = 50.3 \text{ ft/s} \qquad \textbf{Ans.}$$

Acceleration: Applying Eq. 12–29, we have

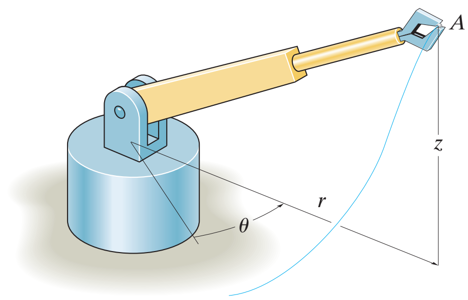
$$a_r = \ddot{r} - r\dot{\theta}^2 = 40 - 4\pi\left(4^2\right) = -161 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 4\pi(5) + 2(32.0)(4) = 319 \text{ ft/s}^2 \qquad \textbf{Ans.}$$



12–195.

The arm of the robot has a length of $r = 3$ ft grip A moves along the path $z = (3 \sin 4\theta)$ ft, where θ is in radians. If $\theta = (0.5t)$ rad, where t is in seconds, determine the magnitudes of the grip's velocity and acceleration when $t = 3$ s.



SOLUTION

$\theta = 0.5\,t \qquad r = 3 \qquad z = 3 \sin 2t$

$\dot{\theta} = 0.5 \qquad \dot{r} = 0 \qquad \dot{z} = 6 \cos 2t$

$\ddot{\theta} = 0 \qquad \ddot{r} = 0 \qquad \ddot{z} = -12 \sin 2t$

At $t = 3$ s,

$z = -0.8382$

$\dot{z} = 5.761$

$\ddot{z} = 3.353$

$v_r = 0$

$v_\theta = 3(0.5) = 1.5$

$v_z = 5.761$

$v = \sqrt{(0)^2 + (1.5)^2 + (5.761)^2} = 5.95 \text{ ft/s}$ **Ans.**

$a_r = 0 - 3(0.5)^2 = -0.75$

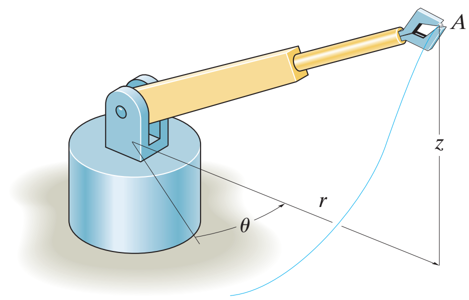
$a_\theta = 0 + 0 = 0$

$a_z = 3.353$

$a = \sqrt{(-0.75)^2 + (0)^2 + (3.353)^2} = 3.44 \text{ ft/s}^2$ **Ans.**

***12–196.**

For a short time the arm of the robot is extending at a constant rate such that $\dot{r} = 1.5$ ft/s when $r = 3$ ft, $z = (4t^2)$ ft, and $\theta = 0.5t$ rad, where t is in seconds. Determine the magnitudes of the velocity and acceleration of the grip A when $t = 3$ s.



SOLUTION

$\theta = 0.5\,t$ rad $\qquad r = 3$ ft $\qquad z = 4\,t^2$ ft

$\dot{\theta} = 0.5$ rad/s $\qquad \dot{r} = 1.5$ ft/s $\qquad \dot{z} = 8\,t$ ft/s

$\ddot{\theta} = 0$ $\qquad \ddot{r} = 0$ $\qquad \ddot{z} = 8$ ft/s^2

At $t = 3$ s,

$\theta = 1.5 \qquad r = 3 \qquad z = 36$

$\dot{\theta} = 0.5 \qquad \dot{r} = 1.5 \qquad \dot{z} = 24$

$\ddot{\theta} = 0 \qquad \ddot{r} = 0 \qquad \ddot{z} = 8$

$v_r = 1.5$

$v_\theta = 3(0.5) = 1.5$

$v_z = 24$

$v = \sqrt{(1.5)^2 + (1.5)^2 + (24)^2} = 24.1$ ft/s **Ans.**

$a_r = 0 - 3(0.5)^2 = -0.75$

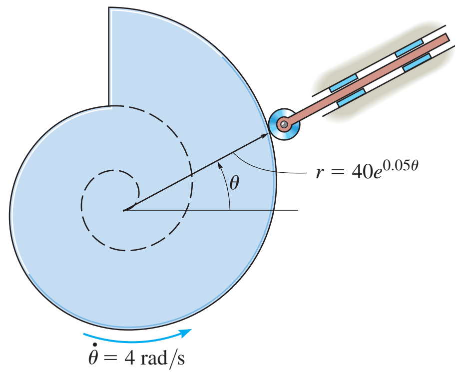
$a_\theta = 0 + 2(1.5)(0.5) = 1.5$

$a_z = 8$

$a = \sqrt{(-0.75)^2 + (1.5)^2 + (8)^2} = 8.17$ ft/s^2 **Ans.**

12–197.

The partial surface of the cam is that of a logarithmic spiral $r = (40e^{0.05\theta})$ mm, where θ is in radians. If the cam is rotating at a constant angular rate of $\dot{\theta} = 4$ rad/s, determine the magnitudes of the velocity and acceleration of the follower rod at the instant $\theta = 30°$.



SOLUTION

$$r = 40e^{0.05\,\theta}$$

$$\dot{r} = 2e^{0.05\theta}\dot{\theta}$$

$$\ddot{r} = 0.1e^{0.05\,\theta}\left(\dot{\theta}\right)^2 + 2e^{0.05\,\theta}\ddot{\theta}$$

$$\theta = \frac{\pi}{6}$$

$$\dot{\theta} = -4$$

$$\ddot{\theta} = 0$$

$$r = 40e^{\,0.05\left(\frac{\pi}{6}\right)} = 41.0610$$

$$\dot{r} = 2e^{\,0.05\left(\frac{\pi}{6}\right)}(-4) = -8.2122$$

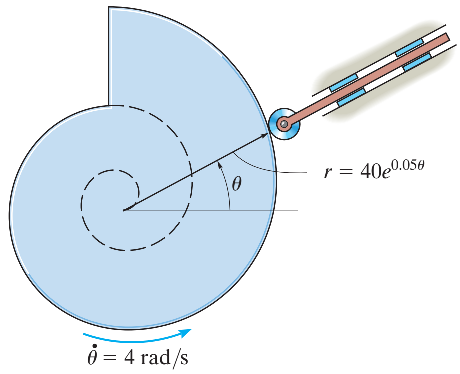
$$\ddot{r} = 0.1e^{\,0.05\left(\frac{\pi}{6}\right)}(-4)^2 + 0 = 1.64244$$

$$v = \dot{r} = -8.2122 = 8.21 \text{ mm/s}$$ **Ans.**

$$a = \ddot{r} - r\dot{\theta}^2 = 1.642\,44 - 41.0610(-4)^2 = -665.33 = -665 \text{ mm/s}^2$$ **Ans.**

12–198.

Solve Prob. 12–197, if the cam has an angular acceleration of $\ddot{\theta} = 2\text{ rad/s}^2$ when its angular velocity is $\dot{\theta} = 4\text{ rad/s}$ at $\dot{\theta} = 30°$.



SOLUTION

$r = 40e^{0.05\theta}$

$\dot{r} = 2e^{0.05\theta}\dot{\theta}$

$\ddot{r} = 0.1e^{0.05\theta}\left(\dot{\theta}\right)^2 + 2e^{0.05\theta}\ddot{\theta}$

$\theta = \dfrac{\pi}{6}$

$\dot{\theta} = -4$

$\ddot{\theta} = -2$

$r = 40e^{0.05\left(\frac{\pi}{6}\right)} = 41.0610$

$\dot{r} = 2e^{0.05\left(\left(\frac{\pi}{6}\right)\right)}(-4) = -8.2122$

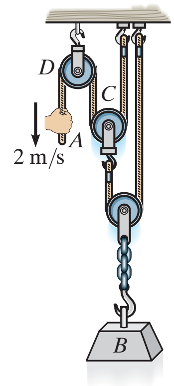
$\ddot{r} = 0.1e^{0.05\left(\frac{\pi}{6}\right)}(-4)^2 + 2e^{0.05\left(\frac{\pi}{6}\right)}(-2) = -2.4637$

$v = \dot{r} = 8.2122 = 8.21\text{ mm/s}$

$a = \ddot{r} - r\dot{\theta}^2 = -2.4637 - 41.0610(-4)^2 = -659\text{ mm/s}^2$

12–199.

If the end of the cable at A is pulled down with a speed of 2 m/s, determine the speed at which block B rises.

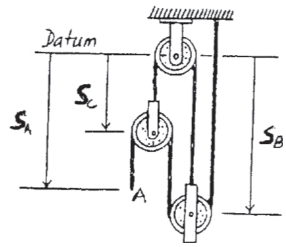


SOLUTION

Position-Coordinate Equation: Datum is established at fixed pulley D. The position of point A, block B and pulley C with respect to datum are s_A, s_B, and s_C respectively. Since the system consists of two cords, two position-coordinate equations can be derived.

$$(s_A - s_C) + (s_B - s_C) + s_B = l_1 \qquad \textbf{(1)}$$

$$s_B + s_C = l_2 \qquad \textbf{(2)}$$



Eliminating s_C from Eqs. (1) and (2) yields

$$s_A + 4s_B = l_1 = 2l_2$$

Time Derivative: Taking the time derivative of the above equation yields

$$v_A + 4v_B = 0 \qquad \textbf{(3)}$$

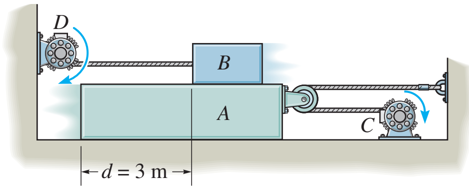
Since $v_A = 2$ m/s, from Eq. (3)

$(+\downarrow)$ $$2 + 4v_B = 0$$

$$v_B = -0.5 \text{ m/s} = 0.5 \text{ m/s} \uparrow \qquad \textbf{Ans.}$$

***12–200.**

The motor at C pulls in the cable with an acceleration $a_C = (3t^2)\ \text{m/s}^2$, where t is in seconds. The motor at D draws in its cable at $a_D = 5\ \text{m/s}^2$. If both motors start at the same instant from rest when $d = 3\ \text{m}$, determine (a) the time needed for $d = 0$, and (b) the relative velocity of block A with respect to block B when this occurs.



SOLUTION

For A:

$s_A + (s_A - s_C) = l$

$2v_A = v_C$

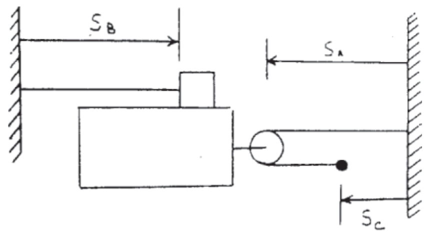
$2a_A = a_C = -3t^2$

$a_A = -1.5t^2 = 1.5t^2 \quad \rightarrow$

$v_A = 0.5t^3 \quad \rightarrow$

$s_A = 0.125\,t^4 \quad \rightarrow$

For B:



$a_B = 5\ \text{m/s}^2 \quad \leftarrow$

$v_B = 5t \quad \leftarrow$

$s_B = 2.5t^2 \quad \leftarrow$

Require $s_A + s_B = d$

$0.125t^4 + 2.5t^2 = 3$

Set $u = t^2 \qquad 0.125u^2 + 2.5u = 3$

The positive root is $u = 1.1355$. Thus,

$t = 1.0656 = 1.07\ \text{s}$ **Ans.**

$v_A = 0.5(1.0656)^3 = 0.6050$

$v_B = 5(1.0656) = 5.3281\ \text{m/s}$

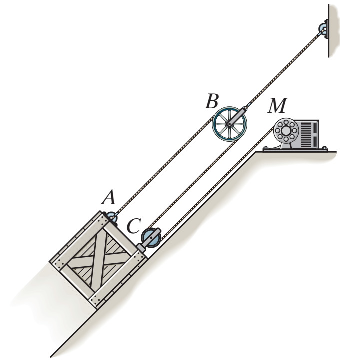
$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$

$0.6050\mathbf{i} = -5.3281\mathbf{i} + v_{A/B}\,\mathbf{i}$

$v_{A/B} = 5.93\ \text{m/s} \quad \rightarrow$ **Ans.**

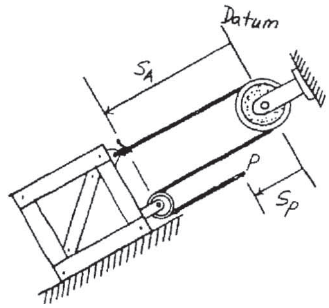
12–201.

The crate is being lifted up the inclined plane using the motor M and the rope and pulley arrangement shown. Determine the speed at which the cable must be taken up by the motor in order to move the crate up the plane with a constant speed of 4 ft/s.



SOLUTION

Position-Coordinate Equation: Datum is established at fixed pulley B. The position of point P and crate A with respect to datum are s_P and s_A, respectively.



$$2s_A + (s_A - s_P) = l$$

$$3s_A - s_P = 0$$

Time Derivative: Taking the time derivative of the above equation yields

$$3v_A - v_P = 0 \qquad \textbf{(1)}$$

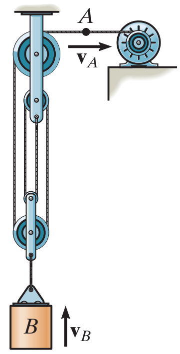
Since $v_A = 4$ ft/s, from Eq. [1]

$(+\ )$ $$3(4) - v_P = 0$$

$$v_P = 12 \text{ ft/s} \qquad \textbf{Ans.}$$

12–202.

Determine the time needed for the load at B to attain a speed of 8 m/s, starting from rest, if the cable is drawn into the motor with an acceleration of 0.2 m/s^2.



SOLUTION

$$4s_B + s_A = l$$

$$4v_B = -v_A$$

$$4a_B = -a_A$$

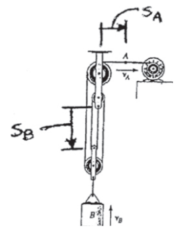
$$4a_B = -0.2$$

$$a_B = -0.05 \text{ m/s}^2$$

$$(+\downarrow) \qquad v_B = (v_B)_0 + a_B t$$

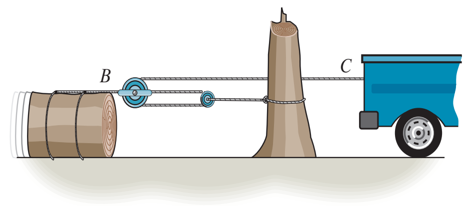
$$-8 = 0 - (0.05)(t)$$

$$t = 160 \text{ s} \qquad \textbf{Ans.}$$

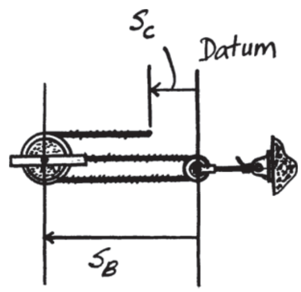


12–203.

Determine the displacement of the log if the truck at C pulls the cable 4 ft to the right.



SOLUTION



$2s_B + (s_B - s_C) = l$

$3s_B - s_C = l$

$3\Delta s_B - \Delta s_C = 0$

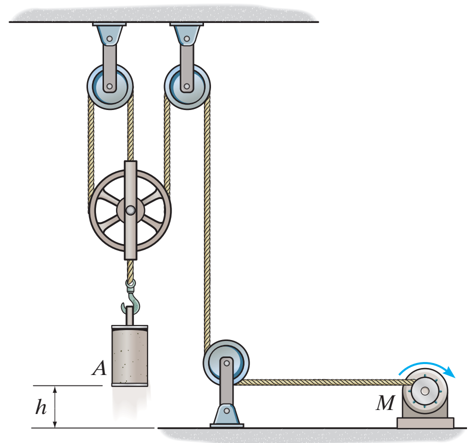
Since $\Delta s_C = -4$, then

$3\Delta s_B = -4$

$\Delta s_B = -1.33 \text{ ft} = 1.33 \text{ ft} \rightarrow$ **Ans.**

***12–204.**

Determine the speed of cylinder A, if the rope is drawn towards the motor M at a constant rate of 10 m/s.



SOLUTION

Position Coordinates: By referring to Fig. a, the length of the rope written in terms of the position coordinates s_A and s_M is

$$3s_A + s_M = l$$

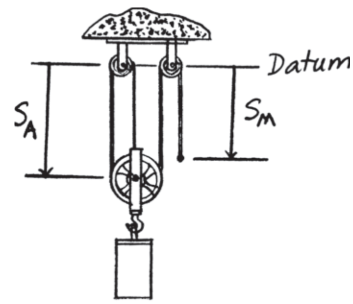
Time Derivative: Taking the time derivative of the above equation,

$$(+\downarrow) \qquad 3v_A + v_M = 0$$

Here, $v_M = 10$ m/s. Thus,

$$3v_A + 10 = 0$$

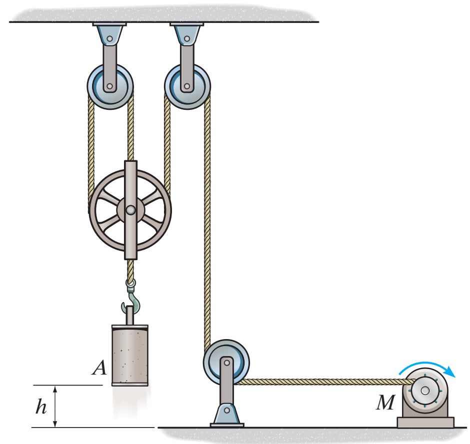
$$v_A = -3.33 \text{ m/s} = 3.33 \text{ m/s} \uparrow$$ **Ans.**



(a)

12–205.

If the rope is drawn toward the motor M at a speed of $v_M = (5t^{3/2})$ m/s, where t is in seconds, determine the speed of cylinder A when $t = 1$ s.



SOLUTION

Position Coordinates: By referring to Fig. a, the length of the rope written in terms of the position coordinates s_A and s_M is

$$3s_A + s_M = l$$

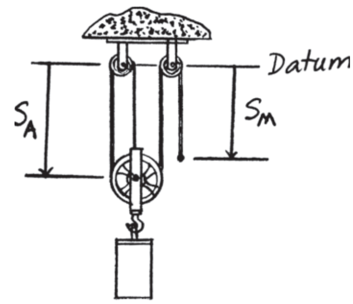
Time Derivative: Taking the time derivative of the above equation,

$$(+\downarrow) \qquad 3v_A + v_M = 0$$

Here, $v_M = \left(5t^{3/2}\right)$ m/s. Thus,

$$3v_A + 5t^{3/2} = 0$$

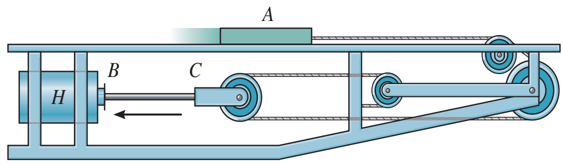
$$v_A = \left(-\frac{5}{3}t^{3/2}\right) \text{m/s} = \left(\frac{5}{3}t^{3/2}\right) \text{m/s}\bigg|_{t=1\text{ s}} = 1.67 \text{ m/s} \qquad \textbf{Ans.}$$



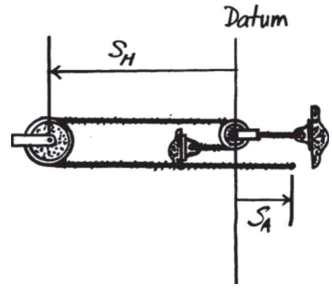
(a)

12–206.

If the hydraulic cylinder H draws in rod BC at 2 ft/s, determine the speed of slider A.



SOLUTION



$$2s_H + s_A = l$$

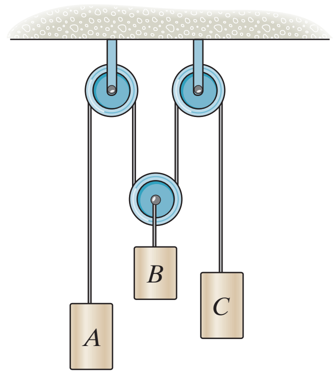
$$2v_H = -v_A$$

$$2(2) = -v_A$$

$$v_A = -4\text{ ft/s} = 4\text{ ft/s} \leftarrow$$ **Ans.**

12–207.

If block A is moving downward with a speed of 4 ft/s while C is moving up at 2 ft/s, determine the speed of block B.



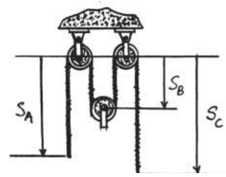
SOLUTION

$s_A + 2s_B + s_C = l$

$v_A + 2v_B + v_C = 0$

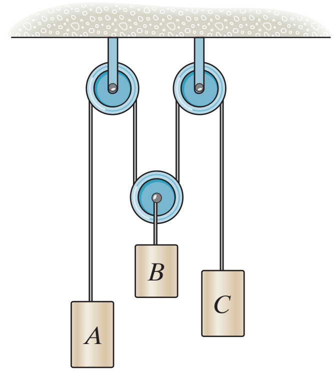
$4 + 2v_B - 2 = 0$

$v_B = -1 \text{ ft/s} = 1 \text{ ft/s} \uparrow$ **Ans.**



***12–208.**

If block A is moving downward at 6 ft/s while block C is moving down at 18 ft/s, determine the speed of block B.



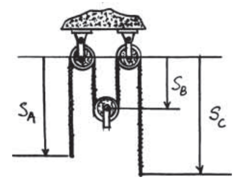
SOLUTION

$s_A + 2s_B + s_C = l$

$v_A + 2v_B + v_C = 0$

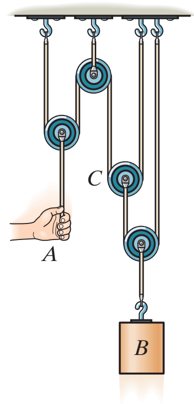
$6 + 2v_B + 18 = 0$

$v_B = -12 \text{ ft/s} = 12 \text{ ft/s} \uparrow$ **Ans.**



12–209.

Determine the displacement of the block B if A is pulled down 4 ft.



SOLUTION

$2s_A + 2s_C = l_1$

$\Delta s_A = -\Delta s_C$

$s_B - s_C + s_B = l_2$

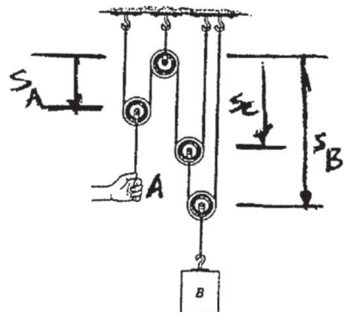
$2\,\Delta s_B = \Delta s_C$

Thus,

$2\,\Delta s_B = -\Delta s_A$

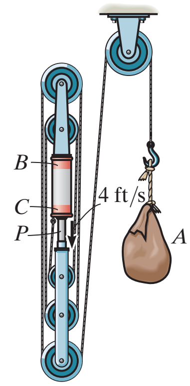
$2\,\Delta s_B = -4$

$\Delta s_B = \ -2 \text{ ft} = 2 \text{ ft} \uparrow$ **Ans.**



12–210.

The pulley arrangement shown is designed for hoisting materials. If *BC remains fixed* while the plunger P is pushed downward with a speed of 4 ft/s, determine the speed of the load at A.



SOLUTION

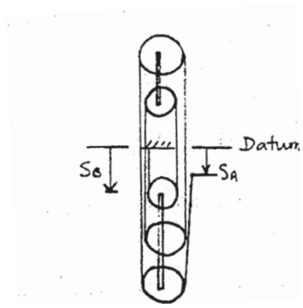
$$5\,s_B + (s_B - s_A) = l$$

$$6\,s_B - s_A = l$$

$$6\,v_B - v_A = 0$$

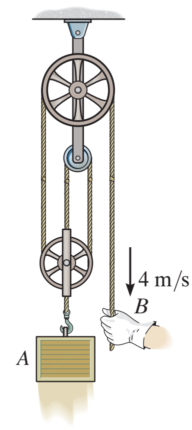
$$6(4) = v_A$$

$$v_A = 24 \text{ ft/s}$$ **Ans.**



12–211.

Determine the speed of block A if the end of the rope is pulled down with a speed of 4 m/s.



SOLUTION

Position Coordinates: By referring to Fig. a, the length of the cord written in terms of the position coordinates s_A and s_B is

$$s_B + s_A + 2(s_A - a) = l$$

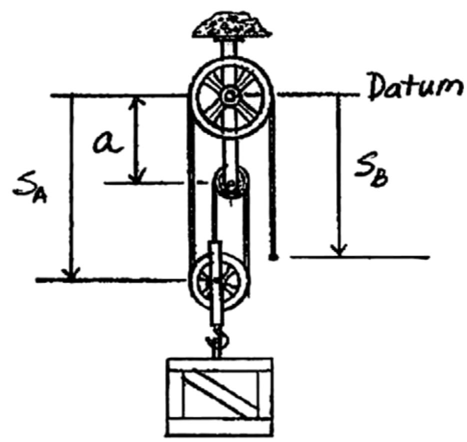
$$s_B + 3s_A = l + 2a$$

Time Derivative: Taking the time derivative of the above equation,

$$(+\downarrow) \qquad v_B + 3v_A = 0$$

Here, $v_B = 4$ m/s. Thus,

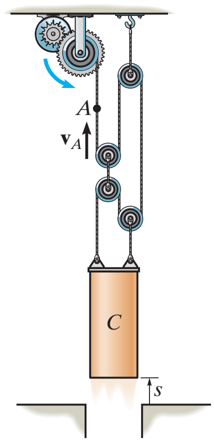
$$4 + 3v_A = 0 \qquad v_A = -133 \text{ m/s} = 1.33 \text{ m/s} \uparrow \qquad \textbf{Ans.}$$



(a)

***12–212.**

The cylinder C is being lifted using the cable and pulley system shown. If point A on the cable is being drawn toward the drum with a speed of 2 m/s, determine the speed of the cylinder.



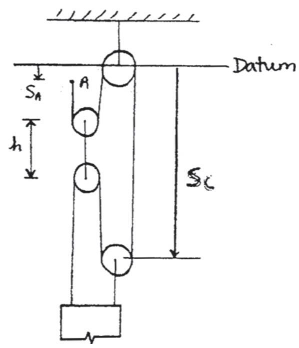
SOLUTION

$$l = s_C + (s_C - h) + (s_C - h - s_A)$$

$$l = 3s_C - 2h - s_A$$

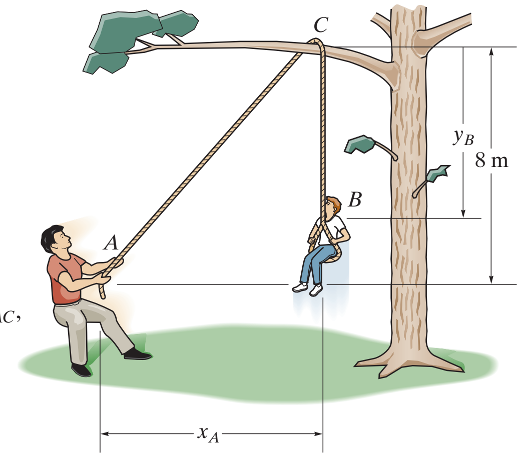
$$0 = 3v_C - v_A$$

$$v_C = \frac{v_A}{3} = \frac{-2}{3} = -0.667 \text{ m/s} = 0.667 \text{ m/s} \uparrow$$ **Ans.**



12–213.

The man pulls the boy up to the tree limb C by walking backward at a constant speed of 1.5 m/s. Determine the speed at which the boy is being lifted at the instant $x_A = 4$ m. Neglect the size of the limb. When $x_A = 0$, $y_B = 8$ m, so that A and B are coincident, i.e., the rope is 16 m long.



SOLUTION

Position-Coordinate Equation: Using the Pythagorean theorem to determine l_{AC}, we have $l_{AC} = \sqrt{x_A^2 + 8^2}$. Thus,

$$l = l_{AC} + y_B$$

$$16 = \sqrt{x_A^2 + 8^2} + y_B$$

$$y_B = 16 - \sqrt{x_A^2 + 64} \qquad \textbf{(1)}$$

Time Derivative: Taking the time derivative of Eq. (1) and realizing that $v_A = \dfrac{dx_A}{dt}$ and $v_B = \dfrac{dy_B}{dt}$, we have

$$v_B = \frac{dy_B}{dt} = -\frac{x_A}{\sqrt{x_A^2 + 64}}\frac{dx_A}{dt}$$

$$v_B = -\frac{x_A}{\sqrt{x_A^2 + 64}}v_A \qquad \textbf{(2)}$$

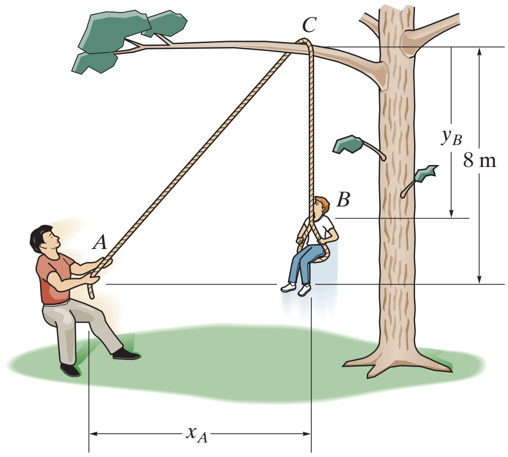
At the instant $x_A = 4$ m, from Eq. [2]

$$v_B = -\frac{4}{\sqrt{4^2 + 64}}(1.5) = -0.671 \text{ m/s} = 0.671 \text{ m/s} \uparrow \qquad \textbf{Ans.}$$

Note: The negative sign indicates that velocity v_B is in the opposite direction to that of positive y_B.

12–214.

The man pulls the boy up to the tree limb C by walking backward. If he starts from rest when $x_A = 0$ and moves backward with a constant acceleration $a_A = 0.2\ \text{m/s}^2$, determine the speed of the boy at the instant $y_B = 4$ m. Neglect the size of the limb. When $x_A = 0$, $y_B = 8$ m, so that A and B are coincident, i.e., the rope is 16 m long.



SOLUTION

Position-Coordinate Equation: Using the Pythagorean theorem to determine l_{AC}, we have $l_{AC} = \sqrt{x_A^2 + 8^2}$. Thus,

$$l = l_{AC} + y_B$$

$$16 = \sqrt{x_A^2 + 8^2} + y_B$$

$$y_B = 16 - \sqrt{x_A^2 + 64} \qquad \textbf{(1)}$$

Time Derivative: Taking the time derivative of Eq. (1) Where $v_A = \dfrac{dx_A}{dt}$ and $v_B = \dfrac{dy_B}{dt}$, we have

$$v_B = \frac{dy_B}{dt} = -\frac{x_A}{\sqrt{x_A^2 + 64}}\frac{dx_A}{dt}$$

$$v_B = -\frac{x_A}{\sqrt{x_A^2 + 64}}v_A \qquad \textbf{(2)}$$

At the instant $y_B = 4$ m, from Eq. (1), $4 = 16 - \sqrt{x_A^2 + 64}$, $x_A = 8.944$ m. The velocity of the man at that instant can be obtained.

$$v_A^2 = (v_0)_A^2 + 2(a_c)_A\left[s_A - (s_0)_A\right]$$

$$v_A^2 = 0 + 2(0.2)(8.944 - 0)$$

$$v_A = 1.891\ \text{m/s}$$

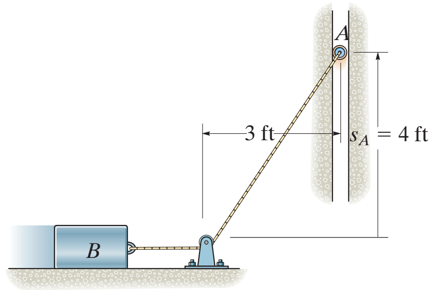
Substitute the above results into Eq. (2) yields

$$v_B = -\frac{8.944}{\sqrt{8.944^2 + 64}}(1.891) = -1.41\ \text{m/s} = 1.41\ \text{m/s}\uparrow \qquad \textbf{Ans.}$$

Note: The negative sign indicates that velocity v_B is in the opposite direction to that of positive y_B.

12–215.

The roller at A is moving upward with a velocity of $v_A = 3$ ft/s and has an acceleration of $a_A = 4$ ft/s^2 when $s_A = 4$ ft. Determine the velocity and acceleration of block B at this instant.



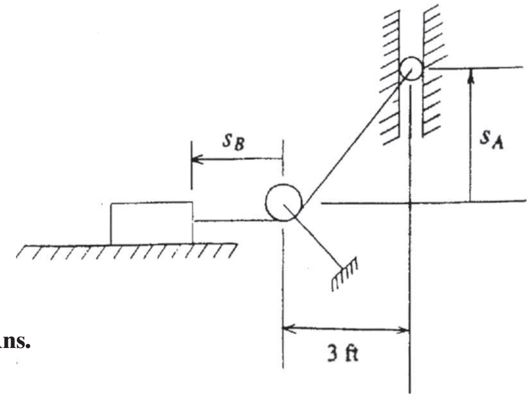
SOLUTION

$$s_B + \sqrt{(s_A)^2 + 3^2} = l$$

$$\dot{s}_B + \frac{1}{2}\left[(s_A)^2 + 3^2\right]^{-\frac{1}{2}}(2s_A)\,\dot{s}_A = 0$$

$$\dot{s}_B + \left[s_A^2 + 9\right]^{-\frac{1}{2}}\left(s_A\dot{s}_A\right) = 0$$

$$\ddot{s}_B - \left[(s_A)^2 + 9\right]^{-\frac{3}{2}}\left(s_A^2\,\dot{s}_A^2\right) + \left[s_A^2 + 9\right]^{-\frac{1}{2}}\left(\dot{s}_A^2\right) + \left[s_A^2 + 9\right]^{-\frac{1}{2}}\left(s_A\ddot{s}_A\right) = 0$$



At $s_A = 4$ ft, $\dot{s}_A = 3$ ft/s, $\ddot{s}_A = 4$ ft/s^2

$$\dot{s}_B + \left(\frac{1}{5}\right)(4)(3) = 0$$

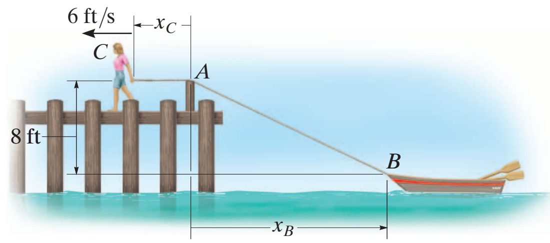
$v_B = -2.4$ ft/s $= 2.40$ ft/s $\rightarrow$ **Ans.**

$$\ddot{s}_B - \left(\frac{1}{5}\right)^3(4)^2(3)^2 + \left(\frac{1}{5}\right)(3)^2 + \left(\frac{1}{5}\right)(4)(4) = 0$$

$a_B = -3.85$ ft/s^2 $= 3.85$ ft/s$^2 \rightarrow$ **Ans.**

*12–216.

The girl at C stands near the edge of the pier and pulls in the rope *horizontally* at a constant speed of 6 ft/s. Determine how fast the boat approaches the pier at the instant the rope length AB is 50 ft.



SOLUTION

The length l of cord is

$$\sqrt{(8)^2 + x_B^2} + x_C = l$$

Taking the time derivative:

$$\frac{1}{2}[(8)^2 + x_B^2]^{-1/2}\, 2\, x_B \dot{x}_B + \dot{x}_C = 0 \qquad \textbf{(1)}$$

$$\dot{x}_C = 6 \text{ ft/s}$$

When $AB = 50$ ft,

$$x_B = \sqrt{(50)^2 - (8)^2} = 49.356 \text{ ft}$$

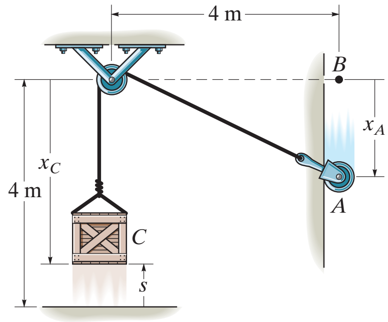
From Eq. (1)

$$\frac{1}{2}[(8)^2 + (49.356)^2]^{-1/2}\, 2(49.356)(\dot{x}_B) + 6 = 0$$

$$\dot{x}_B = -6.0783 = 6.08 \text{ ft/s} \leftarrow \qquad \textbf{Ans.}$$

12–217.

The crate C is being lifted by moving the roller at A downward with a constant speed of $v_A = 2$ m/s along the guide. Determine the velocity and acceleration of the crate at the instant $s = 1$ m. When the roller is at B, the crate rests on the ground. Neglect the size of the pulley in the calculation. *Hint:* Relate the coordinates x_C and x_A using the problem geometry, then take the first and second time derivatives.

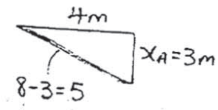


SOLUTION

$$x_C + \sqrt{x_A^2 + (4)^2} = l$$

$$\dot{x}_C + \frac{1}{2}(x_A^2 + 16)^{-1/2}(2x_A)(\dot{x}_A) = 0$$

$$\ddot{x}_C - \frac{1}{2}(x_A^2 + 16)^{-3/2}(2x_A^2)(\dot{x}_A^2) + (x_A^2 + 16)^{-1/2}(\dot{x}_A)^2 + (x_A^2 + 16)^{-1/2}(x_A)(\ddot{x}_A) = 0$$



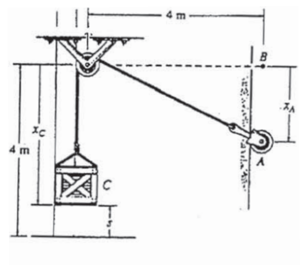
$l = 8$ m, and when $s = 1$ m,

$x_C = 3$ m

$x_A = 3$ m

$v_A = \dot{x}_A = 2$ m/s

$a_A = \ddot{x}_A = 0$



Thus,

$v_C + [(3)^2 + 16]^{-1/2}(3)(2) = 0$

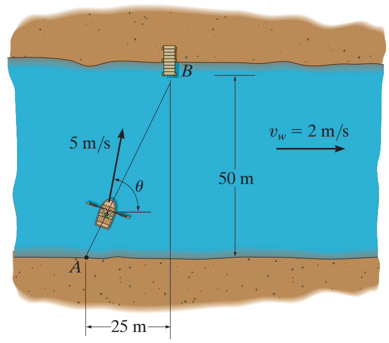
$v_C = -1.2 \text{ m/s} = 1.2 \text{ m/s} \uparrow$ **Ans.**

$a_C - [(3)^2 + 16]^{-3/2}(3)^2(2)^2 + [(3)^2 + 16]^{-1/2}(2)^2 + 0 = 0$

$a_C = -0.512 \text{ m/s}^2 = 0.512 \text{ m/s}^2 \uparrow$ **Ans.**

12–218.

The man can row the boat in still water with a speed of 5 m/s. If the river is flowing at 2 m/s, determine the speed of the boat and the angle θ he must direct the boat so that it travels from A to B.



SOLUTION

Solution I

Vector Analysis: Here, the velocity $\mathbf{v}_b$ of the boat is directed from A to B. Thus, $\phi = \tan^{-1}\left(\frac{50}{25}\right) = 63.43°$. The magnitude of the boat's velocity relative to the flowing river is $v_{b/w} = 5$ m/s. Expressing $\mathbf{v}_b$, $\mathbf{v}_w$, and $\mathbf{v}_{b/w}$ in Cartesian vector form, we have $\mathbf{v}_b = v_b \cos 63.43\mathbf{i} + v_b \sin 63.43\mathbf{j} = 0.4472v_b\mathbf{i} + 0.8944v_b\mathbf{j}$, $\mathbf{v}_w = [2\mathbf{i}]$ m/s, and $\mathbf{v}_{b/w} = 5\cos\theta\mathbf{i} + 5\sin\theta\mathbf{j}$. Applying the relative velocity equation, we have

$$\mathbf{v}_b = \mathbf{v}_w + \mathbf{v}_{b/w}$$

$$0.4472v_b\mathbf{i} + 0.8944v_b\mathbf{j} = 2\mathbf{i} + 5\cos\theta\mathbf{i} + 5\sin\theta\mathbf{j}$$

$$0.4472v_b\mathbf{i} + 0.8944v_b\mathbf{j} = (2 + 5\cos\theta)\mathbf{i} + 5\sin\theta\mathbf{j}$$

Equating the **i** and **j** components, we have

$$0.4472v_b = 2 + 5\cos\theta \qquad \textbf{(1)}$$

$$0.8944v_b = 5\sin\theta \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$v_b = 5.56 \text{ m/s} \qquad \theta = 84.4° \qquad \textbf{Ans.}$$

Solution II

Scalar Analysis: Referring to the velocity diagram shown in Fig. *a* and applying the law of cosines,

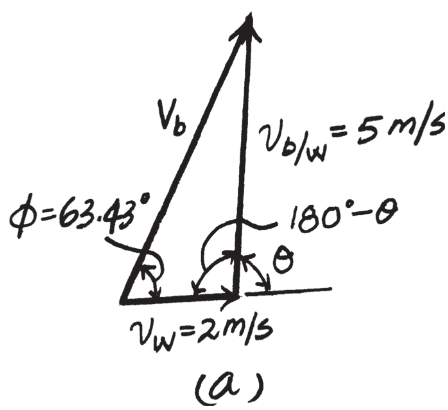
$$5^2 = 2^2 + {v_b}^2 - 2(2)(v_b)\cos 63.43°$$

$${v_b}^2 - 1.789v_b - 21 = 0$$

$$v_b = \frac{-(-1.789) \pm \sqrt{(-1.789)^2 - 4(1)(-21)}}{2(1)}$$

Choosing the positive root,

$$v_b = 5.563 \text{ m/s} = 5.56 \text{ m/s} \qquad \textbf{Ans.}$$



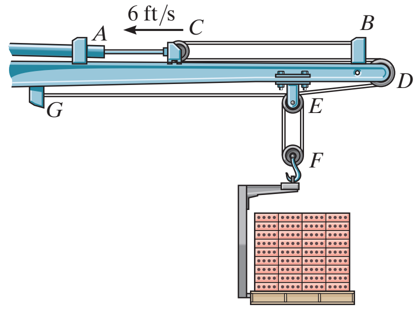
Using the result of v_b and applying the law of sines,

$$\frac{\sin(180° - \theta)}{5.563} = \frac{\sin 63.43°}{5}$$

$$\theta = 84.4° \qquad \textbf{Ans.}$$

12–219.

Vertical motion of the load is produced by movement of the piston at A on the boom. Determine the distance the piston or pulley at C must move to the left in order to lift the load 2 ft. The cable is attached at B, passes over the pulley at C, then D, E, F, and again around E, and is attached at G.



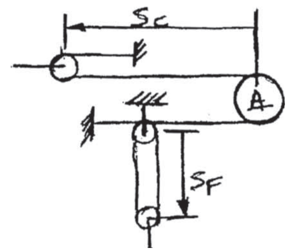
SOLUTION

$2\,s_C + 2\,s_F = l$

$2\,\Delta s_C = -2\,\Delta s_F$

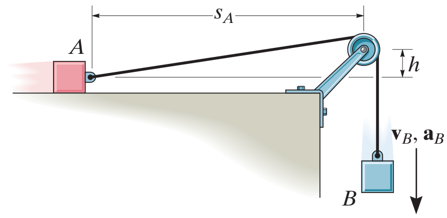
$\Delta s_C = -\Delta s_F$

$\Delta s_C = -(-2\text{ ft}) = 2\text{ ft}$ **Ans.**



12–220.

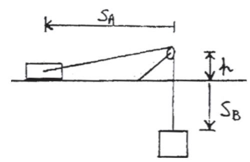
If block B is moving down with a velocity v_B and has an acceleration a_B, determine the velocity and acceleration of block A in terms of the parameters shown.



SOLUTION

$$l = s_B + \sqrt{s_B^2 + h^2}$$

$$0 = \dot{s}_B + \frac{1}{2}(s_A^2 + h^2)^{-1/2}2s_A\,\dot{s}_A$$



$$v_A = \dot{s}_A = \frac{-\,\dot{s}_B(s_A^2 + h^2)^{1/2}}{s_A}$$

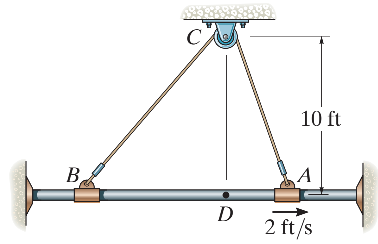
$$v_A = -v_B\,(1 + \left(\frac{h}{s_A}\right)^2)^{1/2}$$ **Ans.**

$$a_A = \dot{v}_A = -\,\dot{v}_B(1 + \left(\frac{h}{s_A}\right)^2)^{1/2} - v_B\left(\frac{1}{2}\right)(1 + \left(\frac{h}{s_A}\right)^2)^{-1/2}(h^2)(-2)(s_A)^{-3}\dot{s}_A$$

$$a_A = -a_B(1+\left(\frac{h}{s_B}\right)^2)^{1/2} + \frac{v_A v_B h^2}{s_A^3}(1+\left(\frac{h}{s_A}\right)^2)^{-1/2}$$ **Ans.**

12–221.

Collars A and B are connected to the cord that passes over the small pulley at C. When A is located at D, B is 24 ft to the left of D. If A moves at a constant speed of 2 ft/s to the right, determine the speed of B when A is 4 ft to the right of D.



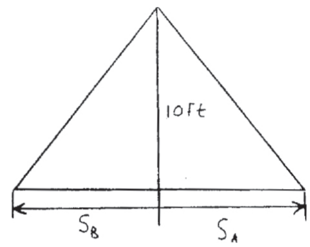
SOLUTION

$$l = \sqrt{(24)^2 + (10)^2} + 10 = 36 \text{ ft}$$

$$\sqrt{(10)^2 + s_B^2} + \sqrt{(10)^2 + s_A^2} = 36$$

$$\frac{1}{2}(100 + s_B^2)^{-\frac{1}{2}}(2s_B\dot{s}_B) + \frac{1}{2}(100 + s_A^2)^{-\frac{1}{2}}(2s_A\dot{s}_A) = 0$$

$$\dot{s}_B = -\left(\frac{s_A\dot{s}_A}{s_B}\right)\left(\frac{100 + s_B^2}{100 + s_A^2}\right)^{\frac{1}{2}}$$



At $s_A = 4$,

$$\sqrt{(10)^2 + s_B^2} + \sqrt{(10)^2 + (4)^2} = 36$$

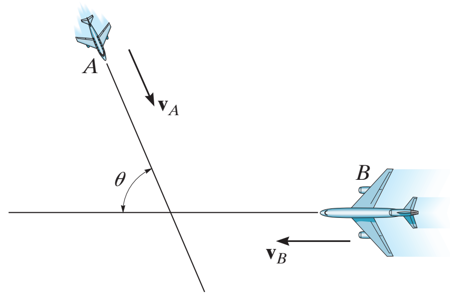
$$s_B = 23.163 \text{ ft}$$

Thus,

$$\dot{s}_B = -\left(\frac{4(2)}{23.163}\right)\left(\frac{100 + (23.163)^2}{100 + 4^2}\right)^{\frac{1}{2}} = -0.809 \text{ ft/s} = 0.809 \text{ ft/s} \rightarrow \qquad \textbf{Ans.}$$

12–222.

Two planes, A and B, are flying at the same altitude. If their velocities are $v_A = 600$ km/h and $v_B = 500$ km/h such that the angle between their straight-line courses is $\theta = 75°$, determine the velocity of plane B with respect to plane A.



SOLUTION

$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$

$[500 \leftarrow] = [600 \overset{75°}{\measuredangle}\theta] + v_{B/A}$

$(\overset{+}{\leftarrow}) \qquad 500 = -600 \cos 75° + (v_{B/A})_x$

$\qquad (v_{B/A})_x = 655.29 \leftarrow$

$(+\uparrow) \qquad 0 = -600 \sin 75° + (v_{B/A})_y$

$\qquad (v_{B/A})_y = 579.56 \uparrow$

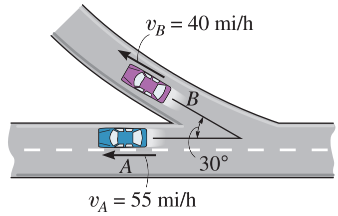
$\qquad (v_{B/A}) = \sqrt{(655.29)^2 + (579.56)^2}$

$\qquad v_{B/A} = 875 \text{ km/h}$ **Ans.**

$\qquad \theta = \tan^{-1}\left(\dfrac{579.56}{655.29}\right) = 41.5°$ **Ans.**

12–223.

At the instant shown, cars A and B are traveling at speeds of 55 mi/h and 40 mi/h, respectively. If B is increasing its speed by 1200 mi/h^2, while A maintains a constant speed, determine the velocity and acceleration of B with respect to A. Car B moves along a curve having a radius of curvature of 0.5 mi.



SOLUTION

$$v_B = -40\cos 30°\mathbf{i} + 40\sin 30°\mathbf{j} = \{-34.64\mathbf{i} + 20\mathbf{j}\}\text{ mi/h}$$

$$v_A = \{-55\mathbf{i}\}\text{ mi/h}$$

$$v_{B/A} = \nu_B - \nu_A$$

$$= (-34.64\mathbf{i} + 20\mathbf{j}) - (-55\mathbf{i}) = \{20.36\mathbf{i} + 20\mathbf{j}\}\text{ mi/h}$$

$$v_{B/A} = \sqrt{20.36^2 + 20^2} = 28.5\text{ mi/h}$$ **Ans.**

$$\theta = \tan^{-1}\frac{20}{20.36} = 44.5° \measuredangle$$ **Ans.**

$$(a_B)_n = \frac{v_A^2}{\rho} = \frac{40^2}{0.5} = 3200\text{ mi/h}^2 \qquad (a_B)_t = 1200\text{ mi/h}^2$$

$$\mathbf{a}_B = (3200\cos 60° - 1200\cos 30°)\mathbf{i} + (3200\sin 60° + 1200\sin 30°)\mathbf{j}$$

$$= \{560.77\mathbf{i} + 3371.28\mathbf{j}\}\quad\text{mi/h}^2$$

$$\mathbf{a}_A = 0$$

$$\mathbf{a}_{B/A} = \mathbf{a}_B - \mathbf{a}_A$$

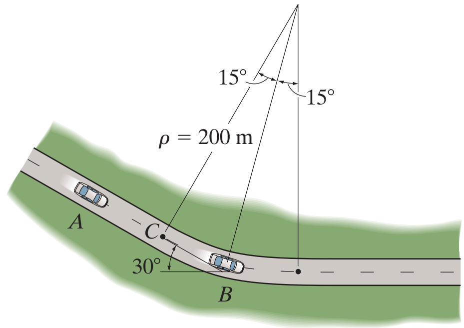
$$= \{560.77\mathbf{i} + 3371.28\mathbf{j}\} - 0 = \{560.77\mathbf{i} + 3371.28\mathbf{j}\}\quad\text{mi/h}^2$$

$$a_{B/A} = \sqrt{(560.77)^2 + (3371.28)^2} = 3418\text{ mi/h}^2$$ **Ans.**

$$\theta = \tan^{-1}\frac{3371.28}{560.77} = 80.6° \measuredangle$$ **Ans.**

12–224.

At the instant shown, car A travels along the straight portion of the road with a speed of 25 m/s. At this same instant car B travels along the circular portion of the road with a speed of 15 m/s. Determine the velocity of car B relative to car A.

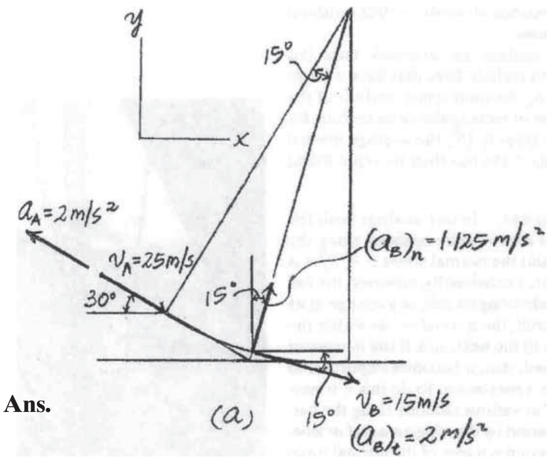


SOLUTION

Velocity: Referring to Fig. a, the velocity of cars A and B expressed in Cartesian vector form are

$$\mathbf{v}_A = [25 \cos 30^\circ\, \mathbf{i} - 25 \sin 30^\circ\, \mathbf{j}] \text{ m/s} = [21.65\mathbf{i} - 12.5\mathbf{j}] \text{ m/s}$$

$$\mathbf{v}_B = [15 \cos 15^\circ\, \mathbf{i} - 15 \sin 15^\circ\, \mathbf{j}] \text{ m/s} = [14.49\mathbf{i} - 3.882\mathbf{j}] \text{ m/s}$$



Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$14.49\mathbf{i} - 3.882\mathbf{j} = 21.65\mathbf{i} - 12.5\mathbf{j} + \mathbf{v}_{B/A}$$

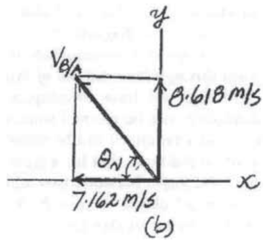
$$\mathbf{v}_{B/A} = [-7.162\mathbf{i} + 8.618\mathbf{j}] \text{ m/s}$$

Thus, the magnitude of $\mathbf{v}_{B/A}$ is given by

$$v_{B/A} = \sqrt{(-7.162)^2 + 8.618^2} = 11.2 \text{ m/s} \qquad \textbf{Ans.}$$

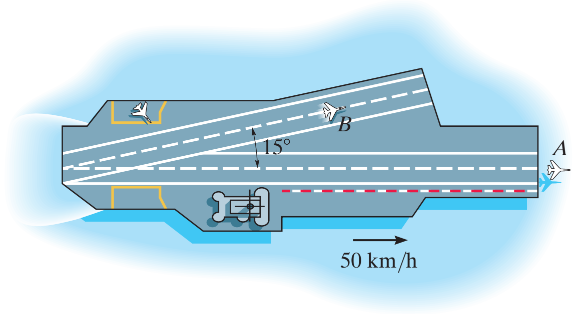
The direction angle θ_v of $\mathbf{v}_{B/A}$ measured down from the negative x axis, Fig. b is

$$\theta_v = \tan^{-1}\left(\frac{8.618}{7.162}\right) = 50.3^\circ \qquad \textbf{Ans.}$$



12–225.

An aircraft carrier is traveling forward with a velocity of 50 km/h. At the instant shown, the plane at A has just taken off and has attained a forward horizontal air speed of 200 km/h, measured from still water. If the plane at B is traveling along the runway of the carrier at 175 km/h in the direction shown, determine the velocity of A with respect to B.



SOLUTION

$$\mathbf{v}_B = \mathbf{v}_C + \mathbf{v}_{B/C}$$

$$\mathbf{v}_B = 50\mathbf{i} + 175 \cos 15°\mathbf{i} + 175 \sin 15°\mathbf{j} = 219.04\mathbf{i} + 45.293\mathbf{j}$$

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

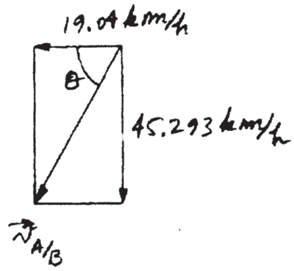
$$200\mathbf{i} = 219.04\mathbf{i} + 45.293\mathbf{j} + (v_{A/B})_x\mathbf{i} + (v_{A/B})_y\,\mathbf{j}$$

$$200 = 219.04 + (v_{A/B})_x$$

$$0 = 45.293 + (v_{A/B})_y$$

$$(v_{A/B})_x = -19.04$$

$$(v_{A/B})_y = -45.293$$



$$v_{A/B} = \sqrt{(-19.04)^2 + (-45.293)^2} = 49.1 \text{ km/h}$$ **Ans.**

$$\theta = \tan^{-1}\left(\frac{45.293}{19.04}\right) = 67.2°$$ **Ans.**

12–226.

A car is traveling north along a straight road at 50 km/h. An instrument in the car indicates that the wind is directed toward the east. If the car's speed is 80 km/h, the instrument indicates that the wind is directed toward the north-east. Determine the speed and direction of the wind.

SOLUTION

Solution I

Vector Analysis: For the first case, the velocity of the car and the velocity of the wind relative to the car expressed in Cartesian vector form are $\mathbf{v}_c = [50\mathbf{j}]$ km/h and $\mathbf{v}_{W/C} = (v_{W/C})_1\,\mathbf{i}$. Applying the relative velocity equation, we have

$$\mathbf{v}_w = \mathbf{v}_c + \mathbf{v}_{w/c}$$

$$\mathbf{v}_w = 50\mathbf{j} + (v_{w/c})_1\,\mathbf{i}$$

$$\mathbf{v}_w = (v_{w/c})_1\mathbf{i} + 50\mathbf{j} \qquad \textbf{(1)}$$

For the second case, $v_C = [80\mathbf{j}]$ km/h and $\mathbf{v}_{W/C} = (v_{W/C})_2 \cos 45°\mathbf{i} + (v_{W/C})_2 \sin 45°\,\mathbf{j}$. Applying the relative velocity equation, we have

$$\mathbf{v}_w = \mathbf{v}_c + \mathbf{v}_{w/c}$$

$$\mathbf{v}_w = 80\mathbf{j} + (v_{w/c})_2 \cos 45°\mathbf{i} + (v_{w/c})_2 \sin 45°\,\mathbf{j}$$

$$\mathbf{v}_w = (v_{w/c})_2 \cos 45°\,\mathbf{i} + \left[80 + (v_{w/c})_2 \sin 45°\right]\mathbf{j} \qquad \textbf{(2)}$$

Equating Eqs. (1) and (2) and then the **i** and **j** components,

$$(v_{w/c})_1 = (v_{w/c})_2 \cos 45° \qquad \textbf{(3)}$$

$$50 = 80 + (v_{w/c})_2 \sin 45° \qquad \textbf{(4)}$$

Solving Eqs. (3) and (4) yields

$$(v_{w/c})_2 = -42.43 \text{ km/h} \qquad (v_{w/c})_1 = -30 \text{ km/h}$$

Substituting the result of $(v_{w/c})_1$ into Eq. (1),

$$\mathbf{v}_w = [-30\mathbf{i} + 50\mathbf{j}] \text{ km/h}$$

Thus, the magnitude of $\mathbf{v}_W$ is

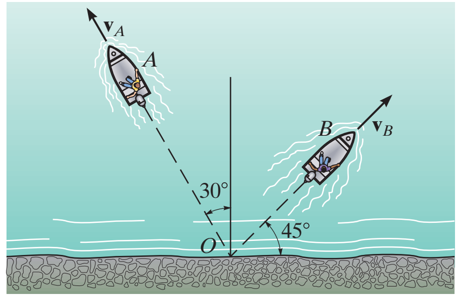
$$v_w = \sqrt{(-30)^2 + 50^2} = 58.3 \text{ km/h} \qquad \textbf{Ans.}$$

and the directional angle θ that $\mathbf{v}_W$ makes with the x axis is

$$\theta = \tan^{-1}\left(\frac{50}{30}\right) = 59.0° \qquad \textbf{Ans.}$$

12–227.

Two boats leave the shore at the same time and travel in the directions shown. If $v_A = 20\,\text{ft/s}$ and $v_B = 15\,\text{ft/s}$, determine the velocity of boat A with respect to boat B. How long after leaving the shore will the boats be 800 ft apart?



SOLUTION

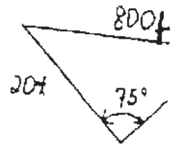
$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

$$-20 \sin 30°\mathbf{i} + 20 \cos 30°\mathbf{j} = 15 \cos 45°\mathbf{i} + 15 \sin 45°\mathbf{j} + \mathbf{v}_{A/B}$$

$$\mathbf{v}_{A/B} = \{-20.61\mathbf{i} + 6.714\mathbf{j}\}\ \text{ft/s}$$

$$v_{A/B} = \sqrt{(-20.61)^2 + (+6.714)^2} = 21.7\ \text{ft/s}$$ **Ans.**

$$\theta = \tan^{-1}\left(\frac{6.714}{20.61}\right) = 18.0°$$ **Ans.**



$$(800)^2 = (20\,t)^2 + (15\,t)^2 - 2(20\,t)(15\,t) \cos 75°$$

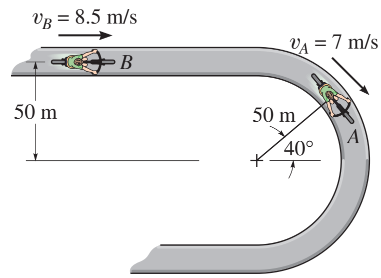
$$t = 36.9\ \text{s}$$ **Ans.**

Also

$$t = \frac{800}{v_{A/B}} = \frac{800}{21.68} = 36.9\ \text{s}$$ **Ans.**

***12–228.**

At the instant shown, the bicyclist at A is traveling at 7 m/s around the curve on the race track while increasing his speed at 0.5 m/s^2. The bicyclist at B is traveling at 8.5 m/s along the straight-a-way and increasing his speed at 0.7 m/s^2. Determine the relative velocity and relative acceleration of A with respect to B at this instant.



SOLUTION

$$v_A = v_B + v_{A/B}$$

$$[7 \searrow_{40°}] = [8.5 \rightarrow] + [(v_{A/B})_x \rightarrow] + [(v_{A/B})_y \downarrow]$$

$$(\overset{+}{\rightarrow}) \qquad 7 \sin 40° = 8.5 + (v_{A/B})_x$$

$$(+\downarrow) \qquad 7 \cos 40° = (v_{A/B})_y$$

Thus,

$$(v_{A/B})_x = 4.00 \text{ m/s} \leftarrow$$

$$(v_{A/B})_y = 5.36 \text{ m/s} \downarrow$$

$$(v_{A/B}) = \sqrt{(4.00)^2 + (5.36)^2}$$

$$v_{A/B} = 6.69 \text{ m/s} \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\left(\frac{5.36}{4.00}\right) = 53.3° \qquad \textbf{Ans.}$$

$$(a_A)_n = \frac{7^2}{50} = 0.980 \text{ m/s}^2$$

$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B}$$

$$[0.980]\,\overset{\theta}{\swarrow}_{40°} + [0.5] \searrow_{40°} = [0.7 \rightarrow] + [(a_{A/B})_x \rightarrow] + [(a_{A/B})_y \downarrow]$$

$$(+\rightarrow) \qquad -0.980 \cos 40° + 0.5 \sin 40° = 0.7 + (a_{A/B})_x$$

$$(a_{A/B})_x = 1.129 \text{ m/s}^2 \leftarrow$$

$$(+\downarrow) \qquad 0.980 \sin 40° + 0.5 \cos 40° = (a_{A/B})_y$$

$$(a_{A/B})_y = 1.013 \text{ m/s}^2 \downarrow$$

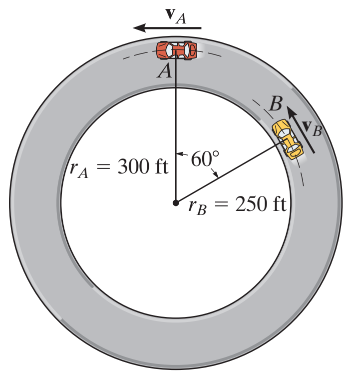
$$(a_{A/B}) = \sqrt{(1.129)^2 + (1.013)^2}$$

$$a_{A/B} = 1.52 \text{ m/s}^2 \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\left(\frac{1.013}{1.129}\right) = 41.9° \qquad \textbf{Ans.}$$

12–229.

Cars A and B are traveling around the circular race track. At the instant shown, A has a speed of 90 ft/s and is increasing its speed at the rate of 15 ft/s^2, whereas B has a speed of 105 ft/s and is decreasing its speed at 25 ft/s^2. Determine the relative velocity and relative acceleration of car A with respect to car B at this instant.



SOLUTION

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

$$-90\mathbf{i} = -105 \sin 30°\,\mathbf{i} + 105 \cos 30°\,\mathbf{j} + \mathbf{v}_{A/B}$$

$$\mathbf{v}_{A/B} = \{-37.5\mathbf{i} - 90.93\mathbf{j}\} \text{ ft/s}$$

$$v_{A/B} = \sqrt{(-37.5)^2 + (-90.93)^2} = 98.4 \text{ ft/s} \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\left(\frac{90.93}{37.5}\right) = 67.6° \qquad \textbf{Ans.}$$

$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B}$$

$$-15\mathbf{i} - \frac{(90)^2}{300}\mathbf{j} = 25 \cos 60°\mathbf{i} - 25 \sin 60°\mathbf{j} - 44.1 \sin 60°\mathbf{i} - 44.1 \cos 60°\mathbf{j} + \mathbf{a}_{A/B}$$

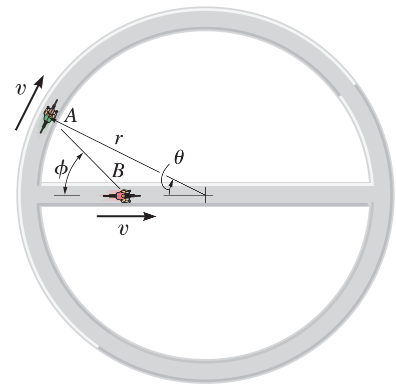
$$\mathbf{a}_{A/B} = \{10.69\mathbf{i} + 16.70\mathbf{j}\} \text{ ft/s}^2$$

$$a_{A/B} = \sqrt{(10.69)^2 + (16.70)^2} = 19.8 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\left(\frac{16.70}{10.69}\right) = 57.4° \qquad \textbf{Ans.}$$

12–230.

The two cyclists A and B travel at the same constant speed v. Determine the speed of A with respect to B if A travels along the circular track, while B travels along the diameter of the circle.



SOLUTION

$$\mathbf{v}_A = v\sin\theta\mathbf{i} + v\cos\theta\mathbf{j} \qquad \mathbf{v}_B = v\mathbf{i}$$

$$\mathbf{v}_{A/B} = \mathbf{v}_A - \mathbf{v}_B$$

$$= (v\sin\theta\mathbf{i} + v\cos\theta\mathbf{j}) - v\mathbf{i}$$

$$= (v\sin\theta - v)\mathbf{i} + v\cos\theta\mathbf{j}$$

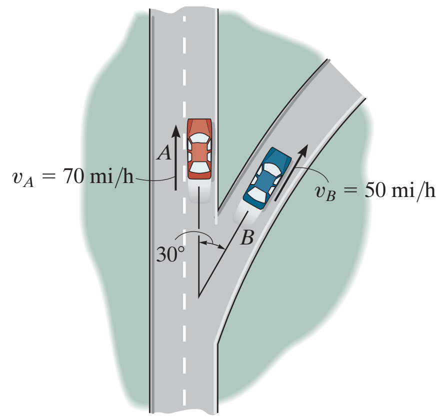
$$v_{A/B} = \sqrt{(v\sin\theta - v)^2 + (v\cos\theta)^2}$$

$$= \sqrt{2v^2 - 2v^2\sin\theta}$$

$$= v\sqrt{2(1 - \sin\theta)} \qquad \textbf{Ans.}$$

12–231.

At the instant shown, cars A and B travel at speeds of 70 mi/h and 50 mi/h, respectively. If B is increasing its speed by 1100 mi/h^2, while A maintains a constant speed, determine the velocity and acceleration of B with respect to A. Car B moves along a curve having a radius of curvature of 0.7 mi.



SOLUTION

Relative Velocity:

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$50 \sin 30°\mathbf{i} + 50 \cos 30°\mathbf{j} = 70\mathbf{j} + \mathbf{v}_{B/A}$$

$$\mathbf{v}_{B/A} = \{25.0\mathbf{i} - 26.70\mathbf{j}\} \text{ mi/h}$$

Thus, the magnitude of the relative velocity $\mathbf{v}_{B/A}$ is

$$v_{B/A} = \sqrt{25.0^2 + (-26.70)^2} = 36.6 \text{ mi/h} \qquad \textbf{Ans.}$$

The direction of the relative velocity is the same as the direction of that for relative acceleration. Thus

$$\theta = \tan^{-1}\frac{26.70}{25.0} = 46.9° \searrow \qquad \textbf{Ans.}$$

Relative Acceleration: Since car B is traveling along a curve, its normal acceleration is $(a_B)_n = \dfrac{v_B^2}{\rho} = \dfrac{50^2}{0.7} = 3571.43$ mi/h^2. Applying Eq. 12–35 gives

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

$$(1100 \sin 30° + 3571.43 \cos 30°)\mathbf{i} + (1100 \cos 30° - 3571.43 \sin 30°)\mathbf{j} = 0 + \mathbf{a}_{B/A}$$

$$\mathbf{a}_{B/A} = \{3642.95\mathbf{i} - 833.09\mathbf{j}\} \text{ mi/h}^2$$

Thus, the magnitude of the relative velocity $\mathbf{a}_{B/A}$ is

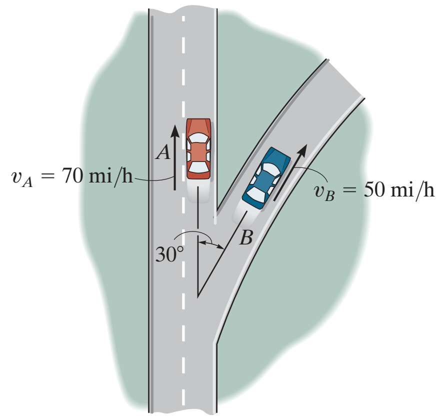
$$a_{B/A} = \sqrt{3642.95^2 + (-833.09)^2} = 3737 \text{ mi/h}^2 \qquad \textbf{Ans.}$$

And its direction is

$$\phi = \tan^{-1}\frac{833.09}{3642.95} = 12.9° \searrow \qquad \textbf{Ans.}$$

12–232.

At the instant shown, cars A and B travel at speeds of 70 mi/h and 50 mi/h, respectively. If B is decreasing its speed at 1400 mi/h^2 while A is increasing its speed at 800 mi/h^2, determine the acceleration of B with respect to A. Car B moves along a curve having a radius of curvature of 0.7 mi.



SOLUTION

Relative Acceleration: Since car B is traveling along a curve, its normal acceleration is $(a_B)_n = \dfrac{v_B^2}{\rho} = \dfrac{50^2}{0.7} = 3571.43$ mi/h^2. Applying Eq. 12–35 gives

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

$$(3571.43 \cos 30° - 1400 \sin 30°)\mathbf{i} + (-1400 \cos 30° - 3571.43 \sin 30°)\mathbf{j} = 800\mathbf{j} + \mathbf{a}_{B/A}$$

$$\mathbf{a}_{B/A} = \{2392.95\mathbf{i} - 3798.15\mathbf{j}\} \text{ mi/h}^2$$

Thus, the magnitude of the relative acc. $\mathbf{a}_{B/A}$ is

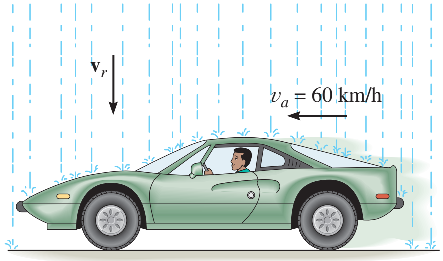
$$a_{B/A} = \sqrt{2392.95^2 + (-3798.15)^2} = 4489 \text{ mi/h}^2 \qquad \textbf{Ans.}$$

And its direction is

$$\phi = \tan^{-1}\frac{3798.15}{2392.95} = 57.8° \qquad \textbf{Ans.}$$

12–233.

A passenger in an automobile observes that raindrops make an angle of 30° with the horizontal as the auto travels forward with a speed of 60 km/h. Compute the terminal (constant)velocity $\mathbf{v}_r$ of the rain if it is assumed to fall vertically.



SOLUTION

$v_r = v_a + v_{r/a}$

$-v_r\mathbf{j} = -60\mathbf{i} + v_{r/a}\cos 30°\mathbf{i} - v_{r/a}\sin 30°\mathbf{j}$

$(\overset{+}{\rightarrow})\qquad 0 = -60 + v_{r/a}\cos 30°$

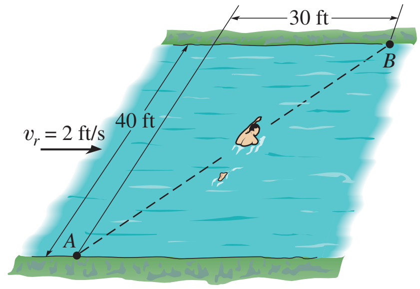
$(+\uparrow)\qquad -v_r = 0 - v_{r/a}\sin 30°$

$v_{r/a} = 69.3$ km/h

$v_r = 34.6$ km/h **Ans.**

12–234.

A man can swim at 4 ft/s in still water. He wishes to cross the 40-ft-wide river to point B, 30 ft downstream. If the river flows with a velocity of 2 ft/s, determine the speed of the man and the time needed to make the crossing. *Note*: While in the water he must not direct himself toward point B to reach this point. Why?



SOLUTION

Relative Velocity:

$$v_m = v_r + v_{m/r}$$

$$\frac{3}{5}v_m\mathbf{i} + \frac{4}{5}v_m\mathbf{j} = 2\mathbf{i} + 4\sin\theta\mathbf{i} + 4\cos\theta\mathbf{j}$$

Equating the i and j components, we have

$$\frac{3}{5}v_m = 2 + 4\sin\theta \qquad \textbf{(1)}$$

$$\frac{4}{5}v_m = 4\cos\theta \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

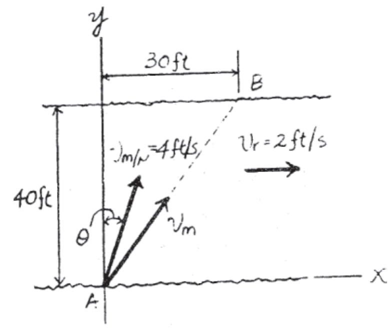
$$\theta = 13.29°$$

$$v_m = 4.866 \text{ ft/s} = 4.87 \text{ ft/s} \qquad \textbf{Ans.}$$

Thus, the time t required by the boat to travel from points A to B is

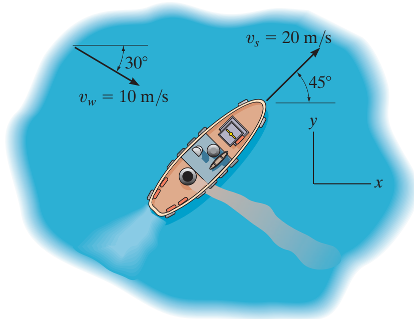
$$t = \frac{s_{AB}}{v_b} = \frac{\sqrt{40^2 + 30^2}}{4.866} = 10.3 \text{ s} \qquad \textbf{Ans.}$$

In order for the man to reached point B, the man has to direct himself at an angle $\theta = 13.3°$ with y axis.



12–235.

The ship travels at a constant speed of $v_s = 20$ m/s and the wind is blowing at a speed of $v_w = 10$ m/s, as shown. Determine the magnitude and direction of the horizontal component of velocity of the smoke coming from the smoke stack as it appears to a passenger on the ship.



SOLUTION

Solution I

Vector Analysis: The velocity of the smoke as observed from the ship is equal to the velocity of the wind relative to the ship. Here, the velocity of the ship and wind expressed in Cartesian vector form are $\mathbf{v}_s = [20\cos 45°\,\mathbf{i} + 20\sin 45°\,\mathbf{j}]$ m/s $= [14.14\mathbf{i} + 14.14\mathbf{j}]$ m/s and $\mathbf{v}_w = [10\cos 30°\,\mathbf{i} - 10\sin 30°\,\mathbf{j}] = [8.660\mathbf{i} - 5\mathbf{j}]$ m/s. Applying the relative velocity equation,

$$\mathbf{v}_w = \mathbf{v}_s + \mathbf{v}_{w/s}$$

$$8.660\mathbf{i} - 5\mathbf{j} = 14.14\mathbf{i} + 14.14\mathbf{j} + \mathbf{v}_{w/s}$$

$$\mathbf{v}_{w/s} = [-5.482\mathbf{i} - 19.14\mathbf{j}] \text{ m/s}$$

Thus, the magnitude of $\mathbf{v}_{w/s}$ is given by

$$v_w = \sqrt{(-5.482)^2 + (-19.14)^2} = 19.9 \text{ m/s} \qquad \textbf{Ans.}$$

and the direction angle θ that $\mathbf{v}_{w/s}$ makes with the x axis is

$$\theta = \tan^{-1}\left(\frac{19.14}{5.482}\right) = 74.0° \qquad \textbf{Ans.}$$

Solution II

Scalar Analysis: Applying the law of cosines by referring to the velocity diagram shown in Fig. a,

$$v_{w/s} = \sqrt{20^2 + 10^2 - 2(20)(10)\cos 75°}$$

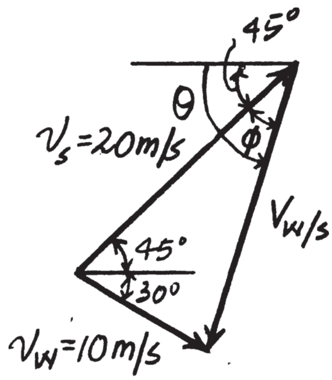
$$= 19.91 \text{ m/s} = 19.9 \text{ m/s} \qquad \textbf{Ans.}$$

Using the result of $v_{w/s}$ and applying the law of sines,

$$\frac{\sin\phi}{10} = \frac{\sin 75°}{19.91} \qquad \phi = 29.02°$$

Thus,

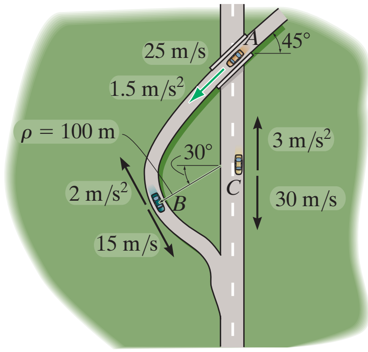
$$\theta = 45° + \phi = 74.0° \qquad \textbf{Ans.}$$



(a)

***12–236.**

Car A travels along a straight road at a speed of 25 m/s while accelerating at 1.5 m/s^2. At this same instant car C is traveling along the straight road with a speed of 30 m/s while decelerating at 3 m/s^2. Determine the velocity and acceleration of car A relative to car C.



SOLUTION

Velocity: The velocity of cars A and C expressed in Cartesian vector form are

$$\mathbf{v}_A = [-25\cos 45°\mathbf{i} - 25\sin 45°\mathbf{j}]\text{ m/s} = [-17.68\mathbf{i} - 17.68\mathbf{j}]\text{ m/s}$$

$$\mathbf{v}_C = [-30\mathbf{j}]\text{ m/s}$$

Applying the relative velocity equation, we have

$$\mathbf{v}_A = \mathbf{v}_C + \mathbf{v}_{A/C}$$

$$-17.68\mathbf{i} - 17.68\mathbf{j} = -30\mathbf{j} + \mathbf{v}_{A/C}$$

$$\mathbf{v}_{A/C} = [-17.68\mathbf{i} + 12.32\mathbf{j}]\text{ m/s}$$

Thus, the magnitude of $\mathbf{v}_{A/C}$ is given by

$$v_{A/C} = \sqrt{(-17.68)^2 + 12.32^2} = 21.5\text{ m/s} \qquad \textbf{Ans.}$$

and the direction angle θ_v that $\mathbf{v}_{A/C}$ makes with the x axis is

$$\theta_v = \tan^{-1}\left(\frac{12.32}{17.68}\right) = 34.9° \text{ ⦣} \qquad \textbf{Ans.}$$

Acceleration: The acceleration of cars A and C expressed in Cartesian vector form are

$$\mathbf{a}_A = [-1.5\cos 45°\mathbf{i} - 1.5\sin 45°\mathbf{j}]\text{ m/s}^2 = [-1.061\mathbf{i} - 1.061\mathbf{j}]\text{ m/s}^2$$

$$\mathbf{a}_C = [3\mathbf{j}]\text{ m/s}^2$$

Applying the relative acceleration equation,

$$\mathbf{a}_A = \mathbf{a}_C + \mathbf{a}_{A/C}$$

$$-1.061\mathbf{i} - 1.061\mathbf{j} = 3\mathbf{j} + \mathbf{a}_{A/C}$$

$$\mathbf{a}_{A/C} = [-1.061\mathbf{i} - 4.061\mathbf{j}]\text{ m/s}^2$$

Thus, the magnitude of $\mathbf{a}_{A/C}$ is given by

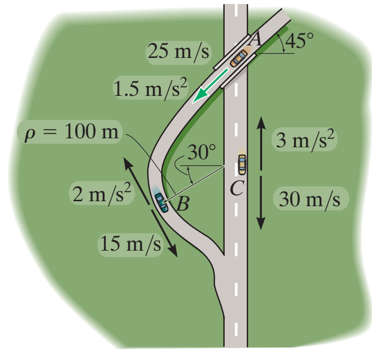
$$a_{A/C} = \sqrt{(-1.061)^2 + (-4.061)^2} = 4.20\text{ m/s}^2 \qquad \textbf{Ans.}$$

and the direction angle θ_a that $\mathbf{a}_{A/C}$ makes with the x axis is

$$\theta_a = \tan^{-1}\left(\frac{4.061}{1.061}\right) = 75.4° \qquad \textbf{Ans.}$$

12–237.

Car B is traveling along the curved road with a speed of 15 m/s while decreasing its speed at 2 m/s^2. At this same instant car C is traveling along the straight road with a speed of 30 m/s while decelerating at 3 m/s^2. Determine the velocity and acceleration of car B relative to car C.



SOLUTION

Velocity: The velocity of cars B and C expressed in Cartesian vector form are

$$\mathbf{v}_B = [15 \cos 60° \,\mathbf{i} - 15 \sin 60° \,\mathbf{j}] \text{ m/s} = [7.5\mathbf{i} - 12.99\mathbf{j}] \text{ m/s}$$

$$\mathbf{v}_C = [-30\mathbf{j}] \text{ m/s}$$

Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_C + \mathbf{v}_{B/C}$$

$$7.5\mathbf{i} - 12.99\mathbf{j} = -30\mathbf{j} + \mathbf{v}_{B/C}$$

$$\mathbf{v}_{B/C} = [7.5\mathbf{i} + 17.01\mathbf{j}] \text{ m/s}$$

Thus, the magnitude of $\mathbf{v}_{B/C}$ is given by

$$v_{B/C} = \sqrt{7.5^2 + 17.01^2} = 18.6 \text{ m/s} \qquad \textbf{Ans.}$$

and the direction angle θ_v that $\mathbf{v}_{B/C}$ makes with the x axis is

$$\theta_v = \tan^{-1}\left(\frac{17.01}{7.5}\right) = 66.2° \qquad \textbf{Ans.}$$

Acceleration: The normal component of car B's acceleration is $(a_B)_n = \dfrac{{v_B}^2}{\rho} = \dfrac{15^2}{100} = 2.25 \text{ m/s}^2$. Thus, the tangential and normal components of car B's acceleration and the acceleration of car C expressed in Cartesian vector form are

$$(\mathbf{a}_B)_t = [-2 \cos 60° \,\mathbf{i} + 2 \sin 60° \mathbf{j}] = [-1\mathbf{i} + 1.732\mathbf{j}] \text{ m/s}^2$$

$$(\mathbf{a}_B)_n = [2.25 \cos 30° \,\mathbf{i} + 2.25 \sin 30° \,\mathbf{j}] = [1.9486\mathbf{i} + 1.125\mathbf{j}] \text{ m/s}^2$$

$$\mathbf{a}_C = [3\mathbf{j}] \text{ m/s}^2$$

Applying the relative acceleration equation,

$$\mathbf{a}_B = \mathbf{a}_C + \mathbf{a}_{B/C}$$

$$(-1\mathbf{i} + 1.732\mathbf{j}) + (1.9486\mathbf{i} + 1.125\mathbf{j}) = 3\mathbf{j} + \mathbf{a}_{B/C}$$

$$\mathbf{a}_{B/C} = [0.9486\mathbf{i} - 0.1429\mathbf{j}] \text{ m/s}^2$$

Thus, the magnitude of $\mathbf{a}_{B/C}$ is given by

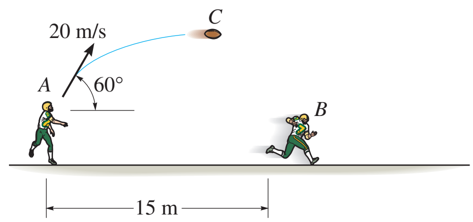
$$a_{B/C} = \sqrt{0.9486^2 + (-0.1429)^2} = 0.959 \text{ m/s}^2 \qquad \textbf{Ans.}$$

and the direction angle θ_a that $\mathbf{a}_{B/C}$ makes with the x axis is

$$\theta_a = \tan^{-1}\left(\frac{0.1429}{0.9486}\right) = 8.57° \qquad \textbf{Ans.}$$

12–238.

At a given instant the football player at A throws a football C with a velocity of 20 m/s in the direction shown. Determine the constant speed at which the player at B must run so that he can catch the football at the same elevation at which it was thrown. Also calculate the relative velocity and relative acceleration of the football with respect to B at the instant the catch is made. Player B is 15 m away from A when A starts to throw the football.



SOLUTION

Ball:

$(\xrightarrow{+})\, s = s_0 + v_0 t$

$s_C = 0 + 20 \cos 60° \, t$

$(+\uparrow) \qquad v = v_0 + a_c t$

$-20 \sin 60° = 20 \sin 60° - 9.81\, t$

$t = 3.53 \text{ s}$

$s_C = 35.31 \text{ m}$

Player B:

$(\xrightarrow{+})\; s_B = s_0 + v_B\, t$

Require,

$35.31 = 15 + v_B(3.53)$

$v_B = 5.75 \text{ m/s}$ **Ans.**

At the time of the catch

$(v_C)_x = 20 \cos 60° = 10 \text{ m/s} \rightarrow$

$(v_C)_y = 20 \sin 60° = 17.32 \text{ m/s} \downarrow$

$v_C = \mathbf{v}_B + \mathbf{v}_{C/B}$

$10\mathbf{i} - 17.32\mathbf{j} = 5.751\mathbf{i} + (v_{C/B})_x \mathbf{i} + (v_{C/B})_y \mathbf{j}$

$(\xrightarrow{+}) \qquad 10 = 5.75 + (v_{C/B})_x$

$(+\uparrow) \qquad -17.32 = (v_{C/B})_y$

$(v_{C/B})_x = 4.25 \text{ m/s} \rightarrow$

$(v_{C/B})_y = 17.32 \text{ m/s} \downarrow$

$v_{C/B} = \sqrt{(4.25)^2 + (17.32)^2} = 17.8 \text{ m/s}$ **Ans.**

$\theta = \tan^{-1}\left(\dfrac{17.32}{4.25}\right) = 76.2°$ **Ans.**

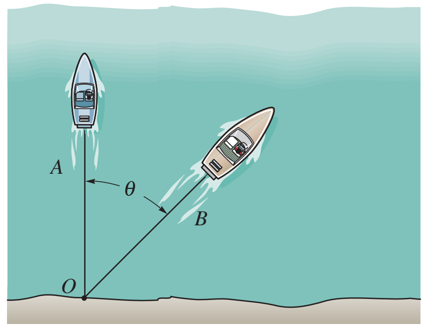
$a_C = \mathbf{a}_B + \mathbf{a}_{C/B}$

$-9.81\,\mathbf{j} = 0 + \mathbf{a}_{C/B}$

$a_{C/B} = 9.81 \text{ m/s}^2 \downarrow$ **Ans.**

12–239.

Both boats A and B leave the shore at O at the same time. If A travels at v_A and B travels at v_B, write a general expression to determine the velocity of A with respect to B.



SOLUTION

Relative Velocity:

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

$$v_A \mathbf{j} = v_B \sin\theta \mathbf{i} + v_B \cos\theta \mathbf{j} + \mathbf{v}_{A/B}$$

$$\mathbf{v}_{A/B} = -v_B \sin\theta \mathbf{i} + (v_A - v_B \cos\theta)\mathbf{j}$$

Thus, the magnitude of the relative velocity $\mathbf{v}_{A/B}$ is

$$v_{A/B} = \sqrt{(-v_B \sin\theta)^2 + (v_A - v_B\cos\theta)^2}$$

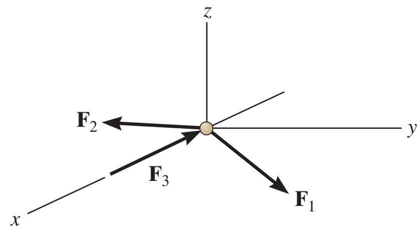
$$= \sqrt{v_A^2 + v_B^2 - 2v_A v_B \cos\theta} \qquad \textbf{Ans.}$$

And its direction is

$$\theta = \tan^{-1}\left(\frac{v_A - v_B\cos\theta}{v_B \sin\theta}\right) \qquad \textbf{Ans.}$$

13–1.

The 6-lb particle is subjected to the action of its weight and forces $\mathbf{F}_1 = \{2\mathbf{i} + 6\mathbf{j} - 2t\mathbf{k}\}$ lb, $\mathbf{F}_2 = \{t^2\mathbf{i} - 4t\mathbf{j} - 1\mathbf{k}\}$ lb, and $\mathbf{F}_3 = \{-2t\mathbf{i}\}$ lb, where t is in seconds. Determine the distance the ball is from the origin 2 s after being released from rest.



SOLUTION

$$\Sigma \mathbf{F} = m\mathbf{a}; \quad (2\mathbf{i} + 6\mathbf{j} - 2t\mathbf{k}) + (t^2\mathbf{i} - 4t\mathbf{j} - 1\mathbf{k}) - 2t\mathbf{i} - 6\mathbf{k} = \left(\frac{6}{32.2}\right)(\mathbf{a}_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k})$$

Equating components:

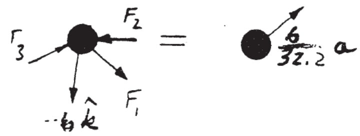
$$\left(\frac{6}{32.2}\right)a_x = t^2 - 2t + 2 \quad \left(\frac{6}{32.2}\right)a_y = -4t + 6 \quad \left(\frac{6}{32.2}\right)a_z = -2t - 7$$

Since $dv = a\,dt$, integrating from $\nu = 0, t = 0$, yields

$$\left(\frac{6}{32.2}\right)v_x = \frac{t^3}{3} - t^2 + 2t \qquad \left(\frac{6}{32.2}\right)v_y = -2t^2 + 6t \qquad \left(\frac{6}{32.2}\right)v_z = -t^2 - 7t$$

Since $ds = v\,dt$, integrating from $s = 0, t = 0$ yields

$$\left(\frac{6}{32.2}\right)s_x = \frac{t^4}{12} - \frac{t^3}{3} + t^2 \quad \left(\frac{6}{32.2}\right)s_y = -\frac{2t^3}{3} + 3t^2 \quad \left(\frac{6}{32.2}\right)s_z = -\frac{t^3}{3} - \frac{7t^2}{2}$$



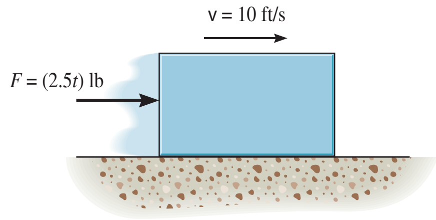
When $t = 2$ s then, $\quad s_x = 14.31$ ft, $\qquad s_y = 35.78$ ft $\qquad s_z = -89.44$ ft

Thus,

$$s = \sqrt{(14.31)^2 + (35.78)^2 + (-89.44)^2} = 97.4 \text{ ft} \qquad \textbf{Ans.}$$

13–2.

The 10-lb block has an initial velocity of 10 ft/s on the smooth plane. If a force $F = (2.5t)$ lb, where t is in seconds, acts on the block for 3 s, determine the final velocity of the block and the distance the block travels during this time.



SOLUTION

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 2.5t = \left(\frac{10}{32.2}\right)a$$

$$a = 8.05t$$

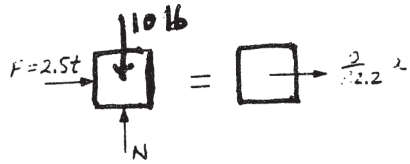
$$dv = a\,dt$$

$$\int_{10}^{v} dv = \int_0^t 8.05t\,dt$$

$$v = 4.025t^2 + 10$$

When $t = 3$ s,

$$v = 46.2 \text{ ft/s}$$ **Ans.**



$$ds = v\,dt$$

$$\int_0^s ds = \int_0^t (4.025t^2 + 10)\,dt$$

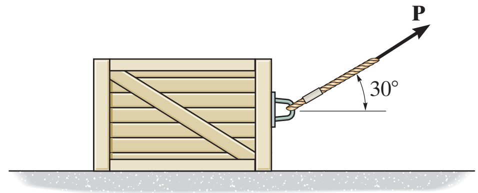
$$s = 1.3417t^3 + 10t$$

When $t = 3\,s$,

$$s = 66.2 \text{ ft}$$ **Ans.**

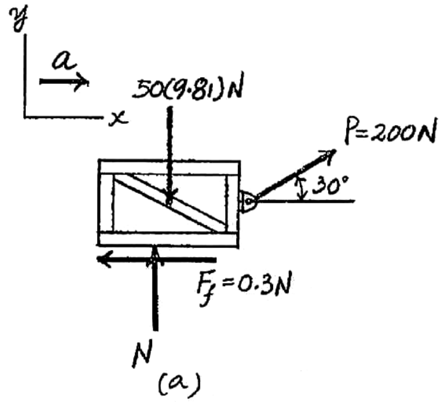
13–3.

If the coefficient of kinetic friction between the 50-kg crate and the ground is $\mu_k = 0.3$, determine the distance the crate travels and its velocity when $t = 3$ s. The crate starts from rest, and $P = 200$ N.



SOLUTION

Free-Body Diagram: The kinetic friction $F_f = \mu_k N$ is directed to the left to oppose the motion of the crate which is to the right, Fig. *a*.



Equations of Motion: Here, $a_y = 0$. Thus,

$$+\uparrow \Sigma F_y = 0; \qquad N - 50(9.81) + 200 \sin 30° = 0$$

$$N = 390.5 \text{ N}$$

$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \quad 200 \cos 30° - 0.3(390.5) = 50a$$

$$a = 1.121 \text{ m/s}^2$$

Kinematics: Since the acceleration **a** of the crate is constant,

$$(\overset{+}{\rightarrow}) \qquad v = v_0 + a_c t$$

$$v = 0 + 1.121(3) = 3.36 \text{ m/s} \qquad \textbf{Ans.}$$

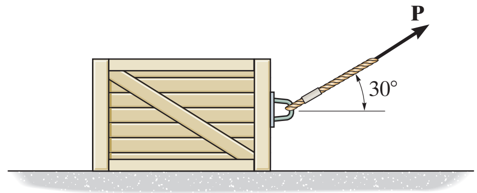
and

$$(\overset{+}{\rightarrow}) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s = 0 + 0 + \frac{1}{2}(1.121)\left(3^2\right) = 5.04 \text{ m} \qquad \textbf{Ans.}$$

*13–4.

If the 50-kg crate starts from rest and achieves a velocity of $v = 4$ m/s when it travels a distance of 5 m to the right, determine the magnitude of force **P** acting on the crate. The coefficient of kinetic friction between the crate and the ground is $\mu_k = 0.3$.



SOLUTION

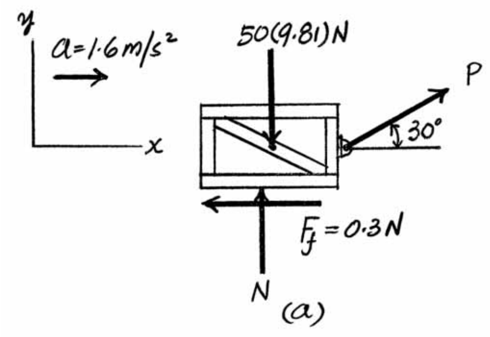
Kinematics: The acceleration **a** of the crate will be determined first since its motion is known.

$(\xrightarrow{+})$ $$v^2 = v_0^2 + 2a_c(s - s_0)$$

$$4^2 = 0^2 + 2a(5 - 0)$$

$$a = 1.60 \text{ m/s}^2 \rightarrow$$

Free-Body Diagram: Here, the kinetic friction $F_f = \mu_k N = 0.3N$ is required to be directed to the left to oppose the motion of the crate which is to the right, Fig. *a*.



Equations of Motion:

$+\uparrow \Sigma F_y = ma_y;$ $$N + P\sin 30° - 50(9.81) = 50(0)$$

$$N = 490.5 - 0.5P$$

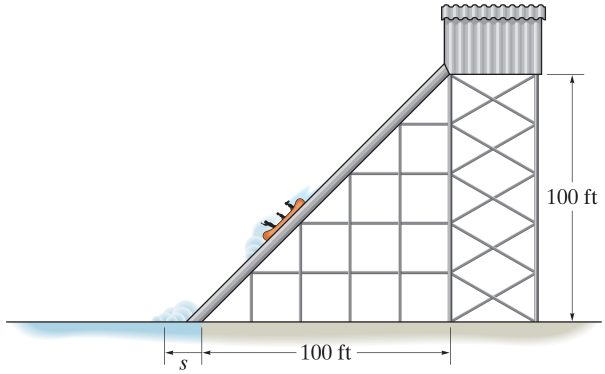
Using the results of **N** and **a**,

$\xrightarrow{+} \Sigma F_x = ma_x;$ $$P\cos 30° - 0.3(490.5 - 0.5P) = 50(1.60)$$

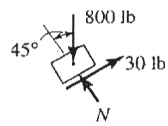
$$P = 224 \text{ N}$$ **Ans.**

13–5.

The water-park ride consists of an 800-lb sled which slides from rest down the incline and then into the pool. If the frictional resistance on the incline is $F_r = 30$ lb, and in the pool for a short distance $F_r = 80$ lb, determine how fast the sled is traveling when $s = 5$ ft.



SOLUTION



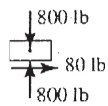
$+\swarrow \sum F_x = ma_x; \qquad 800 \sin 45° - 30 = \frac{800}{32.2}a$

$$a = 21.561 \text{ ft/s}^2$$

$$v_1^2 = v_0^2 + 2a_c(s - s_0)$$

$$v_1^2 = 0 + 2(21.561)(100\sqrt{2} - 0))$$

$$v_1 = \ = 78.093 \text{ ft/s}$$



$\overset{+}{\leftarrow} \sum F_x = ma_x; \qquad -80 = \frac{800}{32.2}a$

$$a = -3.22 \text{ ft/s}^2$$

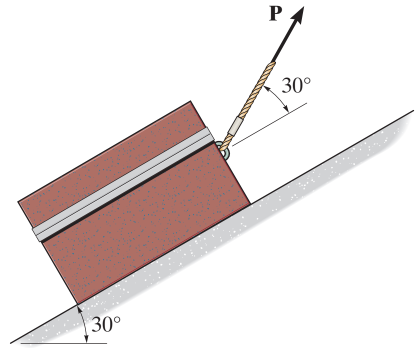
$$v_2^2 = v_1^2 + 2a_c(s_2 - s_1)$$

$$v_2^2 = (78.093)^2 + 2(-3.22)(5 - 0)$$

$$v_2 = 77.9 \text{ ft/s}$$ **Ans.**

13–6.

If $P = 400$ N and the coefficient of kinetic friction between the 50-kg crate and the inclined plane is $\mu_k = 0.25$, determine the velocity of the crate after it travels 6 m up the plane. The crate starts from rest.



SOLUTION

Free-Body Diagram: Here, the kinetic friction $F_f = \mu_k N = 0.25N$ is required to be directed down the plane to oppose the motion of the crate which is assumed to be directed up the plane. The acceleration **a** of the crate is also assumed to be directed up the plane, Fig. *a*.

Equations of Motion: Here, $a_{y'} = 0$. Thus,

$$\Sigma F_{y'} = ma_{y'}; \qquad N + 400 \sin 30° - 50(9.81) \cos 30° = 50(0)$$

$$N = 224.79 \text{ N}$$

Using the result of **N**,

$$\Sigma F_{x'} = ma_{y'}; \qquad 400 \cos 30° - 50(9.81) \sin 30° - 0.25(224.79) = 50a$$

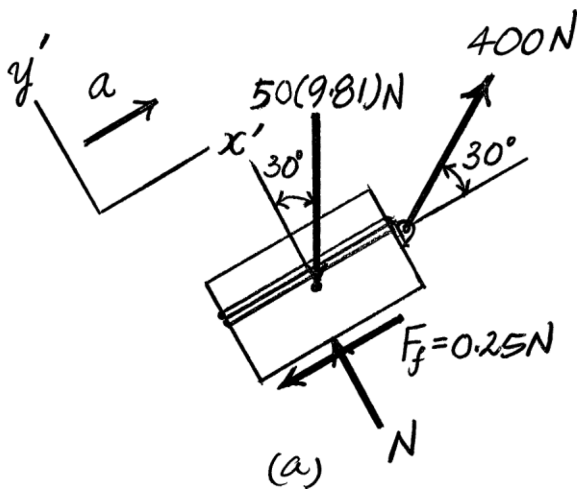
$$a = 0.8993 \text{ m/s}^2$$

Kinematics: Since the acceleration **a** of the crate is constant,

$$v^2 = v_0{}^2 + 2a_c(s - s_0)$$

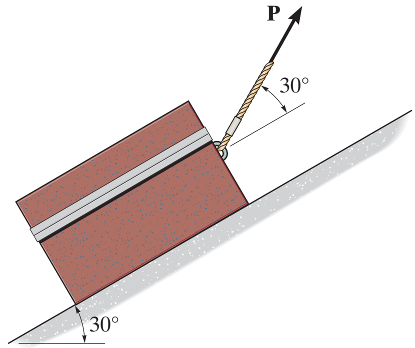
$$v^2 = 0 + 2(0.8993)(6 - 0)$$

$$v = 3.29 \text{ m/s} \qquad \textbf{Ans.}$$



13–7.

If the 50-kg crate starts from rest and travels a distance of 6 m up the plane in 4 s, determine the magnitude of force **P** acting on the crate. The coefficient of kinetic friction between the crate and the ground is $\mu_k = 0.25$.



SOLUTION

Kinematics: Here, the acceleration **a** of the crate will be determined first since its motion is known.

$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$6 = 0 + 0 + \frac{1}{2} a(4^2)$$

$$a = 0.75 \text{ m/s}^2$$

Free-Body Diagram: Here, the kinetic friction $F_f = \mu_k N = 0.25N$ is required to be directed down the plane to oppose the motion of the crate which is directed up the plane, Fig. *a*.

Equations of Motion: Here, $a_{y'} = 0$. Thus,

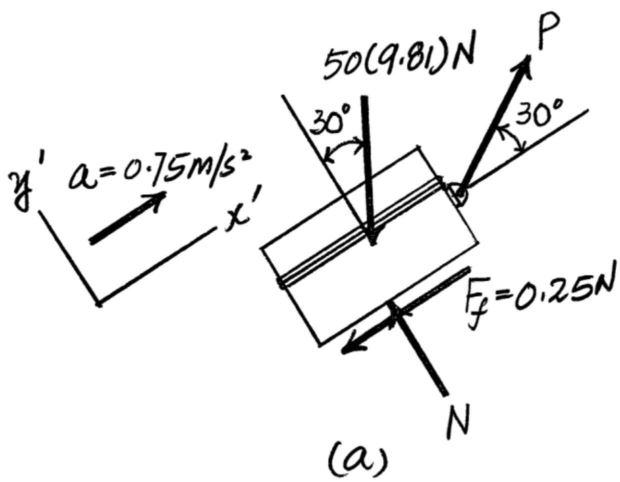
$\Sigma F_{y'} = m a_{y'};$ $\quad N + P \sin 30° - 50(9.81) \cos 30° = 50(0)$

$$N = 424.79 - 0.5P$$

Using the results of **N** and **a**,

$\Sigma F_{x'} = m a_{x'};$ $\quad P \cos 30° - 0.25(424.79 - 0.5P) - 50(9.81) \sin 30° = 50(0.75)$

$P = 392 \text{ N}$ **Ans.**



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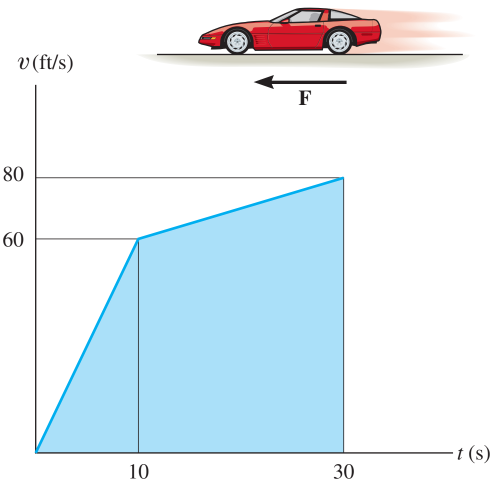
***13–8.**

The speed of the 3500-lb sports car is plotted over the 30-s time period. Plot the variation of the traction force **F** needed to cause the motion.



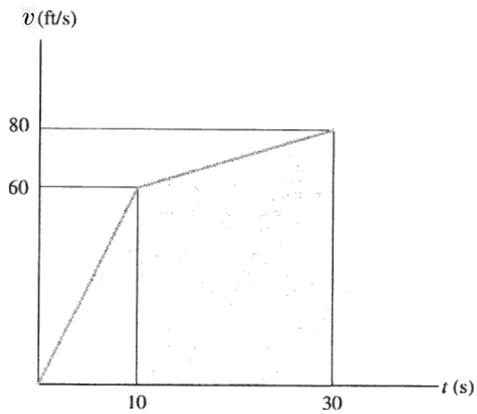
SOLUTION

Kinematics: For $\mathbf{0} \leq \mathbf{t} < \mathbf{10}$ **s**. $v = \dfrac{60}{10}t = \{6t\}$ ft/s. Applying equation $a = \dfrac{dv}{dt}$, we have

$$a = \frac{dv}{dt} = 6 \text{ ft/s}^2$$

For $\mathbf{10} < t \leq \mathbf{30}$ **s**, $\dfrac{v - 60}{t - 10} = \dfrac{80 - 60}{30 - 10}$, $v = \{t + 50\}$ ft/s. Applying equation $a = \dfrac{dv}{dt}$, we have

$$a = \frac{dv}{dt} = 1 \text{ ft/s}^2$$



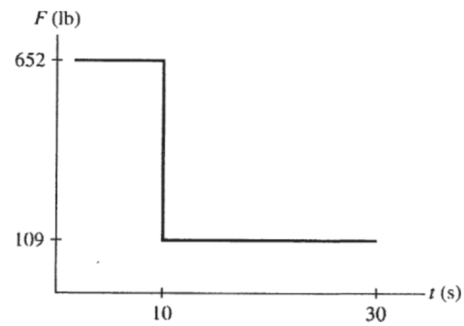
Equation of Motion:

For $\mathbf{0} \leq \mathbf{t} < \mathbf{10}$ **s**

$$\overset{+}{\leftarrow} \sum F_x = ma_x; \quad F = \left(\frac{3500}{32.2}\right)(6) = 652 \text{ lb}$$ **Ans.**

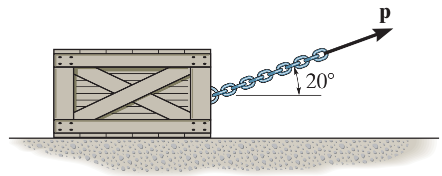
For $\mathbf{10} < \boldsymbol{t} \leq \mathbf{30}$ **s**

$$\overset{+}{\leftarrow} \sum F_x = ma_x; \quad F = \left(\frac{3500}{32.2}\right)(1) = 109 \text{ lb}$$ **Ans.**



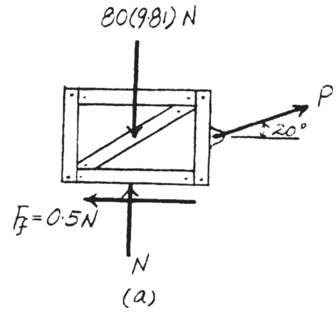
13–9.

The crate has a mass of 80 kg and is being towed by a chain which is always directed at 20° from the horizontal as shown. If the magnitude of **P** is increased until the crate begins to slide, determine the crate's initial acceleration if the coefficient of static friction is $\mu_s = 0.5$ and the coefficient of kinetic friction is $\mu_k = 0.3$.



SOLUTION

Equations of Equilibrium: If the crate is on the verge of slipping, $F_f = \mu_s N = 0.5N$. From FBD(a),



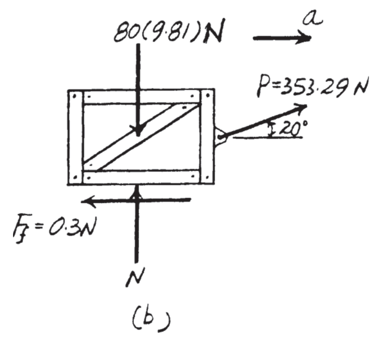
$$+\uparrow \Sigma F_y = 0; \qquad N + P\sin 20° - 80(9.81) = 0 \qquad \textbf{(1)}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad P\cos 20° - 0.5N = 0 \qquad \textbf{(2)}$$

Solving Eqs.(1) and (2) yields

$$P = 353.29\text{ N} \qquad N = 663.97\text{ N}$$

Equations of Motion: The friction force developed between the crate and its contacting surface is $F_f = \mu_k N = 0.3N$ since the crate is moving. From FBD(b),



$$+\uparrow \Sigma F_y = ma_y; \qquad N - 80(9.81) + 353.29\sin 20° = 80(0)$$

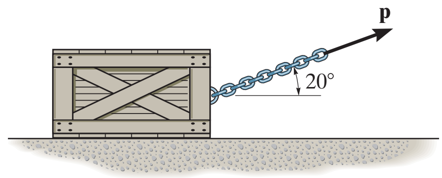
$$N = 663.97\text{ N}$$

$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad 353.29\cos 20° - 0.3(663.97) = 80a$$

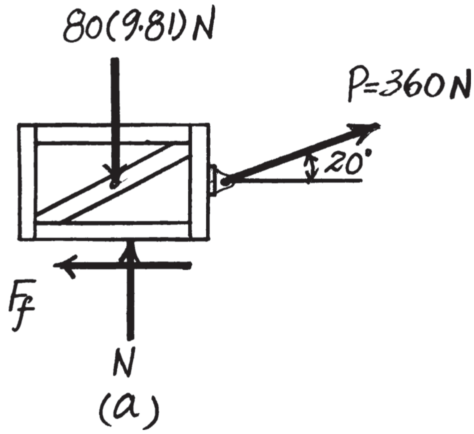
$$a = 1.66\text{ m/s}^2 \qquad \textbf{Ans.}$$

13–10.

The crate has a mass of 80 kg and is being towed by a chain which is always directed at 20° from the horizontal as shown. Determine the crate's acceleration in $t = 2$ s if the coefficient of static friction is $\mu_s = 0.4$, the coefficient of kinetic friction is $\mu_k = 0.3$, and the towing force is $P = (90t^2)$ N, where t is in seconds.



SOLUTION



Equations of Equilibrium: At $t = 2$ s, $P = 90\left(2^2\right) = 360$ N. From FBD(a)

$$+\uparrow \Sigma F_y = 0; \qquad N + 360 \sin 20^\circ - 80(9.81) = 0 \qquad N = 661.67 \text{ N}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 360 \cos 20^\circ - F_f = 0 \qquad F_f = 338.29 \text{ N}$$

Since $F_f > (F_f)_{\max} = \mu_s N = 0.4(661.67) = 264.67$ N, the crate accelerates.

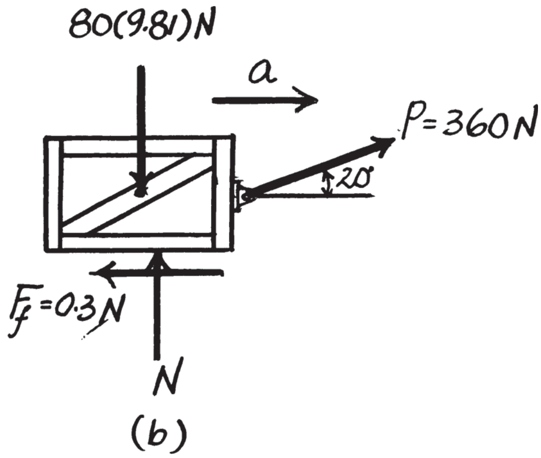
Equations of Motion: The friction force developed between the crate and its contacting surface is $F_f = \mu_k N = 0.3N$ since the crate is moving. From FBD(b),

$$+\uparrow \Sigma F_y = ma_y; \qquad N - 80(9.81) + 360 \sin 20^\circ = 80(0)$$

$$N = 661.67 \text{ N}$$

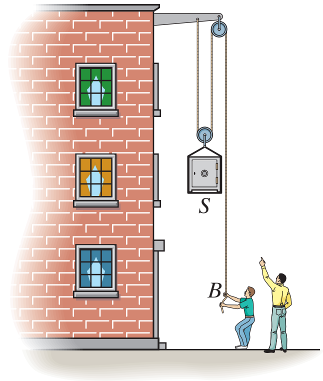
$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad 360 \cos 20^\circ - 0.3(661.67) = 80a$$

$$a = 1.75 \text{ m/s}^2$$ **Ans.**



13–11.

The safe S has a weight of 200 lb and is supported by the rope and pulley arrangement shown. If the end of the rope is given to a boy B of weight 90 lb, determine his acceleration if in the confusion he doesn't let go of the rope. Neglect the mass of the pulleys and rope.

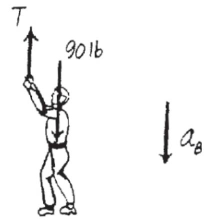


SOLUTION

Equation of Motion: The tension T developed in the cord is the same throughout the entire cord since the cord passes over the smooth pulleys.

From FBD(a),

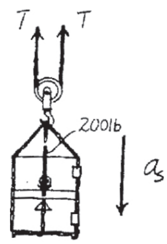
$$+\uparrow \Sigma F_y = ma_y; \qquad T - 90 = -\left(\frac{90}{32.2}\right)a_B \qquad \textbf{(1)}$$



(a)

From FBD(b),

$$+\uparrow \Sigma F_y = ma_y; \qquad 2T - 200 = -\left(\frac{200}{32.2}\right)a_S \qquad \textbf{(2)}$$



(b)

Kinematic: Establish the position-coordinate equation, we have

$$2s_S + s_B = l$$

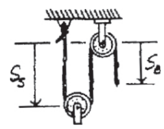
Taking time derivative twice yields

$$(+\downarrow) \qquad 2a_S + a_B = 0 \qquad \textbf{(3)}$$

Solving Eqs.(1), (2), and (3) yields

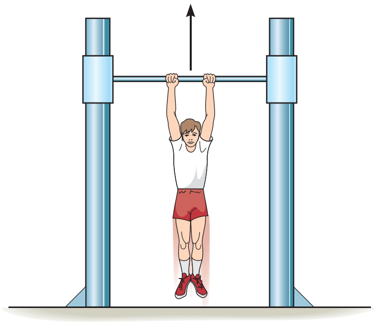
$$a_B = -2.30\ \text{ft/s}^2 = 2.30\ \text{ft/s}^2 \uparrow \qquad \textbf{Ans.}$$

$$a_S = 1.15\ \text{ft/s}^2 \downarrow \qquad T = 96.43\ \text{lb}$$



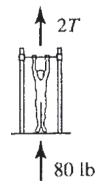
***13–12.**

The boy having a weight of 80 lb hangs uniformly from the bar. Determine the force in each of his arms in $t = 2$ s if the bar is moving upward with (a) a constant velocity of 3 ft/s, and (b) a speed of $v = (4t^2)$ ft/s, where t is in seconds.



SOLUTION

(a) $T = 40$ lb **Ans.**



(b) $v = 4t^2$

$a = 8t$

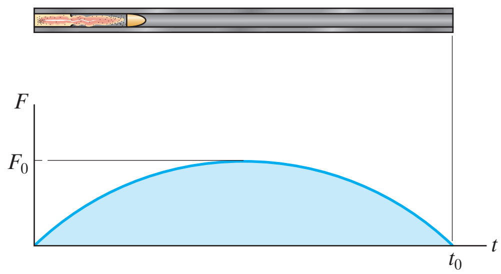
$$+\uparrow \sum F_y = ma_y; \qquad 2T - 80 = \frac{80}{32.2}(8t)$$

At $t = 2$ s.

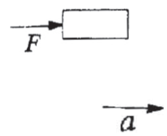
$T = 59.9$ lb **Ans.**

13–13.

The bullet of mass m is given a velocity due to gas pressure caused by the burning of powder within the chamber of the gun. Assuming this pressure creates a force of $F = F_0 \sin(\pi t/t_0)$ on the bullet, determine the velocity of the bullet at any instant it is in the barrel. What is the bullet's maximum velocity? Also, determine the position of the bullet in the barrel as a function of time.



SOLUTION



$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad F_0 \sin\left(\frac{\pi t}{t_0}\right) = ma$$

$$a = \frac{dv}{dt} = \left(\frac{F_0}{m}\right) \sin\left(\frac{\pi t}{t_0}\right)$$

$$\int_0^v dv = \int_0^t \left(\frac{F_0}{m}\right) \sin\left(\frac{\pi t}{t_0}\right) dt \qquad v = -\left(\frac{F_0 t_0}{\pi m}\right) \cos\left(\frac{\pi t}{t_0}\right)\Big]_0^t$$

$$v = \left(\frac{F_0 t_0}{\pi m}\right)\left(1 - \cos\left(\frac{\pi t}{t_0}\right)\right)$$ **Ans.**

v_{max} occurs when $\cos\left(\frac{\pi t}{t_0}\right) = -1$, or $t = t_0$.

$$v_{max} = \frac{2F_0 t_0}{\pi m}$$ **Ans.**

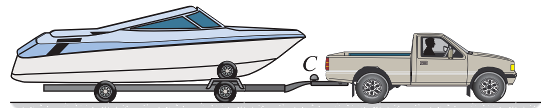
$$\int_0^s ds = \int_0^t \left(\frac{F_0 t_0}{\pi m}\right)\left(1 - \cos\left(\frac{\pi t}{t_0}\right)\right) dt$$

$$s = \left(\frac{F_0 t_0}{\pi m}\right)\left[t - \frac{t_0}{\pi} \sin\left(\frac{\pi t}{t_0}\right)\right]_0^t$$

$$s = \left(\frac{F_0 t_0}{\pi m}\right)\left(t - \frac{t_0}{\pi} \sin\left(\frac{\pi t}{t_0}\right)\right)$$ **Ans.**

13–14.

The 2-Mg truck is traveling at 15 m/s when the brakes on all its wheels are applied, causing it to skid for a distance of 10 m before coming to rest. Determine the constant horizontal force developed in the coupling C, and the frictional force developed between the tires of the truck and the road during this time. The total mass of the boat and trailer is 1 Mg.



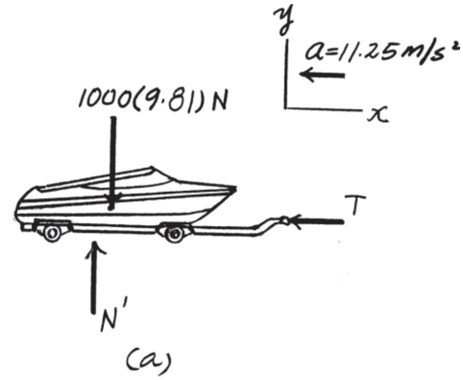
SOLUTION

Kinematics: Since the motion of the truck and trailer is known, their common acceleration **a** will be determined first.

$$\left(\overset{+}{\rightarrow}\right) \qquad v^2 = v_0{}^2 + 2a_c(s - s_0)$$

$$0 = 15^2 + 2a(10 - 0)$$

$$a = -11.25 \text{ m/s}^2 = 11.25 \text{ m/s}^2 \leftarrow$$



Free-Body Diagram: The free-body diagram of the truck and trailer are shown in Figs. (a) and (b), respectively. Here, **F** representes the frictional force developed when the truck skids, while the force developed in coupling C is represented by **T**.

Equations of Motion: Using the result of **a** and referrning to Fig. (a),

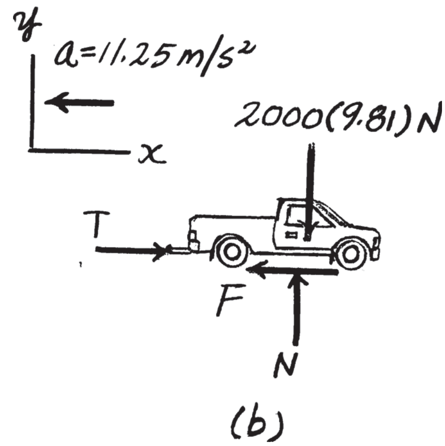
$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad -T = 1000(-11.25)$$

$$T = 11\,250 \text{ N} = 11.25 \text{ kN} \qquad \textbf{Ans.}$$

Using the results of **a** and **T** and referring to Fig. (b),

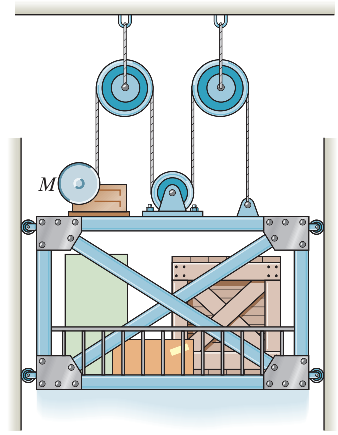
$$+\uparrow \Sigma F_x = ma_x; \qquad 11\,250 - F = 2000(-11.25)$$

$$F = 33\,750 \text{ N} = 33.75 \text{ kN} \qquad \textbf{Ans.}$$

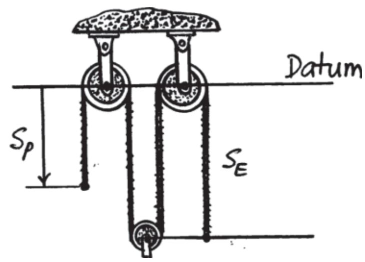


13–15.

A freight elevator, including its load, has a mass of 500 kg. It is prevented from rotating by the track and wheels mounted along its sides. When $t = 2$ s, the motor M draws in the cable with a speed of 6 m/s, *measured relative to the elevator.* If it starts from rest, determine the constant acceleration of the elevator and the tension in the cable. Neglect the mass of the pulleys, motor, and cables.



SOLUTION



$$3s_E + s_P = l$$

$$3v_E = -v_P$$

$$(+\downarrow) \qquad v_P = v_E + v_{P/E}$$

$$-3v_E = v_E + 6$$

$$v_E = -\frac{6}{4} = -1.5 \text{ m/s} = 1.5 \text{ m/s} \uparrow$$

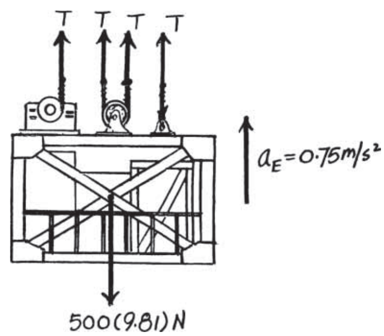
$$(+\uparrow) \qquad v = v_0 + a_c t$$

$$1.5 = 0 + a_E(2)$$

$$a_E = 0.75 \text{ m/s}^2 \uparrow$$ **Ans.**

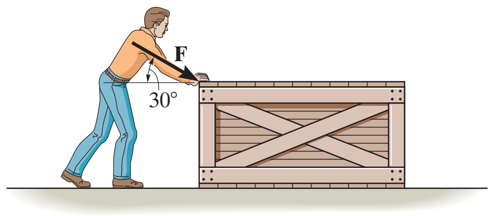
$$+\uparrow \Sigma F_y = ma_y; \qquad 4T - 500(9.81) = 500(0.75)$$

$$T = 1320 \text{ N} = 1.32 \text{ kN}$$ **Ans.**



***13–16.**

The man pushes on the 60-lb crate with a force **F**. The force is always directed down at 30° from the horizontal as shown, and its magnitude is increased until the crate begins to slide. Determine the crate's initial acceleration if the coefficient of static friction is $\mu_s = 0.6$ and the coefficient of kinetic friction is $\mu_k = 0.3$.



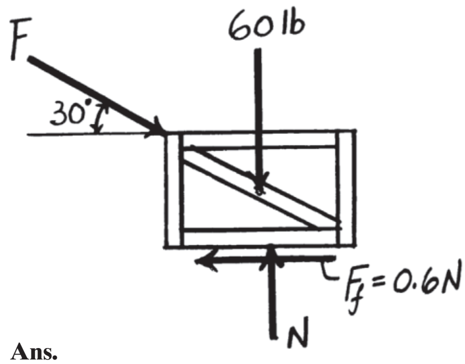
SOLUTION

Force to produce motion:

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F\cos 30^\circ - 0.6N = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad N - 60 - F\sin 30^\circ = 0$$

$$N = 91.80 \text{ lb} \qquad F = 63.60 \text{ lb}$$

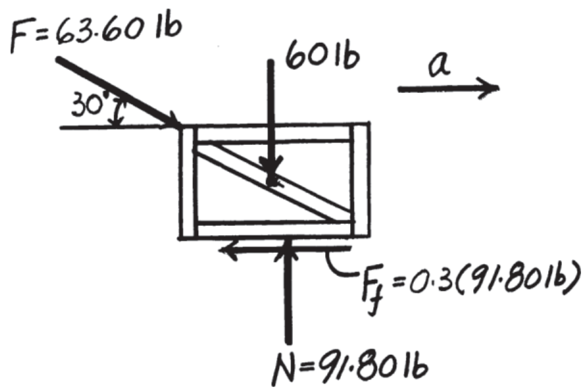


Since $N = 91.80$ lb,

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 63.60\cos 30^\circ - 0.3(91.80) = \left(\frac{60}{32.2}\right)a$$

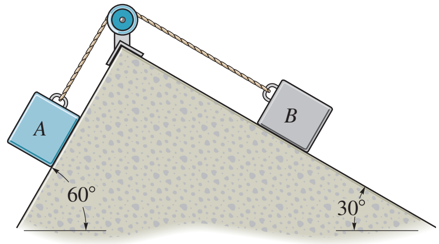
$$a = 14.8 \text{ ft/s}^2$$

Ans.



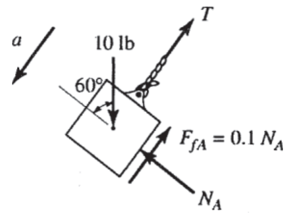
13–17.

The double inclined plane supports two blocks A and B, each having a weight of 10 lb. If the coefficient of kinetic friction between the blocks and the plane is $\mu_k = 0.1$, determine the acceleration of each block.



SOLUTION

Equation of Motion: Since blocks A and B are sliding along the plane, the friction forces developed between the blocks and the plane are $(F_f)_A = \mu_k N_A = 0.1\, N_A$ and $(F_f)_B = \mu_k N_B = 0.1 N_B$. Here, $a_A = a_B = a$. Applying Eq. 13–7 to FBD(a), we have

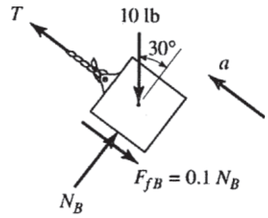


(a)

$$\nwarrow + \sum F_{y'} = ma_{y'}; \quad N_A - 10 \cos 60° = \left(\frac{10}{32.2}\right)(0) \quad N_A = 5.00 \text{ lb}$$

$$\nearrow + \sum F_{x'} = ma_{x'}; \quad T + 0.1(5.00) - 10 \sin 60° = -\left(\frac{10}{32.2}\right) a \qquad \textbf{(1)}$$

From FBD(b),



(b)

$$\nearrow + \sum F_{y'} = ma_{y'}; \quad N_B - 10 \cos 30° = \left(\frac{10}{32.2}\right)(0) \quad N_B = 8.660 \text{ lb}$$

$$\nwarrow + \sum F_{x'} = ma_{x'}; \quad T - 0.1(8.660) - 10 \sin 30° = \left(\frac{10}{32.2}\right) a \qquad \textbf{(2)}$$

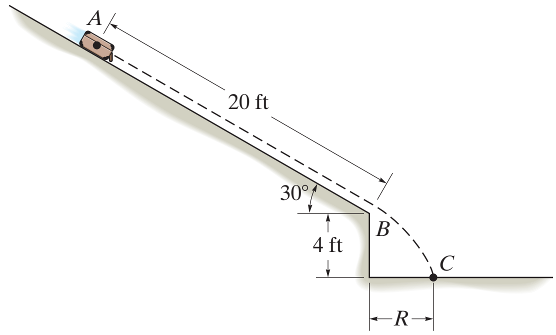
Solving Eqs. (1) and (2) yields

$$a = 3.69 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

$$T = 7.013 \text{ lb}$$

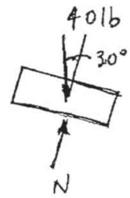
13–18.

A 40-lb suitcase slides from rest 20 ft down the smooth ramp. Determine the point where it strikes the ground at C. How long does it take to go from A to C?



SOLUTION

$+\searrow \Sigma F_x = m\,a_x; \qquad 40 \sin 30^\circ = \dfrac{40}{32.2}a$



$a = 16.1\ \text{ft/s}^2$

$(+\searrow) v^2 = v_0^2 + 2\,a_c(s - s_0);$

$v_B^2 = 0 + 2(16.1)(20)$

$v_B = 25.38\ \text{ft/s}$

$(+\searrow)\ v = v_0 + a_c\,t;$

$25.38 = 0 + 16.1\,t_{AB}$

$t_{AB} = 1.576\ \text{s}$

$(\xrightarrow{+}) s_x = (s_x)_0 + (v_x)_0\,t$

$R = 0 + 25.38 \cos 30^\circ (t_{BC})$

$(+\downarrow)\ s_y = (s_y)_0 + (v_y)_0\,t + \dfrac{1}{2}a_c t^2$

$4 = 0 + 25.38 \sin 30^\circ\, t_{BC} + \dfrac{1}{2}(32.2)(t_{BC})^2$

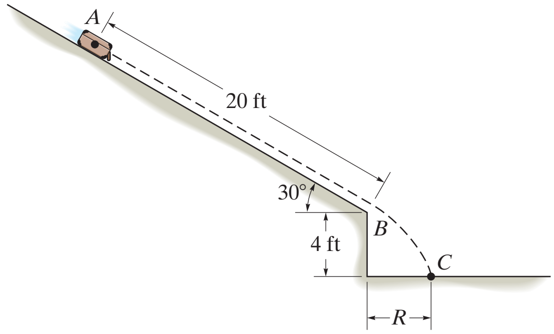
$t_{BC} = 0.2413\ \text{s}$

$R = 5.30\ \text{ft}$ **Ans.**

Total time $= t_{AB} + t_B{}^C = 1.82\ \text{s}$ **Ans.**

13–19.

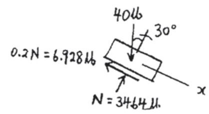
Solve Prob. 13–18 if the suitcase has an initial velocity down the ramp of $v_A = 10$ ft/s and the coefficient of kinetic friction along AB is $\mu_k = 0.2$.



SOLUTION

$+\searrow \Sigma F_x = ma_x; \qquad 40 \sin 30° - 6.928 = \frac{40}{32.2} a$

$a = 10.52 \text{ ft/s}^2$



$(+\searrow)\ v^2 = v_0^2 + 2\,a_c(s - s_0);$

$v_B^2 = (10)^2 + 2(10.52)(20)$

$v_B = 22.82$ ft/s

$(+\searrow)\ v = v_0 + a_c t;$

$22.82 = 10 + 10.52\, t_{AB}$

$t_{AB} = 1.219$ s

$(\overset{+}{\rightarrow})\ s_x = (s_x)_0 + (v_x)_0\, t$

$R = 0 + 22.82 \cos 30° \,(t_{BC})$

$(+\downarrow)\ s_y = (s_y)_0 + (v_y)_0\, t + \frac{1}{2} a_c t^2$

$4 = 0 + 22.82 \sin 30°\, t_{BC} + \frac{1}{2}(32.2)(t_{BC})^2$

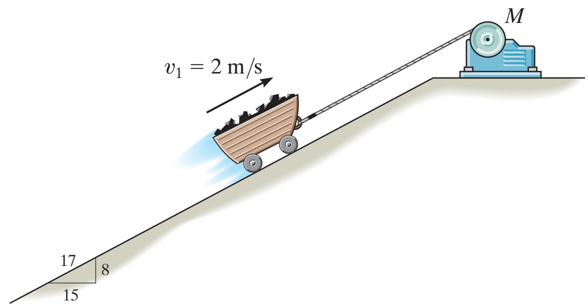
$t_{BC} = 0.2572$ s

$R = 5.08$ ft **Ans.**

Total time $= t_{AB} + t_{BC} = 1.48$ s **Ans.**

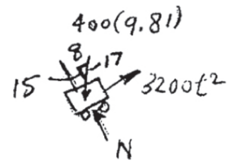
***13–20.**

The 400-kg mine car is hoisted up the incline using the cable and motor M. For a short time, the force in the cable is $F = (3200t^2)$ N, where t is in seconds. If the car has an initial velocity $v_1 = 2$ m/s when $t = 0$, determine its velocity when $t = 2$ s.



SOLUTION

$\nearrow + \Sigma F_{x'} = ma_{x'};\qquad 3200t^2 - 400(9.81)\left(\dfrac{8}{17}\right) = 400a \qquad a = 8t^2 - 4.616$



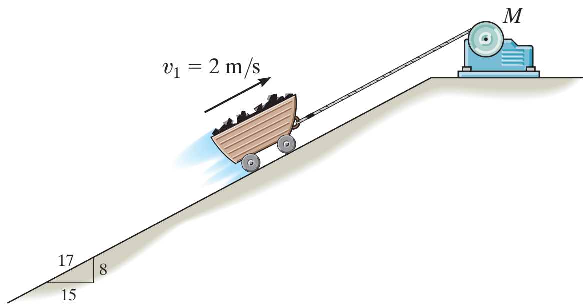
$$dv = a\,dt$$

$$\int_2^v dv = \int_0^2 (8t^2 - 4.616)\,dt$$

$$v = 14.1 \text{ m/s}$$ **Ans.**

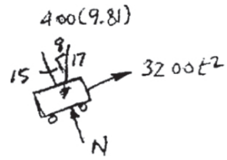
13–21.

The 400-kg mine car is hoisted up the incline using the cable and motor M. For a short time, the force in the cable is $F = (3200t^2)$ N, where t is in seconds. If the car has an initial velocity $v_1 = 2$ m/s at $s = 0$ and $t = 0$, determine the distance it moves up the plane when $t = 2$ s.



SOLUTION

$$\nearrow + \Sigma F_{x'} = ma_{x'}; \qquad 3200t^2 - 400(9.81)\left(\frac{8}{17}\right) = 400a \qquad a = 8t^2 - 4.616$$



$$dv = a\,dt$$

$$\int_2^v dv = \int_0^t (8t^2 - 4.616)\,dt$$

$$v = \frac{ds}{dt} = 2.667t^3 - 4.616t + 2$$

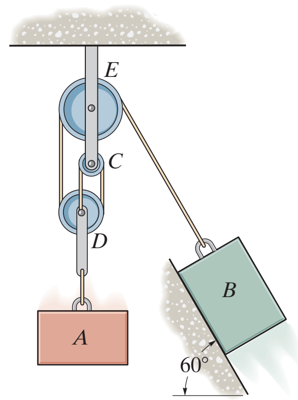
$$\int_0^s ds = \int_0^2 (2.667\,t^3 - 4.616t + 2)\,dt$$

$$s = 5.43 \text{ m}$$

Ans.

13–22.

Determine the required mass of block A so that when it is released from rest it moves the 5-kg block B a distance of 0.75 m up along the smooth inclined plane in $t = 2$ s. Neglect the mass of the pulleys and cords.

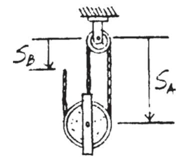


SOLUTION

Kinematic: Applying equation $s = s_0 + v_0 t + \frac{1}{2} a_c t^2$, we have

$$(\nwarrow +) \qquad 0.75 = 0 + 0 + \frac{1}{2} a_B \left(2^2\right) \qquad a_B = 0.375 \text{ m/s}^2$$

Establishing the position - coordinate equation, we have

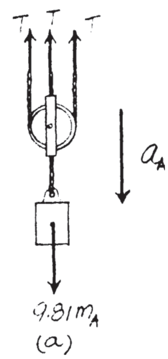


$$2s_A + (s_A - s_B) = l \qquad 3s_A - s_B = l$$

Taking time derivative twice yields

$$3a_A - a_B = 0 \tag{1}$$

From Eq.(1),



$$3a_A - 0.375 = 0 \qquad a_A = 0.125 \text{ m/s}^2$$

Equation of Motion: The tension T developed in the cord is the same throughout the entire cord since the cord passes over the smooth pulleys. From FBD(b),

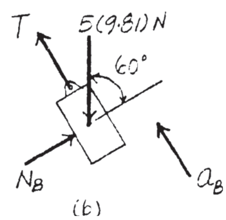
$$\nwarrow + \Sigma F_{y'} = ma_{y'}; \qquad T - 5(9.81)\sin 60° = 5(0.375)$$

$$T = 44.35 \text{ N}$$

From FBD(a),

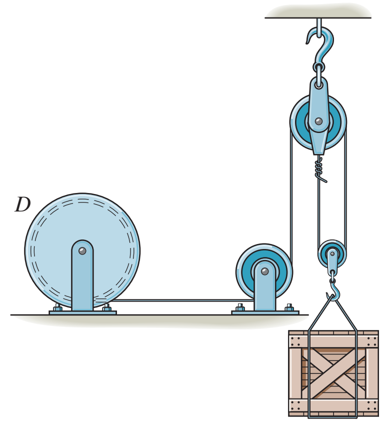
$$+\uparrow \Sigma F_y = ma_y; \qquad 3(44.35) - 9.81m_A = m_A(-0.125)$$

$$m_A = 13.7 \text{ kg} \qquad \textbf{Ans.}$$



13–23.

The winding drum D is drawing in the cable at an accelerated rate of $5\ \text{m/s}^2$. Determine the cable tension if the suspended crate has a mass of 800 kg.



SOLUTION

$$s_A + 2\,s_B = l$$

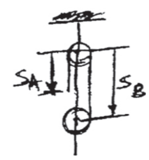
$$a_A = -2\,a_B$$

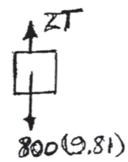
$$5 = -2\,a_B$$

$$a_B = -2.5\ \text{m/s}^2 = 2.5\ \text{m/s}^2 \uparrow$$

$+\uparrow \Sigma F_y = ma_y;\qquad 2T - 800(9.81) = 800(2.5)$

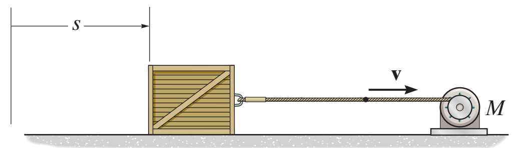
$$T = 4924\ \text{N} = 4.92\ \text{kN}$$ **Ans.**





***13–24.**

If the motor draws in the cable at a rate of $v = (0.05s^{3/2})$ m/s, where s is in meters, determine the tension developed in the cable when $s = 10$ m. The crate has a mass of 20 kg, and the coefficient of kinetic friction between the crate and the ground is $\mu_k = 0.2$.



SOLUTION

Kinematics: Since the motion of the create is known, its acceleration **a** will be determined first.

$$a = v\frac{dv}{ds} = \left(0.05s^{3/2}\right)\left[(0.05)\left(\frac{3}{2}\right)s^{1/2}\right] = 0.00375s^2 \text{ m/s}^2$$

When $s = 10$ m,

$$a = 0.00375(10^2) = 0.375 \text{ m/s}^2 \rightarrow$$

Free-Body Diagram: The kinetic friction $F_f = \mu_k N = 0.2N$ must act to the left to oppose the motion of the crate which is to the right, Fig. a.

Equations of Motion: Here, $a_y = 0$. Thus,

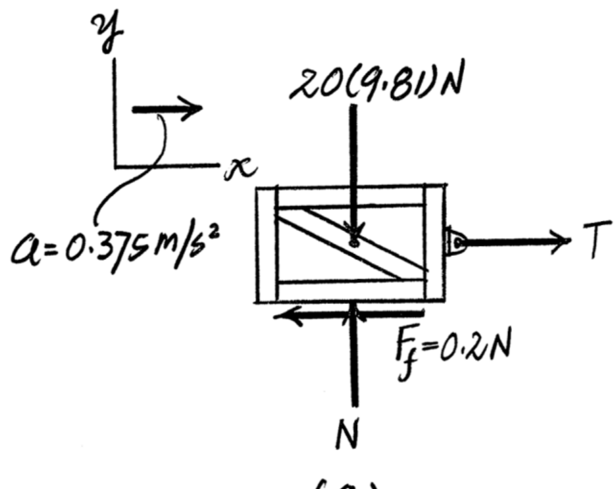
$$+\uparrow \Sigma F_y = ma_y; \qquad N - 20(9.81) = 20(0)$$

$$N = 196.2 \text{ N}$$

Using the results of **N** and **a**,

$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad T - 0.2(196.2) = 20(0.375)$$

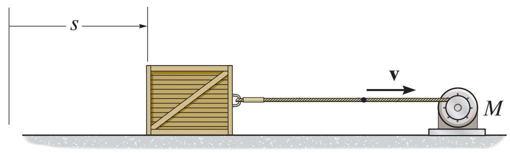
$$T = 46.7 \text{ N}$$ **Ans.**



(a)

13–25.

If the motor draws in the cable at a rate of $v = (0.05t^2)$ m/s, where t is in seconds, determine the tension developed in the cable when $t = 5$ s. The crate has a mass of 20 kg and the coefficient of kinetic friction between the crate and the ground is $\mu_k = 0.2$.



SOLUTION

Kinematics: Since the motion of the crate is known, its acceleration **a** will be determined first.

$$a = \frac{dv}{dt} = 0.05(2t) = (0.1t)\ \text{m/s}^2$$

When $t = 5$ s,

$$a = 0.1(5) = 0.5\ \text{m/s}^2 \rightarrow$$

Free-Body Diagram: The kinetic friction $F_f = \mu_k N = 0.2N$ must act to the left to oppose the motion of the crate which is to the right, Fig. a.

Equations of Motion: Here, $a_y = 0$. Thus,

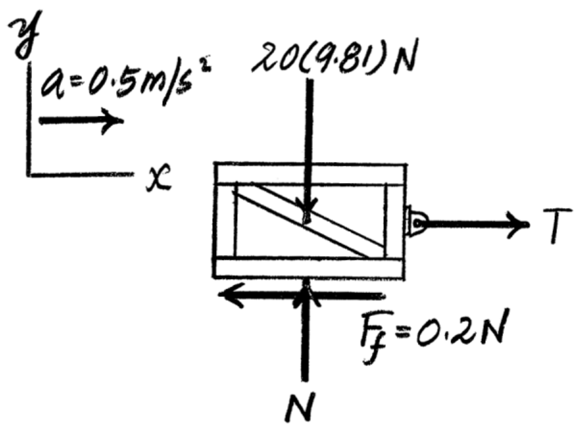
$$+\uparrow \Sigma F_y = ma_y; \qquad N - 20(9.81) = 0$$

$$N = 196.2\ \text{N}$$

Using the results of **N** and **a**,

$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad T - 0.2(196.2) = 20(0.5)$$

$$T = 49.2\ \text{N}$$ **Ans.**

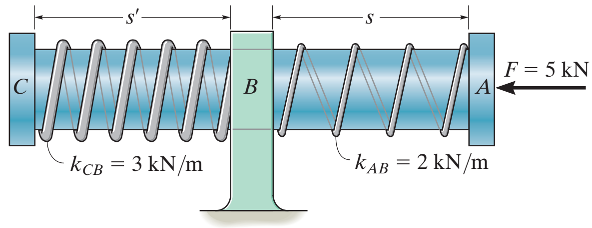


(a)

لجنة الميكانيك - الإتجاه الإسلامي

13–26.

The 2-kg shaft CA passes through a smooth journal bearing at B. Initially, the springs, which are coiled loosely around the shaft, are unstretched when no force is applied to the shaft. In this position $s = s' = 250$ mm and the shaft is at rest. If a horizontal force of $F = 5$ kN is applied, determine the speed of the shaft at the instant $s = 50$ mm, $s' = 450$ mm. The ends of the springs are attached to the bearing at B and the caps at C and A.



SOLUTION

$F_{CB} = k_{CB}x = 3000x \qquad F_{AB} = k_{AB}x = 2000x$

$\stackrel{+}{\leftarrow} \Sigma F_x = ma_x; \qquad 5000 - 3000x - 2000x = 2a$

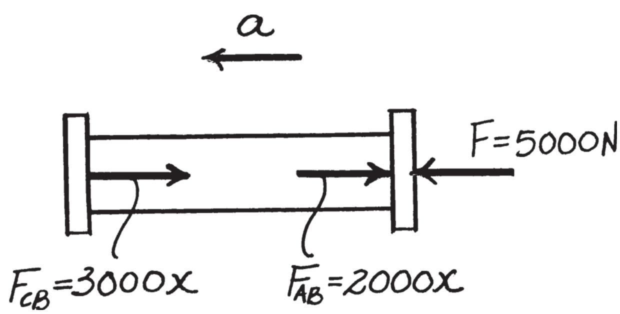
$$2500 - 2500x = a$$

$a\,dx - v\,dv$

$$\int_0^{0.2} (2500 - 2500x)\,dx = \int_0^{v} v\,dv$$

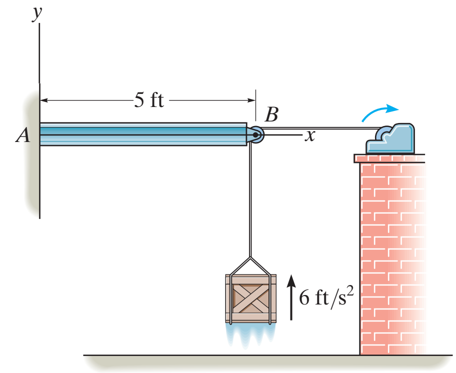
$$2500(0.2) - \left(\frac{2500(0.2)^2}{2}\right) = \frac{v^2}{2}$$

$v = 30$ m/s **Ans.**



13–27.

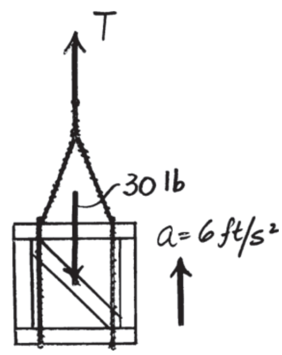
The 30-lb crate is being hoisted upward with a constant acceleration of 6 ft/s^2. If the uniform beam AB has a weight of 200 lb, determine the components of reaction at the fixed support A. Neglect the size and mass of the pulley at B. *Hint:* First find the tension in the cable, then analyze the forces in the beam using statics.



SOLUTION

Crate:

$+\uparrow \Sigma F_y = ma_y; \qquad T - 30 = \left(\dfrac{30}{32.2}\right)(6) \qquad T = 35.59 \text{ lb}$

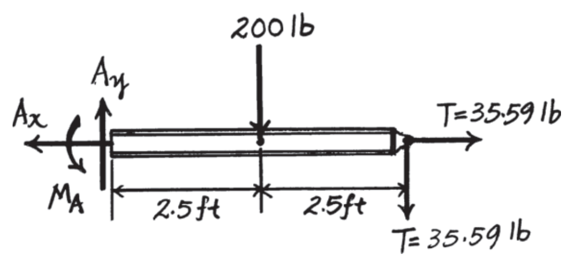


Beam:

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad -A_x + 35.59 = 0 \qquad A_x = 35.6 \text{ lb}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad A_y - 200 - 35.59 = 0 \qquad A_y = 236 \text{ lb}$ **Ans.**

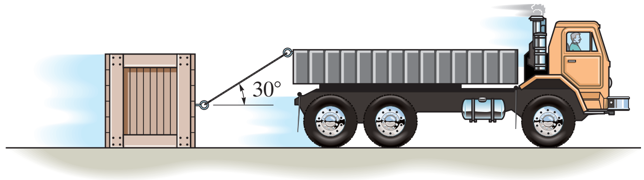
$\circlearrowleft + \Sigma M_A = 0; \qquad M_A - 200(2.5) - (35.59)(5) = 0 \qquad M_A = 678 \text{ lb} \cdot \text{ft}$ **Ans.**



لجنة الميكانيك - الإتجاه الإسلامي

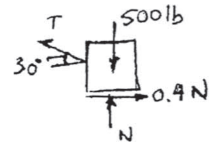
***13–28.**

The driver attempts to tow the crate using a rope that has a tensile strength of 200 lb. If the crate is originally at rest and has a weight of 500 lb, determine the greatest acceleration it can have if the coefficient of static friction between the crate and the road is $\mu_s = 0.4$, and the coefficient of kinetic friction is $\mu_k = 0.3$.



SOLUTION

Equilibrium: In order to slide the crate, the towing force must overcome static friction.

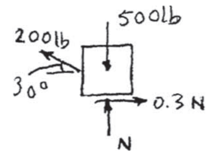


$$\xrightarrow{+} \Sigma F_x = 0; \qquad -T\cos 30° + 0.4N = 0 \qquad \textbf{(1)}$$

$$\xrightarrow{+} \Sigma F = 0; \qquad N + T\sin 30° - 500 = 0 \qquad \textbf{(2)}$$

Solving Eqs.(1) and (2) yields:

$$T = 187.6 \text{ lb} \qquad N = 406.2 \text{ lb}$$



Since $T < 200$ lb, the cord will not break at the moment the crate slides.

After the crate begins to slide, the kinetic friction is used for the calculation.

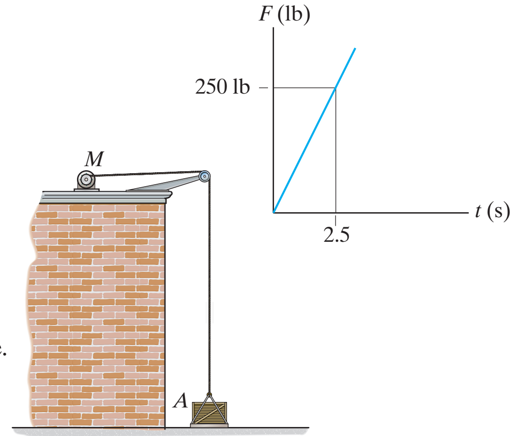
$$+\uparrow \Sigma F_y = ma_y; \qquad N + 200\sin 30° - 500 = 0 \qquad N = 400 \text{ lb}$$

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 200\cos 30° - 0.3(400) = \frac{500}{32.2}a$$

$$a = 3.43 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

13–29.

The force exerted by the motor on the cable is shown in the graph. Determine the velocity of the 200-lb crate when $t = 2.5$ s.



SOLUTION

Free-Body Diagram: The free-body diagram of the crate is shown in Fig. *a*.

Equilibrium: For the crate to move, force **F** must overcome the weight of the crate. Thus, the time required to move the crate is given by

$$+\uparrow \Sigma F_y = 0; \qquad 100t - 200 = 0 \qquad t = 2\text{ s}$$

Equation of Motion: For $2\text{ s} < t < 2.5\text{ s}$, $F = \dfrac{250}{2.5}t = (100t)$ lb. By referring to Fig. *a*,

$$+\uparrow \Sigma F_y = ma_y; \qquad 100t - 200 = \frac{200}{32.2}a$$

$$a = (16.1t - 32.2)\text{ ft/s}^2$$

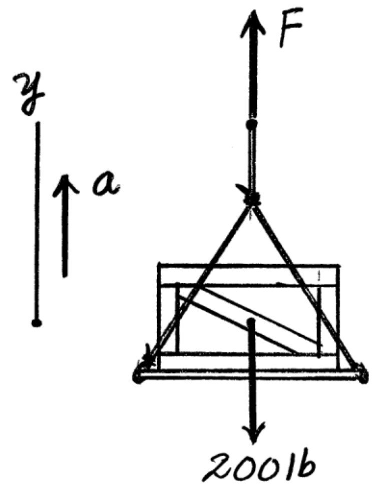
Kinematics: The velocity of the crate can be obtained by integrating the kinematic equation, $dv = a\,dt$. For $2\text{ s} \le t < 2.5\text{ s}$, $v = 0$ at $t = 2$ s will be used as the lower integration limit. Thus,

$$(+\uparrow) \qquad \int dv = \int a\,dt$$

$$\int_0^v dv = \int_{2\text{ s}}^t (16.1t - 32.2)dt$$

$$v = \left(8.05t^2 - 32.2t\right)\Big|_{2\text{ s}}^{t}$$

$$= \left(8.05t^2 - 32.2t + 32.2\right)\text{ft/s}$$



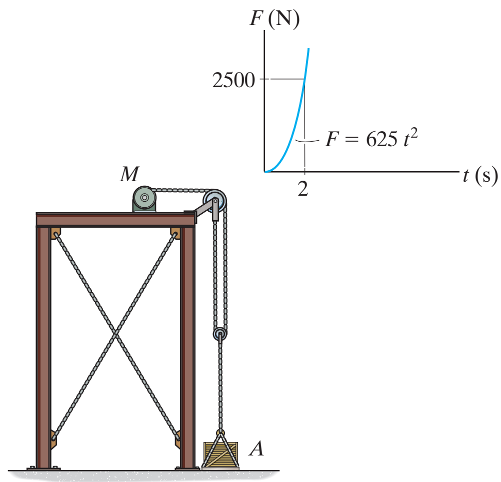
(a)

When $t = 2.5$ s,

$$v = 8.05(2.5^2) - 32.2(2.5) + 32.2 = 2.01\text{ ft/s} \qquad \textbf{Ans.}$$

13–30.

The force of the motor M on the cable is shown in the graph. Determine the velocity of the 400-kg crate A when $t = 2$ s.



SOLUTION

Free-Body Diagram: The free-body diagram of the crate is shown in Fig. a.

Equilibrium: For the crate to move, force 2**F** must overcome its weight. Thus, the time required to move the crate is given by

$$+\uparrow \Sigma F_y = 0; \qquad 2(625t^2) - 400(9.81) = 0$$

$$t = 1.772 \text{ s}$$

Equations of Motion: $F = \left(625t^2\right)$ N. By referring to Fig. a,

$$+\uparrow \Sigma F_y = ma_y; \qquad 2\left(625t^2\right) - 400(9.81) = 400a$$

$$a = (3.125t^2 - 9.81) \text{ m/s}^2$$

Kinematics: The velocity of the crate can be obtained by integrating the kinematic equation, $dv = adt$. For $1.772 \text{ s} \leq t < 2 \text{ s}$, $v = 0$ at $t = 1.772$ s will be used as the lower integration limit. Thus,

$$(+\uparrow) \qquad \int dv = \int adt$$

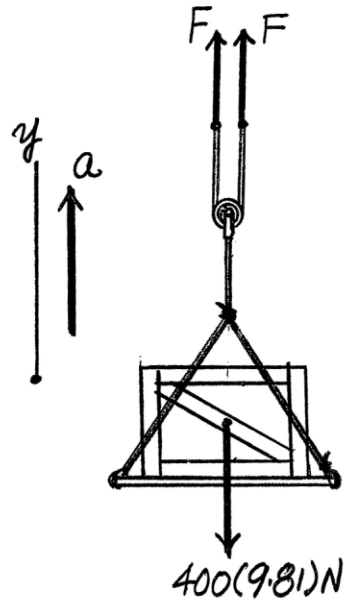
$$\int_0^v dv = \int_{1.772 \text{ s}}^t \left(3.125t^2 - 9.81\right)dt$$

$$v = \left(1.0417t^3 - 9.81t\right)\Big|_{1.772 \text{ s}}^t$$

$$= \left(1.0417t^3 - 9.81t + 11.587\right) \text{ m/s}$$

When $t = 2$ s,

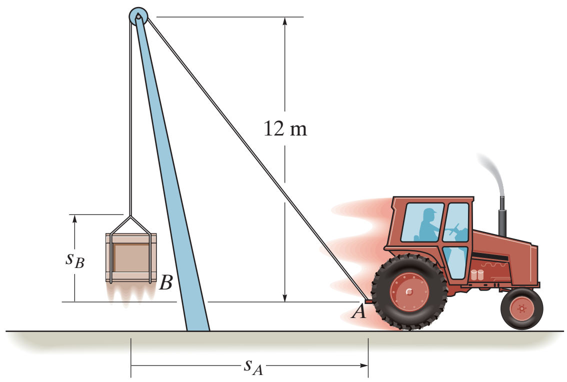
$$v = 1.0417(2^3) - 9.81(2) + 11.587 = 0.301 \text{ m/s} \qquad \textbf{Ans.}$$



(a)

13–31.

The tractor is used to lift the 150-kg load B with the 24-m-long rope, boom, and pulley system. If the tractor travels to the right at a constant speed of 4 m/s, determine the tension in the rope when $s_A = 5$ m. When $s_A = 0$, $s_B = 0$.



SOLUTION

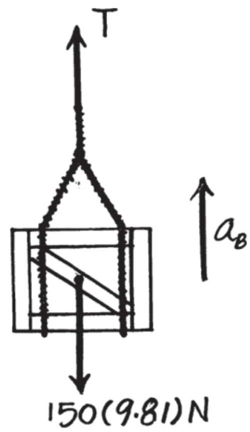
$$12 - s_B + \sqrt{s_A^2 + (12)^2} = 24$$

$$-\dot{s}_B + \left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(s_A\dot{s}_A\right) = 0$$

$$-\ddot{s}_B - \left(s_A^2 + 144\right)^{-\frac{3}{2}}\left(s_A\dot{s}_A\right)^2 + \left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(\dot{s}_A^2\right) + \left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(s_A\ddot{s}_A\right) = 0$$

$$\ddot{s}_B = -\left[\frac{s_A^2\dot{s}_A^2}{\left(s_A^2 + 144\right)^{\frac{3}{2}}} - \frac{\dot{s}_A^2 + s_A\ddot{s}_A}{\left(s_A^2 + 144\right)^{\frac{1}{2}}}\right]$$

$$a_B = -\left[\frac{(5)^2(4)^2}{((5)^2 + 144)^{\frac{3}{2}}} - \frac{(4)^2 + 0}{((5)^2 + 144)^{\frac{1}{2}}}\right] = 1.0487 \text{ m/s}^2$$

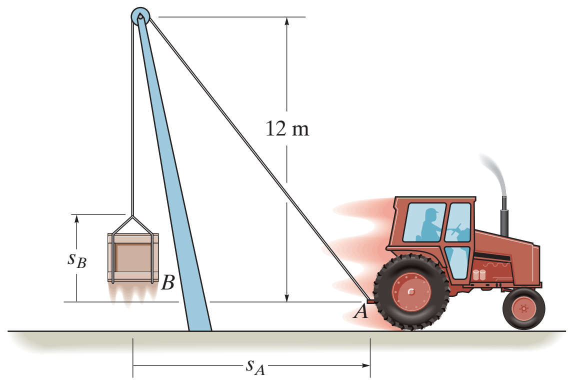


$+\uparrow \Sigma F_y = ma_y;\qquad T - 150(9.81) = 150(1.0487)$

$T = 1.63$ kN **Ans.**

***13–32.**

The tractor is used to lift the 150-kg load B with the 24-m-long rope, boom, and pulley system. If the tractor travels to the right with an acceleration of 3 m/s^2 and has a velocity of 4 m/s at the instant $s_A = 5$ m, determine the tension in the rope at this instant. When $s_A = 0, s_B = 0$.



SOLUTION

$$12 = s_B + \sqrt{s_A^2 + (12)^2} = 24$$

$$-\dot{s}_B + \frac{1}{2}\left(s_A^2 + 144\right)^{-\frac{3}{2}}\left(2s_A\dot{s}_A\right) = 0$$

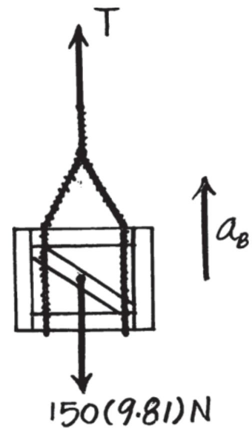
$$-\ddot{s}_B - \left(s_A^2 + 144\right)^{-\frac{3}{2}}\left(s_A\dot{s}_A\right)^2 + \left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(\dot{s}_A^2\right) + \left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(s_A\ddot{s}_A\right) = 0$$

$$\ddot{s}_B = -\left[\frac{s_A^2\,\dot{s}_A^2}{\left(s_A^2 + 144\right)^{\frac{3}{2}}} - \frac{\dot{s}_A^2 + s_A\ddot{s}_A}{\left(s_A^2 + 144\right)^{\frac{1}{2}}}\right]$$

$$a_B = -\left[\frac{(5)^2(4)^2}{((5)^2 + 144)^{\frac{3}{2}}} - \frac{(4)^2 + (5)(3)}{((5)^2 + 144)^{\frac{1}{2}}}\right] = 2.2025 \text{ m/s}^2$$

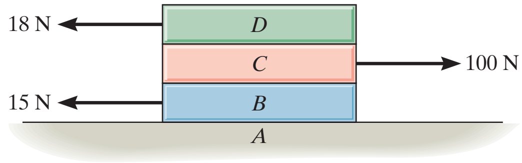
$$+\uparrow\Sigma F_y = ma_y; \qquad T - 150(9.81) = 150(2.2025)$$

$$T = 1.80 \text{ kN} \qquad \textbf{Ans.}$$



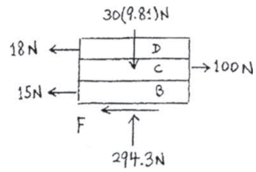
13–33.

Each of the three plates has a mass of 10 kg. If the coefficients of static and kinetic friction at each surface of contact are $\mu_s = 0.3$ and $\mu_k = 0.2$, respectively, determine the acceleration of each plate when the three horizontal forces are applied.



SOLUTION

Plates B, C and D



$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad 100 - 15 - 18 - F = 0$$

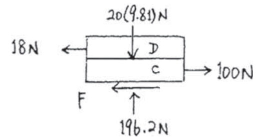
$$F = 67\text{ N}$$

$$F_{max} = 0.3(294.3) = 88.3\text{ N} > 67\text{ N}$$

Plate B will not slip.

$$a_B = 0$$ **Ans.**

Plates D and C



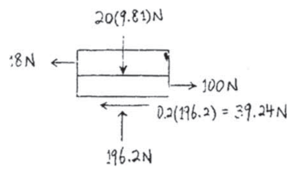
$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad 100 - 18 - F = 0$$

$$F = 82\text{ N}$$

$$F_{max} = 0.3(196.2) = 58.86\text{ N} < 82\text{N}$$

Slipping between B and C.

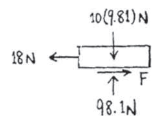
Assume no slipping between D and C,



$$\overset{+}{\rightarrow}\Sigma F_x = ma_x; \qquad 100 - 39.24 - 18 = 20\,a_x$$

$$a_x = 2.138\text{ m/s}^2 \rightarrow$$

Check slipping between D and C.



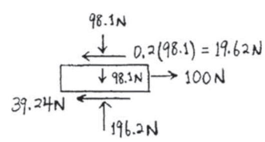
$$\overset{+}{\rightarrow}\Sigma F_x = m\,a_x; \qquad F - 18 = 10(2.138)$$

$$F = 39.38\text{ N}$$

$$F_{max} = 0.3(98.1) = 29.43\text{ N} < 39.38\text{ N}$$

Slipping between D and C.

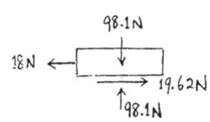
Plate C:



$$\overset{+}{\rightarrow}\Sigma F_x = m\,a_x; \qquad 100 - 39.24 - 19.62 = 10\,a_c$$

$$a_c = 4.11\text{ m/s}^2 \rightarrow$$ **Ans.**

Plate D:

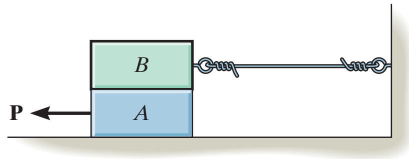


$$\overset{+}{\rightarrow}\Sigma F_x = m\,a_x; \qquad 19.62 - 18 = 10\,a_D$$

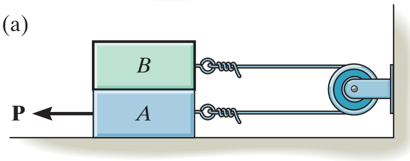
$$a_D = 0.162\text{m/s}^2 \rightarrow$$ **Ans.**

13–34.

Each of the two blocks has a mass m. The coefficient of kinetic friction at all surfaces of contact is μ. If a horizontal force **P** moves the bottom block, determine the acceleration of the bottom block in each case.



(a)



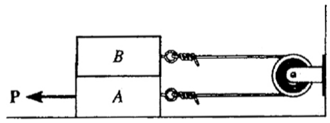
(b)

SOLUTION

Block A:

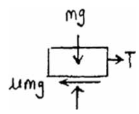
(a) $\overset{+}{\leftarrow} \Sigma F_x = ma_x; \qquad P - 3\mu mg = m\,a_A$

$$a_A = \frac{P}{m} - 3\mu g \qquad \textbf{Ans.}$$



(b) $s_B + s_A = l$

$$a_A = -a_B \qquad \textbf{(1)}$$

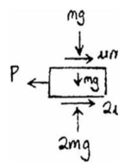


Block A:

$\overset{+}{\leftarrow} \Sigma F_x = m\,a_x; \qquad P - T - 3\mu mg = m\,a_A \qquad$ **(2)**

Block B:

$\overset{+}{\leftarrow} \Sigma F_x = ma_x; \qquad \mu mg - T = ma_B \qquad$ **(3)**



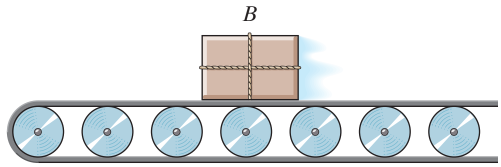
Subtract Eq.(3) from Eq.(2):

$$P - 4\mu mg = m\,(a_A - a_B)$$

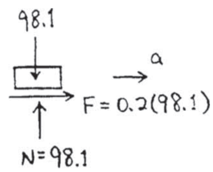
Use Eq.(1); $\qquad a_A = \dfrac{P}{2m} - 2\mu g \qquad$ **Ans.**

13–35.

The conveyor belt is moving at 4 m/s. If the coefficient of static friction between the conveyor and the 10-kg package B is $\mu_s = 0.2$, determine the shortest time the belt can stop so that the package does not slide on the belt.



SOLUTION



$\xrightarrow{+} \Sigma F_x = m\,a_x;\qquad 0.2(98.1) = 10\,a$

$$a = 1.962 \text{ m/s}^2$$

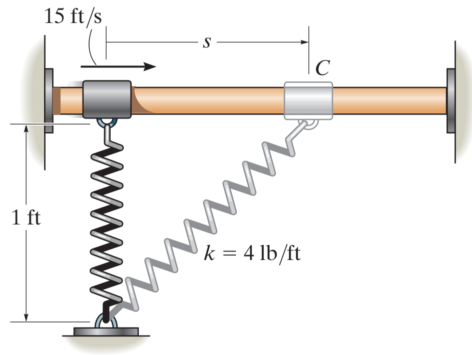
$(\xrightarrow{+})\, v = v_0 + a_c t$

$$4 = 0 + 1.962\,t$$

$$t = 2.04 \text{ s}$$ **Ans.**

***13–36.**

The 2-lb collar C fits loosely on the smooth shaft. If the spring is unstretched when $s = 0$ and the collar is given a velocity of 15 ft/s, determine the velocity of the collar when $s = 1$ ft.



SOLUTION

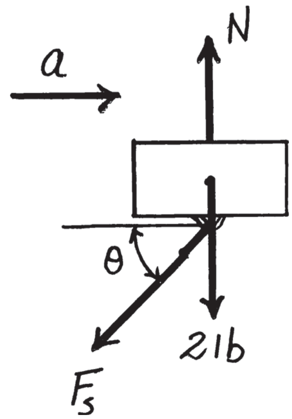
$F_s = kx; \qquad F_s = 4\left(\sqrt{1 + s^2} - 1\right)$

$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad -4\left(\sqrt{1 + s^2} - 1\right)\left(\frac{s}{\sqrt{1 + s^2}}\right) = \left(\frac{2}{32.2}\right)\left(v\frac{dv}{ds}\right)$

$-\int_0^1 \left(4s\, ds - \frac{4s\, ds}{\sqrt{1 + s^2}}\right) = \int_{15}^{v} \left(\frac{2}{32.2}\right) v\, dv$

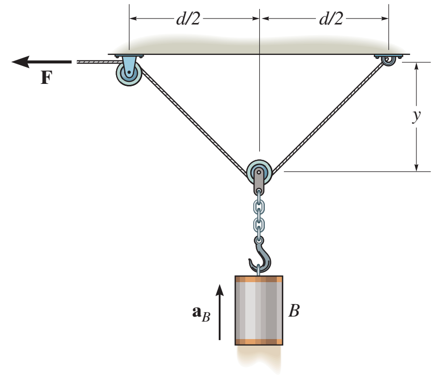
$-\left[2s^2 - 4\sqrt{1 + s^2}\right]_0^1 = \frac{1}{32.2}\left(v^2 - 15^2\right)$

$v = 14.6$ ft/s **Ans.**



13–37.

Cylinder B has a mass m and is hoisted using the cord and pulley system shown. Determine the magnitude of force **F** as a function of the cylinder's vertical position y so that when **F** is applied the cylinder rises with a constant acceleration $\mathbf{a}_B$. Neglect the mass of the cord, pulleys, hook and chain.

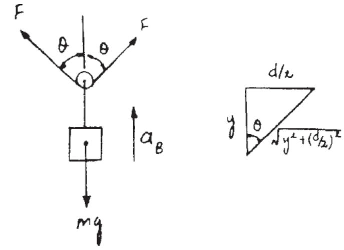


SOLUTION

$$+\uparrow \Sigma F_y = ma_y; \qquad 2F\cos\theta - mg = ma_B \qquad \text{where } \cos\theta = \frac{y}{\sqrt{y^2 + \left(\frac{d}{2}\right)^2}}$$

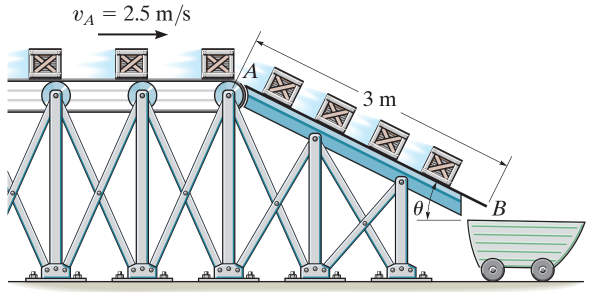
$$2F\left(\frac{y}{\sqrt{y^2 + \left(\frac{d}{2}\right)^2}}\right) - mg = ma_B$$

$$F = \frac{m(a_B + g)\sqrt{4y^2 + d^2}}{4y} \qquad \textbf{Ans.}$$

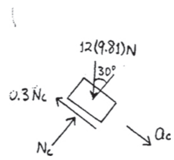


13–38.

The conveyor belt delivers each 12-kg crate to the ramp at A such that the crate's speed is $v_A = 2.5$ m/s, directed down *along* the ramp. If the coefficient of kinetic friction between each crate and the ramp is $\mu_k = 0.3$, determine the speed at which each crate slides off the ramp at B. Assume that no tipping occurs. Take $\theta = 30°$.



SOLUTION



$\nearrow + \Sigma F_y = ma_y;$ $\quad N_C - 12(9.81)\cos 30° = 0$

$$N_C = 101.95 \text{ N}$$

$+\searrow \Sigma F_x = ma_x;$ $\quad 12(9.81)\sin 30° - 0.3(101.95) = 12\,a_C$

$$a_C = 2.356 \text{ m/s}^2$$

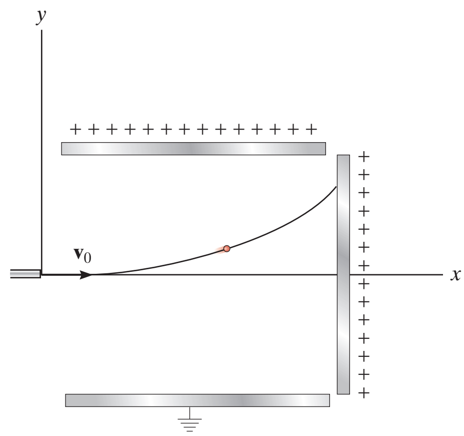
$(+\searrow)$ $\quad v_B^2 = v_A^2 + 2a_c(s_B - s_A)$

$$v_B^2 = (2.5)^2 + 2(2.356)(3 - 0)$$

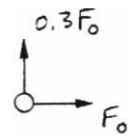
$$v_B = 4.5152 = 4.52 \text{ m/s}$$ **Ans.**

13–39.

An electron of mass m is discharged with an initial horizontal velocity of $\mathbf{v}_0$. If it is subjected to two fields of force for which $F_x = F_0$ and $F_y = 0.3F_0$, where F_0 is constant, determine the equation of the path, and the speed of the electron at any time t.



SOLUTION



$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad F_0 = ma_x$$

$$+\uparrow \Sigma F_y = ma_y; \qquad 0.3\,F_0 = ma_y$$

Thus,

$$\int_{v_0}^{v_x} dv_x = \int_0^t \frac{F_0}{m}\,dt$$

$$v_x = \frac{F_0}{m}t + v_0$$

$$\int_0^{v_y} dv_y = \int_0^t \frac{0.3F_0}{m}\,dt \qquad v_y = \frac{0.3F_0}{m}t$$

$$v = \sqrt{\left(\frac{F_0}{m}t + v_0\right)^2 + \left(\frac{0.3F_0}{m}t\right)^2}$$

$$= \frac{1}{m}\sqrt{1.09F_0^2t^2 + 2F_0tmv_0 + m^2v_0^2}$$ **Ans.**

$$\int_0^x dx = \int_0^t \left(\frac{F_0}{m}t + v_0\right)dt$$

$$x = \frac{F_0\,t^2}{2m} + v_0\,t$$

$$\int_0^y dy = \int_0^t \frac{0.3F_0}{m}\,t\,dt$$

$$y = \frac{0.3F_0\,t^2}{2m}$$

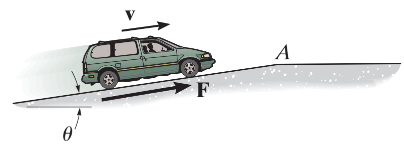
$$t = \left(\sqrt{\frac{2m}{0.3F_0}}\right)y^{\frac{1}{2}}$$

$$x = \frac{F_0}{2m}\left(\frac{2m}{0.3F_0}\right)y + v_0\left(\sqrt{\frac{2m}{0.3F_0}}\right)y^{\frac{1}{2}}$$

$$x = \frac{y}{0.3} + v_0\left(\sqrt{\frac{2m}{0.3F_0}}\right)y^{\frac{1}{2}}$$ **Ans.**

***13–40.**

The engine of the van produces a constant driving traction force **F** at the wheels as it ascends the slope at a constant velocity **v**. Determine the acceleration of the van when it passes point A and begins to travel on a level road, provided that it maintains the *same* traction force.



SOLUTION

Free-Body Diagram: The free-body diagrams of the van up the slope and on the level road are shown in Figs. a and b, respectively.

Equations of Motion: Since the van is travelling up the slope with a constant velocity, its acceleration is $a = 0$. By referring to Fig. a,

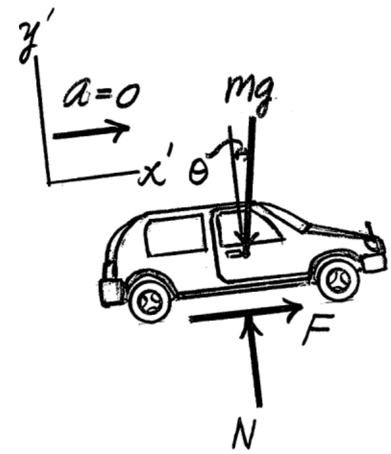
$$\Sigma F_{x'} = ma_{x'}; \qquad F - mg\sin\theta = m(0)$$

$$F = mg\sin\theta$$

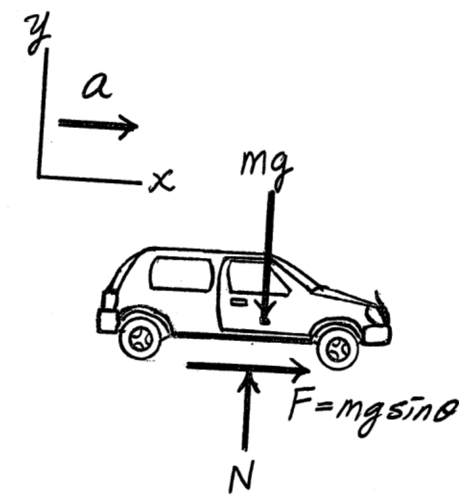
Since the van maintains the same tractive force **F** when it is on level road, from Fig. b,

$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad mg\sin\theta = ma$$

$$a = g\sin\theta \qquad \textbf{Ans.}$$



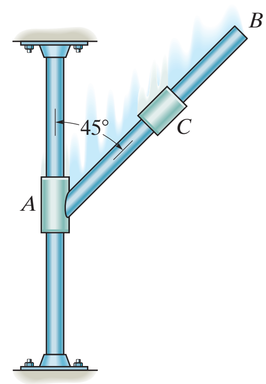
(a)



(b)

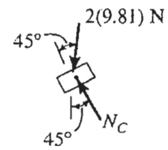
13–41.

The 2-kg collar C is free to slide along the smooth shaft AB. Determine the acceleration of collar C if (a) the shaft is fixed from moving, (b) collar A, which is fixed to shaft AB, moves downward at constant velocity along the vertical rod, and (c) collar A is subjected to a downward acceleration of 2 m/s^2. In all cases, the collar moves in the plane.



SOLUTION

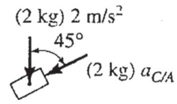
(a) $+\swarrow\Sigma F_{x'} = ma_{x'};\qquad 2(9.81)\sin 45^\circ = 2a_C \qquad a_C = 6.94\text{ m/s}^2$



(b) From part (a) $\mathbf{a}_{C/A} = 6.94\text{ m/s}^2$

$$\mathbf{a}_C = \mathbf{a}_A + \mathbf{a}_{C/A} \qquad \text{Where } \mathbf{a}_A = 0$$

$$= 6.94\text{ m/s}^2$$



(c)

$\mathbf{a}_C = \mathbf{a}_A + \mathbf{a}_{C/A}$

$$= \underset{\downarrow}{2} + \underset{\swarrow}{a_{C/A}} \qquad \textbf{(1)}$$

$+\swarrow\Sigma F_{x'} = ma_{x'};\quad 2(9.81)\sin 45^\circ = 2(2\cos 45^\circ + a_{C/A}) \qquad a_{C/A} = 5.5225\text{ m/s}^2 \swarrow$

From Eq.(1)

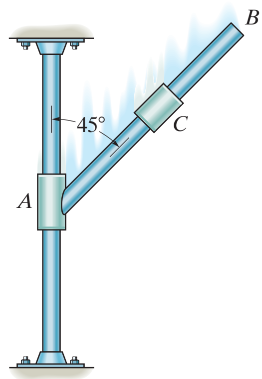
$$\mathbf{a}_C = \underset{\downarrow}{2} + \underset{\swarrow}{5.5225} = \underset{\leftarrow}{3.905} + \underset{\downarrow}{5.905}$$

$$a_C = \sqrt{3.905^2 + 5.905^2} = 7.08\text{ m/s}^2 \qquad \textbf{Ans.}$$

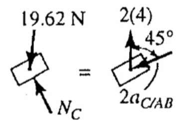
$$\theta = \tan^{-1}\frac{5.905}{3.905} = 56.5^\circ \;\theta\swarrow \qquad \textbf{Ans.}$$

13–42.

The 2-kg collar C is free to slide along the smooth shaft AB. Determine the acceleration of collar C if collar A is subjected to an upward acceleration of 4 m/s^2.



SOLUTION



$\overset{+}{\leftarrow} \Sigma F_x = ma_x;$ $\quad N \sin 45° = 2a_{C/AB} \sin 45°$

$$N = 2\, a_{C/AB}$$

$+\uparrow \Sigma F_y = ma_y;$ $\quad N \cos 45° - 19.62 = 2(4) - 2a_{C/AB} \cos 45°$

$$a_{C/AB} = 9.76514$$

$$\mathbf{a}_C = \mathbf{a}_{AB} + \mathbf{a}_{C/AB}$$

$$(a_C)_x = 0 + 9.76514 \sin 45° = 6.905 \leftarrow$$

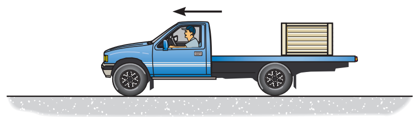
$$(a_C)_y = 4 - 9.76514 \cos 45° = 2.905 \downarrow$$

$$a_C = \sqrt{(6.905)^2 + (2.905)^2} = 7.49 \text{ m/s}^2$$ **Ans.**

$$\theta = \tan^{-1}\left(\frac{2.905}{6.905}\right) = 22.8° \ \theta\measuredangle$$ **Ans.**

13–43.

The coefficient of static friction between the 200-kg crate and the flat bed of the truck is $\mu_s = 0.3$. Determine the shortest time for the truck to reach a speed of 60 km/h, starting from rest with constant acceleration, so that the crate does not slip.



SOLUTION

Free-Body Diagram: When the crate accelerates with the truck, the frictional force F_f develops. Since the crate is required to be on the verge of slipping, $F_f = \mu_s N = 0.3N$.

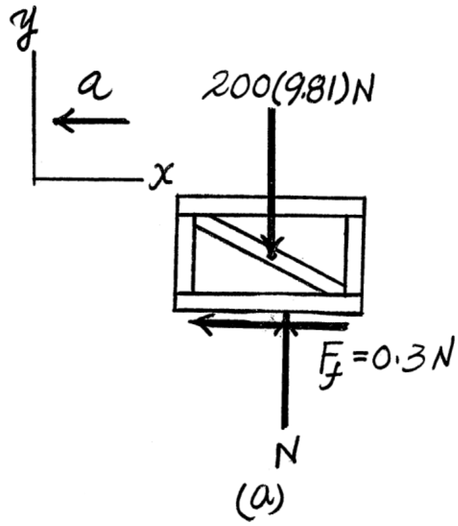
Equations of Motion: Here, $a_y = 0$. By referring to Fig. a,

$$+\uparrow \Sigma F_y = ma_y; \qquad N - 200(9.81) = 200(0)$$

$$N = 1962 \text{ N}$$

$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad -0.3(1962) = 200(-a)$$

$$a = 2.943 \text{ m/s}^2 \leftarrow$$



Kinematics: The final velocity of the truck is $v = \left(60\dfrac{\text{km}}{\text{h}}\right)\left(\dfrac{1000 \text{ m}}{1 \text{ km}}\right)\left(\dfrac{1 \text{ h}}{3600 \text{ s}}\right) =$ 16.67 m/s. Since the acceleration of the truck is constant,

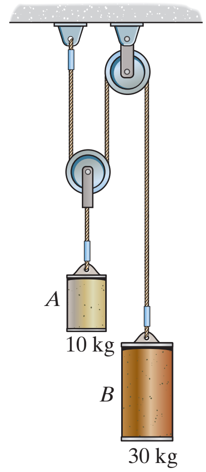
$$(\overset{+}{\leftarrow}) \qquad v = v_0 + a_c t$$

$$16.67 = 0 + 2.943t$$

$$t = 5.66 \text{ s}$$ **Ans.**

***13–44.**

When the blocks are released, determine their acceleration and the tension of the cable. Neglect the mass of the pulley.



SOLUTION

Free-Body Diagram: The free-body diagram of blocks A and B are shown in Figs. b and c, respectively. Here, $\mathbf{a}_A$ and $\mathbf{a}_B$ are assumed to be directed downwards so that they are consistent with the positive sense of position coordinates s_A and s_B of blocks A and B, Fig. a. Since the cable passes over the smooth pulleys, the tension in the cable remains constant throughout.

Equations of Motion: By referring to Figs. b and c,

$$+\uparrow \Sigma F_y = ma_y; \qquad 2T - 10(9.81) = -10a_A \qquad \textbf{(1)}$$

and

$$+\uparrow \Sigma F_y = ma_y; \qquad T - 30(9.81) = -30a_B \qquad \textbf{(2)}$$

Kinematics: We can express the length of the cable in terms of s_A and s_B by referring to Fig. a.

$$2s_A + s_B = l$$

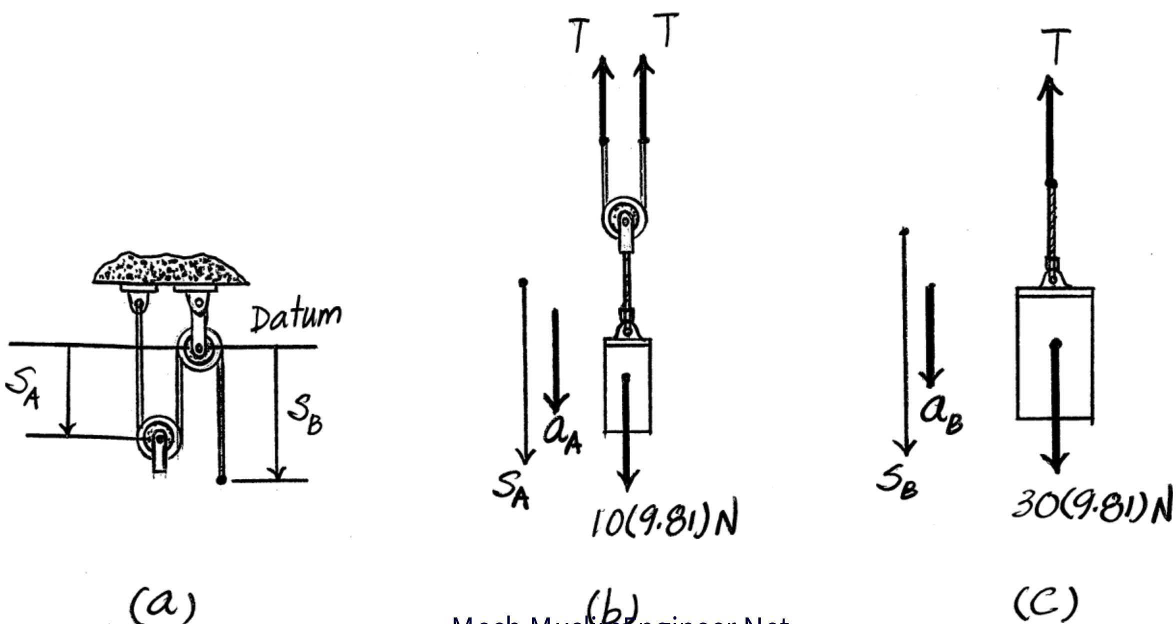
The second derivative of the above equation gives

$$2a_A + a_B = 0 \qquad \textbf{(3)}$$

Solving Eqs. (1), (2), and (3) yields

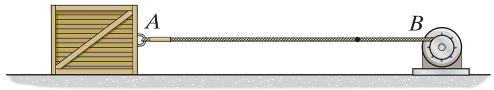
$$a_A = -3.773 \text{ m/s}^2 = 3.77 \text{ m/s}^2 \uparrow \qquad a_B = 7.546 \text{ m/s}^2 = 7.55 \text{ m/s}^2 \downarrow \qquad \textbf{Ans.}$$

$$T = 67.92 \text{ N} = 67.9 \text{ N} \qquad \textbf{Ans.}$$



13–45.

If the force exerted on cable AB by the motor is $F = (100t^{3/2})$ N, where t is in seconds, determine the 50-kg crate's velocity when $t = 5$ s. The coefficients of static and kinetic friction between the crate and the ground are $\mu_s = 0.4$ and $\mu_k = 0.3$, respectively. Initially the crate is at rest.



SOLUTION

Free-Body Diagram: The frictional force $\mathbf{F}_f$ is required to act to the left to oppose the motion of the crate which is to the right.

Equations of Motion: Here, $a_y = 0$. Thus,

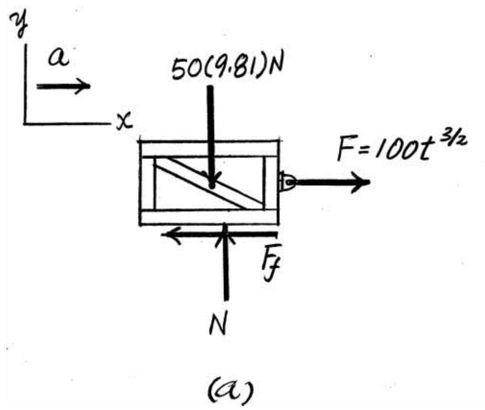
$$+\uparrow \Sigma F_y = ma_y; \qquad N - 50(9.81) = 50(0)$$

$$N = 490.5 \text{ N}$$

Realizing that $F_f = \mu_k N = 0.3(490.5) = 147.15$ N,

$$+\uparrow \Sigma F_x = ma_x; \qquad 100t^{3/2} - 147.15 = 50a$$

$$a = \left(2t^{3/2} - 2.943\right) \text{ m/s}$$



Equilibrium: For the crate to move, force $\mathbf{F}$ must overcome the static friction of $F_f = \mu_s N = 0.4(490.5) = 196.2$ N. Thus, the time required to cause the crate to be on the verge of moving can be obtained from.

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 100t^{3/2} - 196.2 = 0$$

$$t = 1.567 \text{ s}$$

Kinematics: Using the result of **a** and integrating the kinematic equation $dv = a\,dt$ with the initial condition $v = 0$ at $t = 1.567$ as the lower integration limit,

$$(\overset{+}{\rightarrow}) \qquad \int dv = \int a dt$$

$$\int_0^v dv = \int_{1.567 \text{ s}}^t \left(2t^{3/2} - 2.943\right)dt$$

$$v = \left(0.8t^{5/2} - 2.943t\right)\Big|_{1.567 \text{ s}}^t$$

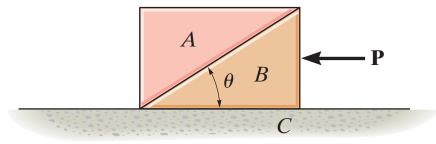
$$v = \left(0.8t^{5/2} - 2.943t + 2.152\right) \text{ m/s}$$

When $t = 5$ s,

$$v = 0.8(5)^{5/2} - 2.943(5) + 2.152 = 32.16 \text{ ft/s} = 32.2 \text{ ft/s} \qquad \textbf{Ans.}$$

13–46.

Blocks A and B each have a mass m. Determine the largest horizontal force **P** which can be applied to B so that A will not move relative to B. All surfaces are smooth.

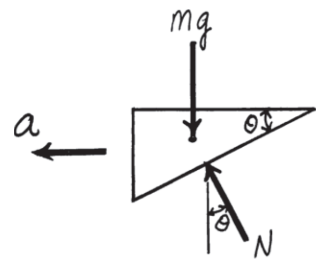


SOLUTION

Require

$a_A = a_B = a$

Block A:

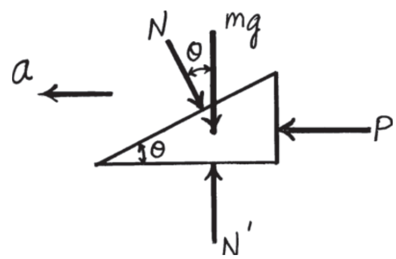


$+\uparrow \Sigma F_y = 0; \qquad N \cos\theta - mg = 0$

$\overset{+}{\leftarrow} \Sigma F_x = ma_x; \qquad N \sin\theta = ma$

$a = g \tan\theta$

Block B:



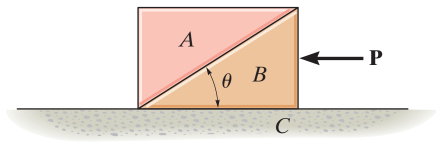
$\overset{+}{\leftarrow} \Sigma F_x = ma_x; \qquad P - N \sin\theta = ma$

$P - mg \tan\theta = mg \tan\theta$

$P = 2mg \tan\theta$ **Ans.**

13–47.

Blocks A and B each have a mass m. Determine the largest horizontal force **P** which can be applied to B so that A will not slip on B. The coefficient of static friction between A and B is μ_s. Neglect any friction between B and C.

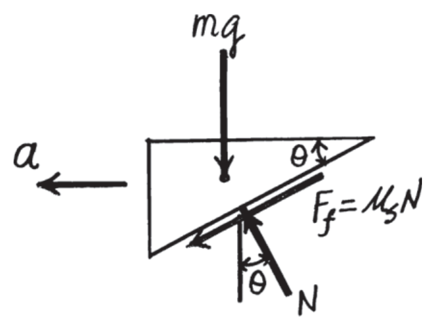


SOLUTION

Require

$a_A = a_B = a$

Block A:



$+\uparrow \Sigma F_y = 0; \qquad N\cos\theta - \mu_s N \sin\theta - mg = 0$

$\overset{+}{\leftarrow} \Sigma F_x = ma_x; \qquad N\sin\theta + \mu_s N\cos\theta = ma$

$$N = \frac{mg}{\cos\theta - \mu_s \sin\theta}$$

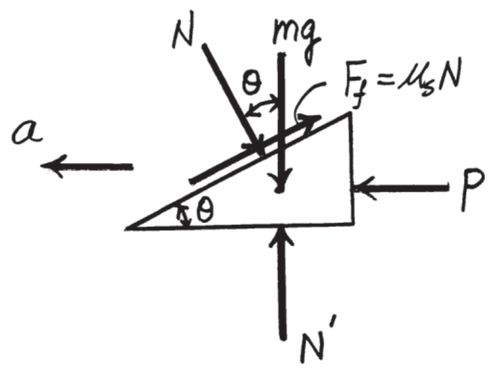
$$a = g\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right)$$

Block B:

$\overset{+}{\leftarrow} \Sigma F_x = ma_x; \qquad P - \mu_s N\cos\theta - N\sin\theta = ma$

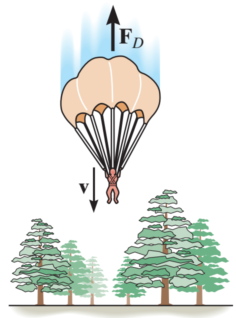
$$P - mg\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right) = mg\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right)$$

$$P = 2mg\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right)$$ **Ans.**



***13–48.**

A parachutist having a mass m opens his parachute from an at-rest position at a very high altitude. If the atmospheric drag resistance is $F_D = kv^2$, where k is a constant, determine his velocity when he has fallen for a time t. What is his velocity when he lands on the ground? This velocity is referred to as the *terminal velocity*, which is found by letting the time of fall $t \rightarrow \infty$.



SOLUTION

$$+\downarrow \Sigma F_z = m\,a_z; \qquad mg - kv^2 = m\frac{dv}{dt}$$

$$m\int_0^v \frac{m\,dv}{(mg - kv^2)} = \int_0^t dt$$

$$\frac{m}{k}\int_0^v \frac{dv}{\frac{mg}{k} - v^2} = t$$

$$\frac{m}{k}\left(\frac{1}{2\sqrt{\frac{mg}{k}}}\right)\ln\left[\frac{\sqrt{\frac{mg}{k}} + v}{\sqrt{\frac{mg}{k}} - v}\right]_0^v = t$$

$$\frac{k}{m}t\left(2\sqrt{\frac{mg}{k}}\right) = \ln\frac{\sqrt{\frac{mg}{k}} + v}{\sqrt{\frac{mg}{k}} - v}$$

$$e^{2t\sqrt{\frac{mg}{k}}} = \frac{\sqrt{\frac{mg}{k}} + v}{\sqrt{\frac{mg}{k}} - v}$$

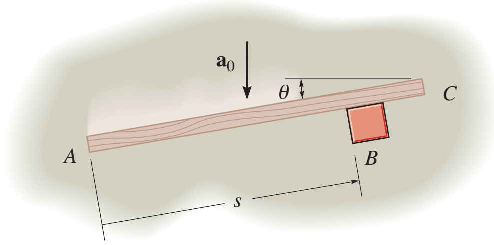
$$\sqrt{\frac{mg}{k}}\,e^{2t\sqrt{\frac{mg}{k}}} - v\,e^{2t\sqrt{\frac{mg}{k}}} = \sqrt{\frac{mg}{k}} + v$$

$$v = \sqrt{\frac{mg}{k}}\left[\frac{e^{2t\sqrt{\frac{mg}{k}}} - 1}{e^{2t\sqrt{\frac{mg}{k}}} + 1}\right] \qquad \textbf{Ans.}$$

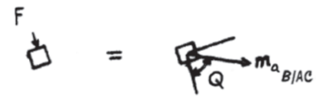
When $t \rightarrow \infty$ $\qquad v_t = \sqrt{\frac{mg}{k}}$ **Ans.**

13–49.

The smooth block B of negligible size has a mass m and rests on the horizontal plane. If the board AC pushes on the block at an angle θ with a constant acceleration $\mathbf{a}_0$, determine the velocity of the block along the board and the distance s the block moves along the board as a function of time t. The block starts from rest when $s = 0$, $t = 0$.



SOLUTION



$\nearrow + \Sigma F_x = m\,a_x;\qquad 0 = m\,a_B \sin\phi$

$$\mathbf{a}_B = \mathbf{a}_{AC} + \mathbf{a}_{B/AC}$$

$$\mathbf{a}_B = \mathbf{a}_0 + \mathbf{a}_{B/AC}$$

$\nearrow +\qquad a_B\ \sin\phi = -a_0\ \sin\theta + a_{B/AC}$

Thus,

$$0 = m(-a_0 \sin\theta + a_{B/AC})$$

$$a_{B/AC} = a_0\ \sin\theta$$

$$\int_0^{v_{B/AC}} dv_{B/AC} = \int_0^t a_0\ \sin\theta\ dt$$

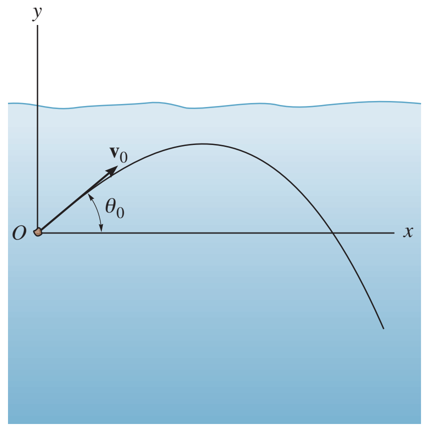
$$v_{B/AC} = a_0 \sin\theta\ t$$ **Ans.**

$$s_{B/AC} = s = \int_0^t a_0\ \sin\theta\ t\ dt$$

$$s = \frac{1}{2}a_0 \sin\theta\ t^2$$ **Ans.**

13–50.

A projectile of mass m is fired into a liquid at an angle θ_0 with an initial velocity $\mathbf{v}_0$ as shown. If the liquid develops a frictional or drag resistance on the projectile which is proportional to its velocity, i.e., $F = kv$, where k is a constant, determine the x and y components of its position at any instant. Also, what is the maximum distance x_{max} that it travels?



SOLUTION

$\xrightarrow{+} \Sigma F_x = ma_x; \qquad -kv\cos\theta = m\,a_x$

$+\uparrow \Sigma F_y = m\,a_y; \qquad -mg - kv\sin\theta = m\,a_y$

or

$$-k\frac{dx}{dt} = m\frac{d^2x}{dt^2}$$

$$-mg - k\frac{dy}{dt} = m\frac{d^2y}{dt^2}$$

Integrating yields

$\text{In}\ \dot{x} = \dfrac{-k}{m}t + C_1$

$\text{In}\ (\dot{y} + \dfrac{mg}{k}) = \dfrac{k}{m}t + C_2$

When $t = 0, \dot{x} = v_0\cos\theta_0, \qquad \dot{y} = v_0\sin\theta_0$

$$\dot{x} = v_0\cos\theta_0\,e^{-(k/m)t}$$

$$\dot{y} = -\frac{mg}{k} + (v_0\sin\theta_0 + \frac{mg}{k})e^{-(k/m)t}$$

Integrating again,

$$x = \frac{-mv_0}{k}\cos\theta_0\,e^{-(k/m)t} + C_3$$

$$y = -\frac{mg}{k}t - (v_0\sin\theta_0 + \frac{mg}{k})(\frac{m}{k})e^{-(k/m)t}$$

When $t = 0, x = y = 0$, thus

$$x = \frac{mv_0}{k}\cos\theta_0(1 - e^{-(k/m)t})$$ **Ans.**

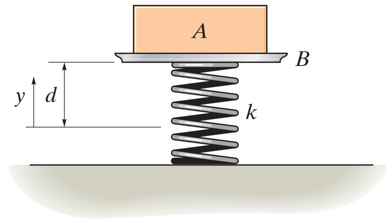
$$y = -\frac{mgt}{k} + \frac{m}{k}(v_0\sin\theta_0 + \frac{mg}{k})(1 - e^{-(k/m)t})$$ **Ans.**

As $t \to \infty$

$$x_{max} = \frac{mv_0\cos\theta_0}{k}$$ **Ans.**

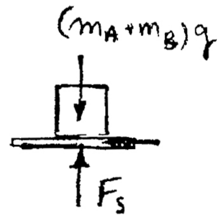
13–51.

The block A has a mass m_A and rests on the pan B, which has a mass m_B. Both are supported by a spring having a stiffness k that is attached to the bottom of the pan and to the ground. Determine the distance d the pan should be pushed down from the equilibrium position and then released from rest so that separation of the block will take place from the surface of the pan at the instant the spring becomes unstretched.



SOLUTION

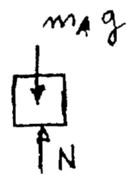
For Equilibrium



$$+\uparrow \Sigma F_y = ma_y; \qquad F_s = (m_A + m_B)g$$

$$y_{eq} = \frac{F_s}{k} = \frac{(m_A + m_B)g}{k}$$

Block:



$$+\uparrow \Sigma F_y = ma_y; \qquad -m_A g + N = m_A a$$

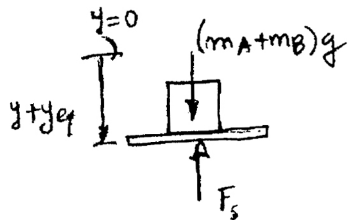
Block and pan

$$+\uparrow \Sigma F_y = ma_y; \qquad -(m_A + m_B)g + k(y_{eq} + y) = (m_A + m_B)a$$

Thus,

$$-(m_A + m_B)g + k\left[\left(\frac{m_A + m_B}{k}\right)g + y\right] = (m_A + m_B)\left(\frac{-m_A g + N}{m_A}\right)$$

Require $y = d, N = 0$



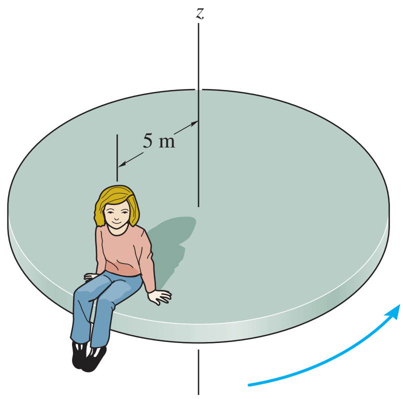
$$kd = -(m_A + m_B)g$$

Since d is downward,

$$d = \frac{(m_A + m_B)g}{k} \qquad \textbf{Ans.}$$

***13–52.**

A girl, having a mass of 15 kg, sits motionless relative to the surface of a horizontal platform at a distance of $r = 5$ m from the platform's center. If the angular motion of the platform is *slowly* increased so that the girl's tangential component of acceleration can be neglected, determine the maximum speed which the girl will have before she begins to slip off the platform. The coefficient of static friction between the girl and the platform is $\mu = 0.2$.



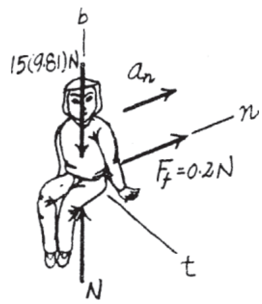
SOLUTION

Equation of Motion: Since the girl is on the verge of slipping, $F_f = \mu_s N = 0.2N$. Applying Eq. 13–8, we have

$$\Sigma F_b = 0; \qquad N - 15(9.81) = 0 \qquad N = 147.15 \text{ N}$$

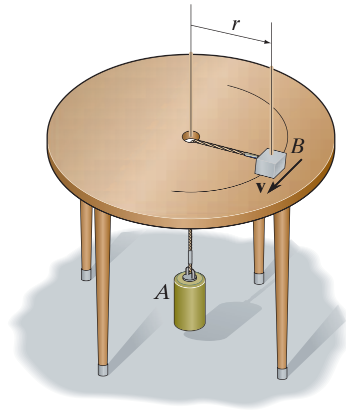
$$\Sigma F_n = ma_n; \qquad 0.2(147.15) = 15\left(\frac{v^2}{5}\right)$$

$$v = 3.13 \text{ m/s}$$ **Ans.**



13–53.

The 2-kg block B and 15-kg cylinder A are connected to a light cord that passes through a hole in the center of the smooth table. If the block is given a speed of $v = 10\,\text{m/s}$, determine the radius r of the circular path along which it travels.



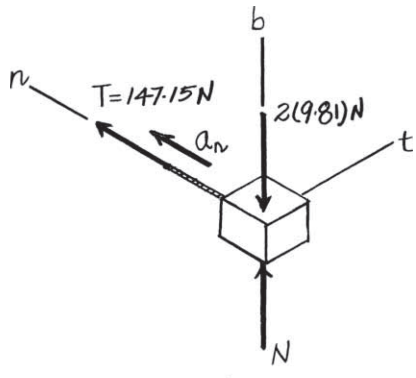
SOLUTION

Free-Body Diagram: The free-body diagram of block B is shown in Fig. (a). The tension in the cord is equal to the weight of cylinder A, i.e., $T = 15(9.81)\text{ N} = 147.15\text{ N}$. Here, $\mathbf{a}_n$ must be directed towards the center of the circular path (positive n axis).

Equations of Motion: Realizing that $a_n = \dfrac{v^2}{\rho} = \dfrac{10^2}{r}$ and referring to Fig. (a),

$$\Sigma F_n = ma_n; \qquad 147.15 = 2\left(\frac{10^2}{r}\right)$$

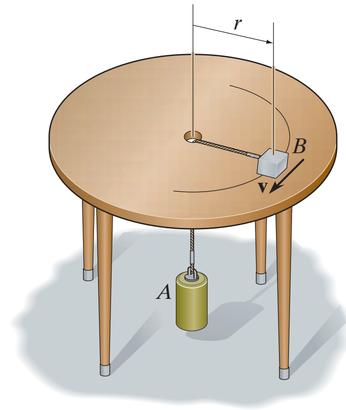
$$r = 1.36\text{ m} \qquad \textbf{Ans.}$$



(a)

13–54.

The 2-kg block B and 15-kg cylinder A are connected to a light cord that passes through a hole in the center of the smooth table. If the block travels along a circular path of radius $r = 1.5\,\text{m}$, determine the speed of the block.



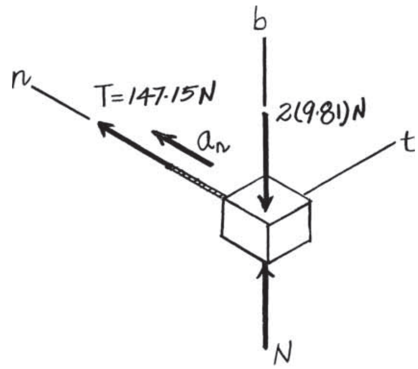
SOLUTION

Free-Body Diagram: The free-body diagram of block B is shown in Fig. (a). The tension in the cord is equal to the weight of cylinder A, i.e., $T = 15(9.81)\text{ N} = 147.15\text{ N}$. Here, $\mathbf{a}_n$ must be directed towards the center of the circular path (positive n axis).

Equations of Motion: Realizing that $a_n = \dfrac{v^2}{r} = \dfrac{v^2}{1.5}$ and referring to Fig. (a),

$$\Sigma F_n = ma_n; \qquad 147.15 = 2\left(\frac{v^2}{1.5}\right)$$

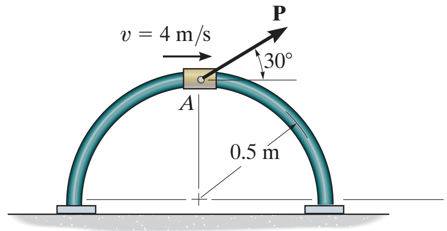
$$v = 10.5\text{ m/s} \qquad \textbf{Ans.}$$



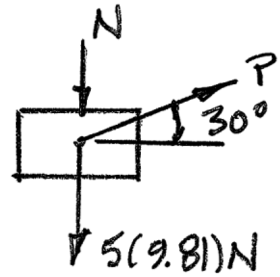
(a)

13–55.

The 5-kg collar A is sliding around a smooth vertical guide rod. At the instant shown, the speed of the collar is $v = 4\text{ m/s}$, which is increasing at 3 m/s^2. Determine the normal reaction of the guide rod on the collar, and force $\mathbf{P}$ at this instant.



SOLUTION



$\overset{+}{\rightarrow} \Sigma F_t = ma_t;\qquad P\cos 30^\circ = 5(3)$

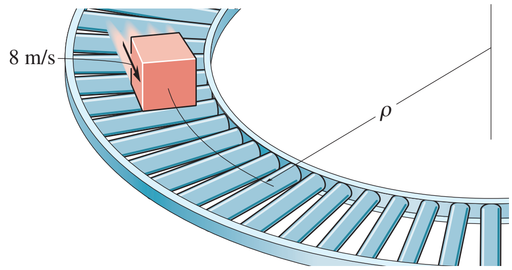
$P = 17.32\text{ N} = 17.3\text{ N}$ **Ans.**

$+\downarrow \Sigma F_n = ma_n;\qquad N + 5(9.81) - 17.32\ \sin 30^\circ = 5\left(\dfrac{4^2}{0.5}\right)$

$N = 119.61\text{ N} = 120\text{ N}\downarrow$ **Ans.**

***13–56.**

Cartons having a mass of 5 kg are required to move along the assembly line at a constant speed of 8 m/s. Determine the smallest radius of curvature, ρ, for the conveyor so the cartons do not slip. The coefficients of static and kinetic friction between a carton and the conveyor are $\mu_s = 0.7$ and $\mu_k = 0.5$, respectively.



SOLUTION

$+\uparrow \Sigma F_b = m\,a_b;$ $\quad N - W = 0$

$$N = W$$

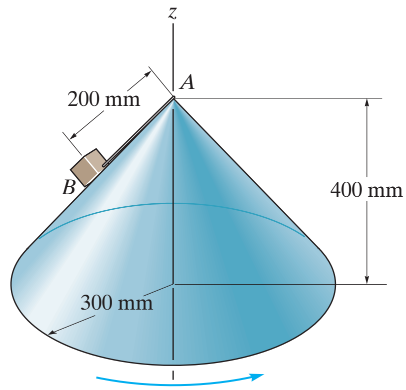
$$F_x = 0.7W$$

$\xleftarrow{+} \Sigma F_n = m a_n;$ $\quad 0.7W = \dfrac{W}{9.81}\left(\dfrac{8^2}{\rho}\right)$

$$\rho = 9.32 \text{ m}$$ **Ans.**

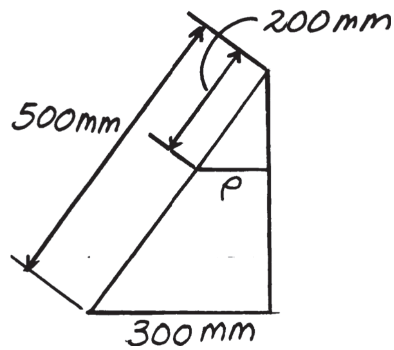
13–57.

The block B, having a mass of 0.2 kg, is attached to the vertex A of the right circular cone using a light cord. If the block has a speed of 0.5 m/s around the cone, determine the tension in the cord and the reaction which the cone exerts on the block and the effect of friction.



SOLUTION

$$\frac{\rho}{200} = \frac{300}{500}; \qquad \rho = 120 \text{ mm} = 0.120 \text{ m}$$



$+\nearrow \Sigma F_y = ma_y;\qquad T - 0.2(9.81)\left(\frac{4}{5}\right) = \left[0.2\left(\frac{(0.5)^2}{0.120}\right)\right]\left(\frac{3}{5}\right)$

$T = 1.82 \text{ N}$ **Ans.**

$+\nwarrow \Sigma F_x = ma_x;\qquad N_B - 0.2(9.81)\left(\frac{3}{5}\right) = -\left[0.2\left(\frac{(0.5)^2}{0.120}\right)\right]\left(\frac{4}{5}\right)$

$N_B = 0.844 \text{ N}$ **Ans.**

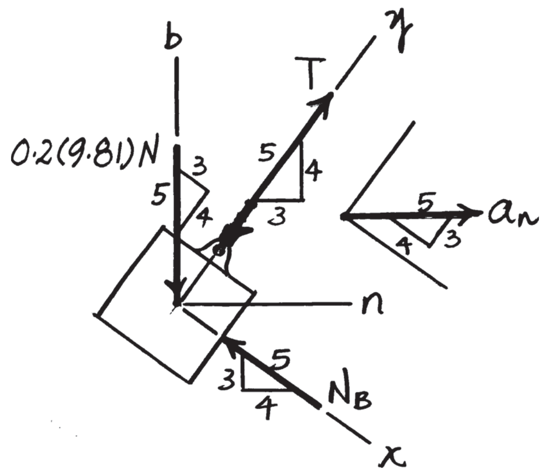
Also,

$\overset{+}{\rightarrow} \Sigma F_n = ma_n;\qquad T\left(\frac{3}{5}\right) - N_B\left(\frac{4}{5}\right) = 0.2\left(\frac{(0.5)^2}{0.120}\right)$

$+\uparrow \Sigma F_b = 0;\qquad T\left(\frac{4}{5}\right) + N_B\left(\frac{3}{5}\right) - 0.2(9.81) = 0$

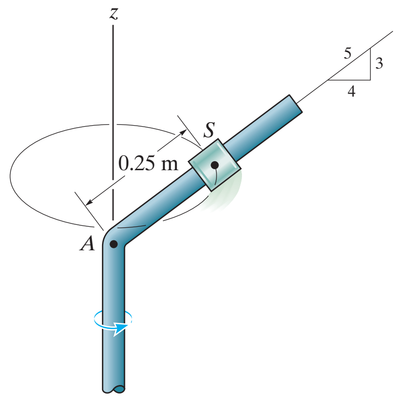
$T = 1.82 \text{ N}$ **Ans.**

$N_B = 0.844 \text{ N}$ **Ans.**



13–58.

The 2-kg spool S fits loosely on the inclined rod for which the coefficient of static friction is $\mu_s = 0.2$. If the spool is located 0.25 m from A, determine the minimum constant speed the spool can have so that it does not slip down the rod.



SOLUTION

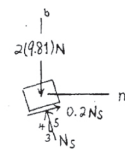
$$\rho = 0.25\left(\frac{4}{5}\right) = 0.2\text{ m}$$

$\overset{+}{\leftarrow} \Sigma F_n = m\,a_n;$ $$N_s\left(\frac{3}{5}\right) - 0.2N_s\left(\frac{4}{5}\right) = 2\left(\frac{v^2}{0.2}\right)$$

$+\uparrow \Sigma F_b = m\,a_b;$ $$N_s\left(\frac{4}{5}\right) + 0.2N_s\left(\frac{3}{5}\right) - 2(9.81) = 0$$

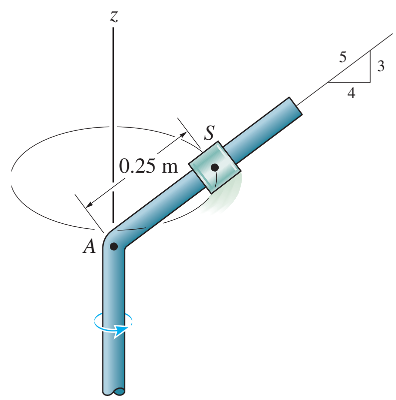
$$N_s = 21.3\text{ N}$$

$$v = 0.969\text{ m/s}$$ **Ans.**

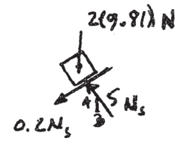


13–59.

The 2-kg spool S fits loosely on the inclined rod for which the coefficient of static friction is $\mu_s = 0.2$. If the spool is located 0.25 m from A, determine the maximum constant speed the spool can have so that it does not slip up the rod.



SOLUTION



$$\rho = 0.25\left(\frac{4}{5}\right) = 0.2\text{ m}$$

$$\overset{+}{\leftarrow} \Sigma F_n = m\,a_n; \qquad N_s\left(\frac{3}{5}\right) + 0.2N_s\left(\frac{4}{5}\right) = 2\left(\frac{v^2}{0.2}\right)$$

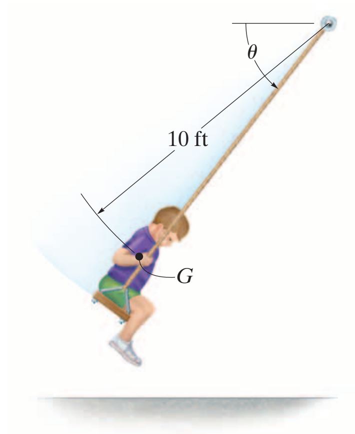
$$+\uparrow \Sigma F_b = m\,a_b; \qquad N_s\left(\frac{4}{5}\right) - 0.2N_s\left(\frac{3}{5}\right) - 2(9.81) = 0$$

$$N_s = 28.85\text{ N}$$

$$v = 1.48\text{ m/s}$$ **Ans.**

***13–60.**

At the instant $\theta = 60°$, the boy's center of mass G has a downward speed $v_G = 15$ ft/s. Determine the rate of increase in his speed and the tension in each of the two supporting cords of the swing at this instant. The boy has a weight of 60 lb. Neglect his size and the mass of the seat and cords.



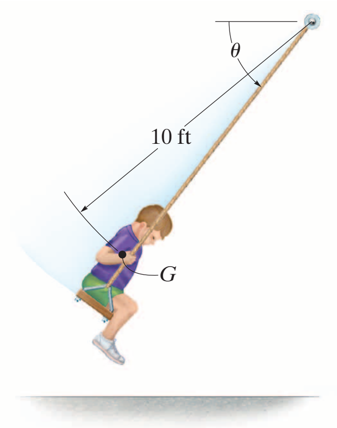
SOLUTION

$$+\searrow \Sigma F_t = ma_t; \qquad 60 \cos 60° = \frac{60}{32.2} a_t \qquad a_t = 16.1 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

$$\nearrow + \Sigma F_n = ma_n; \qquad 2T - 60 \sin 60° = \frac{60}{32.2}\left(\frac{15^2}{10}\right) \qquad T = 46.9 \text{ lb} \qquad \textbf{Ans.}$$

13–61.

At the instant $\theta = 60°$, the boy's center of mass G is momentarily at rest. Determine his speed and the tension in each of the two supporting cords of the swing when $\theta = 90°$. The boy has a weight of 60 lb. Neglect his size and the mass of the seat and cords.



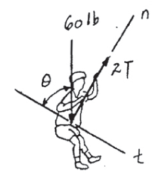
SOLUTION

$$+\searrow\Sigma_t = ma_t; \qquad 60\cos\theta = \frac{60}{32.2}a_t \qquad a_t = 32.2\cos\theta$$

$$\nearrow+\Sigma F_n = ma_n; \qquad 2T - 60\sin\theta = \frac{60}{32.2}\left(\frac{v^2}{10}\right) \qquad \textbf{(1)}$$

$$v\,dv = a\,ds \qquad \text{however } ds = 10d\theta$$

$$\int_0^v v\,dv = \int_{60°}^{90°} 322\cos\theta\,d\theta$$



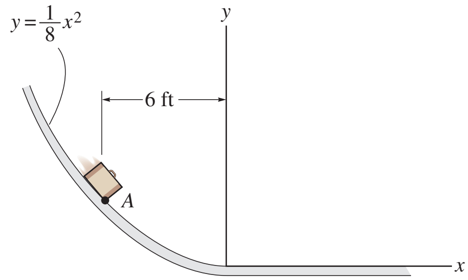
$$v = 9.289 \text{ ft/s} \qquad \textbf{Ans.}$$

From Eq. (1)

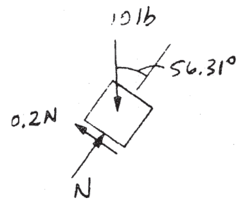
$$2T - 60\sin 90° = \frac{60}{32.2}\left(\frac{9.289^2}{10}\right) \qquad T = 38.0 \text{ lb} \qquad \textbf{Ans.}$$

13–62.

The 10-lb suitcase slides down the curved ramp for which the coefficient of kinetic friction is $\mu_k = 0.2$. If at the instant it reaches point A it has a speed of 5 ft/s, determine the normal force on the suitcase and the rate of increase of its speed.



SOLUTION



$$y = \frac{1}{8}x^2$$

$$\frac{dy}{dx} = \tan\theta = \frac{1}{4}x\bigg|_{x=-6} = -1.5 \qquad \theta = -56.31^\circ$$

$$\frac{d^2y}{dx^2} = \frac{1}{4}$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1 + (-1.5)^2\right]^{\frac{3}{2}}}{\left|\frac{1}{4}\right|} = 23.436 \text{ ft}$$

$$+\nearrow\Sigma F_n = ma_n; \qquad N - 10\cos 56.31^\circ = \left(\frac{10}{32.2}\right)\left(\frac{(5)^2}{23.436}\right)$$

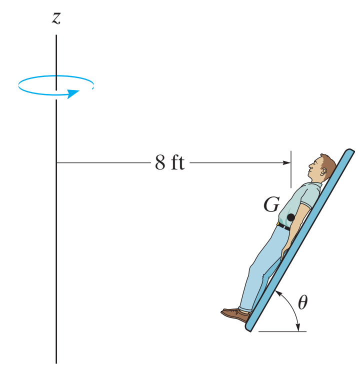
$$N = 5.8783 = 5.88 \text{ lb}$$ **Ans.**

$$+\searrow\Sigma F_t = ma_t; \qquad -0.2(5.8783) + 10\sin 56.31^\circ = \left(\frac{10}{32.2}\right)a_t$$

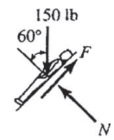
$$a_t = 23.0 \text{ ft/s}^2$$ **Ans.**

13–63.

The 150-lb man lies against the cushion for which the coefficient of static friction is $\mu_s = 0.5$. Determine the resultant normal and frictional forces the cushion exerts on him if, due to rotation about the z axis, he has a constant speed $v = 20$ ft/s. Neglect the size of the man. Take $\theta = 60^\circ$.



SOLUTION

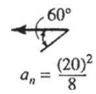


$$+\nwarrow \sum F_y = m(a_n)_y; \qquad N - 150\cos 60^\circ = \frac{150}{32.2}\left(\frac{20^2}{8}\right)\sin\ 60^\circ$$

$$N = 277 \text{ lb} \qquad \textbf{Ans.}$$

$$+\swarrow \sum F_x = m(a_n)_x; \qquad -F + 150\sin\ 60^\circ = \frac{150}{32.2}\left(\frac{20^2}{8}\right)\cos 60^\circ$$

$$F = 13.4 \text{ lb} \qquad \textbf{Ans.}$$

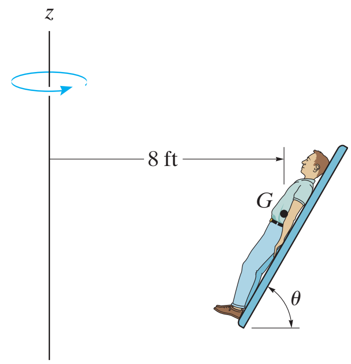


Note: No slipping occurs

Since $\mu_s N = 138.4 \text{ lb} > 13.4 \text{ lb}$

***13–64.**

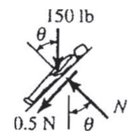
The 150-lb man lies against the cushion for which the coefficient of static friction is $\mu_s = 0.5$. If he rotates about the z axis with a constant speed $v = 30$ ft/s, determine the smallest angle θ of the cushion at which he will begin to slip off.



SOLUTION

$\overset{+}{\leftarrow} \Sigma F_n = ma_n;$ $\quad 0.5N\cos\theta + N\sin\theta = \frac{150}{32.2}\left(\frac{(30)^2}{8}\right)$

$+\uparrow \Sigma F_b = 0;$ $\quad -150 + N\cos\theta - 0.5\,N\sin\theta = 0$



$$N = \frac{150}{\cos\theta - 0.5\sin\theta}$$

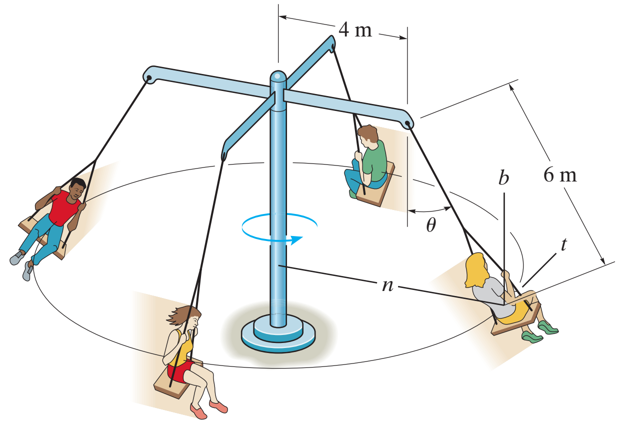
$$\frac{(0.5\cos\theta + \sin\theta)150}{(\cos\theta - 0.5\sin\theta)} = \frac{150}{32.2}\left(\frac{(30)^2}{8}\right)$$

$$0.5\cos\theta + \sin\theta = 3.49378\cos\theta - 1.74689\sin\theta$$

$$\theta = 47.5^\circ$$ **Ans.**

13–65.

Determine the constant speed of the passengers on the amusement-park ride if it is observed that the supporting cables are directed at $\theta = 30°$ from the vertical. Each chair including its passenger has a mass of 80 kg. Also, what are the components of force in the n, t, and b directions which the chair exerts on a 50-kg passenger during the motion?



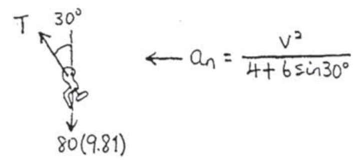
SOLUTION

$\overset{+}{\leftarrow} \Sigma F_n = m\,a_n;$ $\qquad T \sin 30° = 80\left(\dfrac{v^2}{4 + 6\,\sin 30°}\right)$

$+\uparrow \Sigma F_b = 0;$ $\qquad T \cos 30° - 80(9.81) = 0$

$T = 906.2\text{ N}$

$v = 6.30\text{ m/s}$ **Ans.**

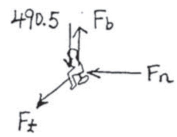


$\Sigma F_n = m\,a_n;$ $\qquad F_n = 50\left(\dfrac{(6.30)^2}{7}\right) = 283\text{ N}$ **Ans.**

$\Sigma F_t = m\,a_t;$ $\qquad F_t = 0$ **Ans.**

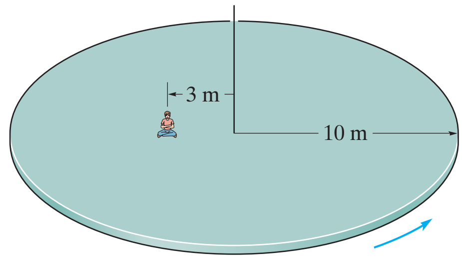
$\Sigma F_b = m\,a_b;$ $\qquad F_b - 490.5 = 0$

$F_b = 490\text{ N}$ **Ans.**



13–66.

The man has a mass of 80 kg and sits 3 m from the center of the rotating platform. Due to the rotation his speed is increased from rest by $\dot{v} = 0.4\ \text{m/s}^2$. If the coefficient of static friction between his clothes and the platform is $\mu_s = 0.3$, determine the time required to cause him to slip.

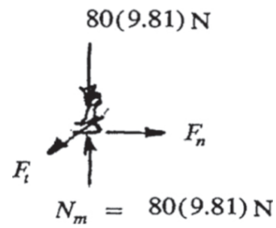


SOLUTION

$\Sigma F_t = m\,a_t;\qquad F_t = 80(0.4)$

$F_t = 32\ \text{N}$

$\Sigma F_n = m\,a_n;\qquad F_n = (80)\dfrac{v^2}{3}$



$F = \mu_s N_m = \sqrt{(F_t)^2 + (F_n)^2}$

$0.3(80)(9.81) = \sqrt{(32)^2 + ((80)\frac{v^2}{3})^2}$

$55\,432 = 1024 + (6400)(\frac{v^4}{9})$

$v = 2.9575\ \text{m/s}$

$a_t = \dfrac{dv}{dt} = 0.4$

$\int_0^v dv = \int_0^t 0.4\,dt$

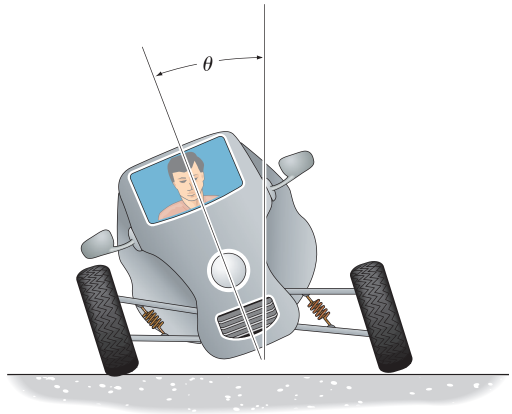
$v = 0.4\,t$

$2.9575 = 0.4\,t$

$t = 7.39\ \text{s}$ **Ans.**

13–67.

The vehicle is designed to combine the feel of a motorcycle with the comfort and safety of an automobile. If the vehicle is traveling at a constant speed of 80 km/h along a circular curved road of radius 100 m, determine the tilt angle θ of the vehicle so that only a normal force from the seat acts on the driver. Neglect the size of the driver.



SOLUTION

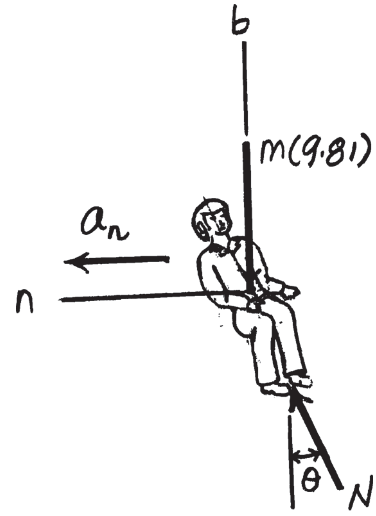
Free-Body Diagram: The free-body diagram of the passenger is shown in Fig. (a). Here, $\mathbf{a}_n$ must be directed towards the center of the circular path (positive n axis).

Equations of Motion: The speed of the passenger is $v = \left(80\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right)$ $= 22.22$ m/s. Thus, the normal component of the passenger's acceleration is given by $a_n = \frac{v^2}{\rho} = \frac{22.22^2}{100} = 4.938\text{ m/s}^2$. By referring to Fig. (a),

$$+\uparrow \Sigma F_b = 0; \qquad N\cos\theta - m(9.81) = 0 \qquad N = \frac{9.81m}{\cos\theta}$$

$$\overset{+}{\leftarrow} \Sigma F_n = ma_n; \qquad \frac{9.81m}{\cos\theta}\sin\theta = m(4.938)$$

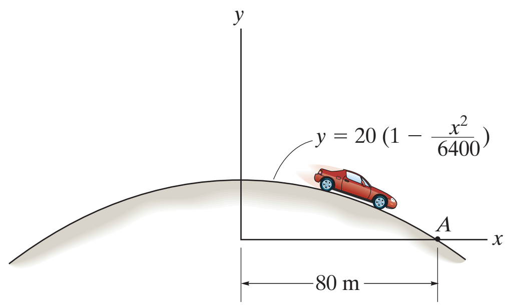
$$\theta = 26.7^\circ$$ **Ans.**



(a)

***13–68.**

The 0.8-Mg car travels over the hill having the shape of a parabola. If the driver maintains a constant speed of 9 m/s, determine both the resultant normal force and the resultant frictional force that all the wheels of the car exert on the road at the instant it reaches point A. Neglect the size of the car.



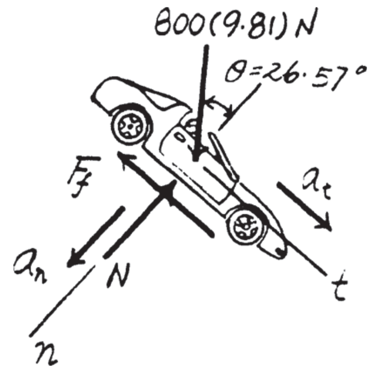
SOLUTION

Geometry: Here, $\frac{dy}{dx} = -0.00625x$ and $\frac{d^2y}{dx^2} = -0.00625$. The slope angle θ at point A is given by

$$\tan\theta = \left.\frac{dy}{dx}\right|_{x=80\text{ m}} = -0.00625(80) \qquad \theta = -26.57^\circ$$

and the radius of curvature at point A is

$$\rho = \frac{[1 + (dy/dx)^2]^{3/2}}{|d^2y/dx^2|} = \left.\frac{[1 + (-0.00625x)^2]^{3/2}}{|-0.00625|}\right|_{x=80\text{ m}} = 223.61\text{ m}$$



Equations of Motion: Here, $a_t = 0$. Applying Eq. 13–8 with $\theta = 26.57^\circ$ and $\rho = 223.61$ m, we have

$\Sigma F_t = ma_t;$ $\qquad 800(9.81)\sin 26.57^\circ - F_f = 800(0)$

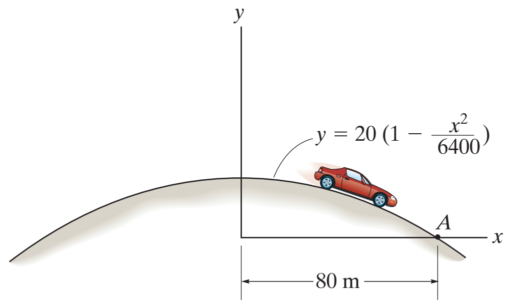
$$F_f = 3509.73\text{ N} = 3.51\text{ kN} \qquad \textbf{Ans.}$$

$\Sigma F_n = ma_n;$ $\qquad 800(9.81)\cos 26.57^\circ - N = 800\left(\frac{9^2}{223.61}\right)$

$$N = 6729.67\text{ N} = 6.73\text{ kN} \qquad \textbf{Ans.}$$

13–69.

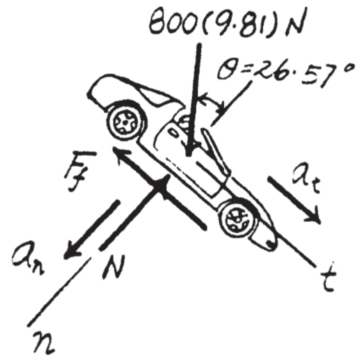
The 0.8-Mg car travels over the hill having the shape of a parabola. When the car is at point A, it is traveling at 9 m/s and increasing its speed at 3 m/s^2. Determine both the resultant normal force and the resultant frictional force that all the wheels of the car exert on the road at this instant. Neglect the size of the car.



SOLUTION

Geometry: Here, $\frac{dy}{dx} = -0.00625x$ and $\frac{d^2y}{dx^2} = -0.00625$. The slope angle θ at point A is given by

$$\tan\theta = \left.\frac{dy}{dx}\right|_{x=80\text{ m}} = -0.00625(80) \qquad \theta = -26.57^\circ$$



and the radius of curvature at point A is

$$\rho = \frac{\left[1 + (dy/dx)^2\right]^{3/2}}{|d^2y/dx^2|} = \left.\frac{\left[1 + (-0.00625x)^2\right]^{3/2}}{|-0.00625|}\right|_{x=80\text{ m}} = 223.61\text{ m}$$

Equation of Motion: Applying Eq. 13–8 with $\theta = 26.57^\circ$ and $\rho = 223.61$ m, we have

$$\Sigma F_t = ma_t; \qquad 800(9.81)\sin 26.57^\circ - F_f = 800(3)$$

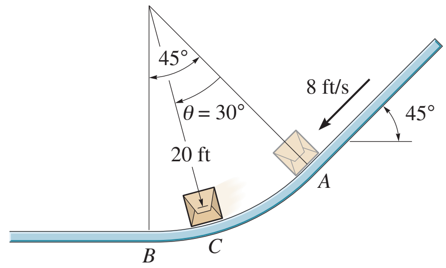
$$F_f = 1109.73\text{ N} = 1.11\text{ kN} \qquad \textbf{Ans.}$$

$$\Sigma F_n = ma_n; \qquad 800(9.81)\cos 26.57^\circ - N = 800\left(\frac{9^2}{223.61}\right)$$

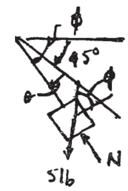
$$N = 6729.67\text{ N} = 6.73\text{ kN} \qquad \textbf{Ans.}$$

13–70.

The package has a weight of 5 lb and slides down the chute. When it reaches the curved portion AB, it is traveling at 8 ft/s ($\theta = 0°$). If the chute is smooth, determine the speed of the package when it reaches the intermediate point C ($\theta = 30°$) and when it reaches the horizontal plane ($\theta = 45°$). Also, find the normal force on the package at C.



SOLUTION



$$+\swarrow \Sigma F_t = ma_t; \qquad 5 \cos \phi = \frac{5}{32.2} a_t$$

$$a_t = 32.2 \cos \phi$$

$$+\nwarrow \Sigma F_n = ma_n; \qquad N - 5 \sin \phi = \frac{5}{32.2} \left(\frac{v^2}{20}\right)$$

$$v\,dv = a_t\,ds$$

$$\int_g^v v\,dv = \int_{45°}^{\phi} 32.2 \cos \phi \,(20\,d\phi)$$

$$\frac{1}{2}v^2 - \frac{1}{2}(8)^2 = 644(\sin \phi - \sin 45°)$$

At $\phi = 45° + 30° = 75°$,

$$v_C = 19.933 \text{ ft/s} = 19.9 \text{ ft/s}$$ **Ans.**

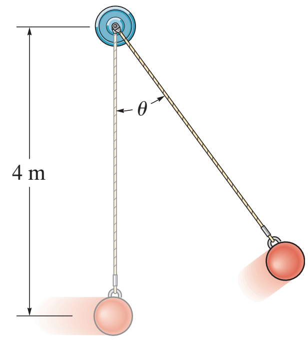
$$N_C = 7.91 \text{ lb}$$ **Ans.**

At $\phi = 45° + 45° = 90°$

$$v_B = 21.0 \text{ ft/s}$$ **Ans.**

13–71.

If the ball has a mass of 30 kg and a speed $v = 4$ m/s at the instant it is at its lowest point, $\theta = 0°$, determine the tension in the cord at this instant. Also, determine the angle θ to which the ball swings and momentarily stops. Neglect the size of the ball.



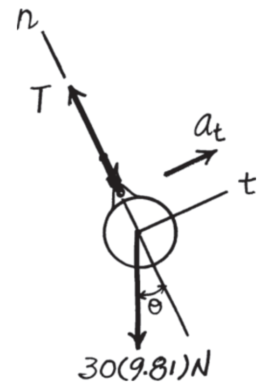
SOLUTION

$+\uparrow \Sigma F_n = ma_n; \qquad T - 30(9.81) = 30\left(\frac{(4)^2}{4}\right)$

$T = 414 \text{ N}$ **Ans.**

$+\nearrow \Sigma F_t = ma_t; \qquad -30(9.81)\sin\theta = 30a_t$

$a_t = -9.81 \sin\theta$



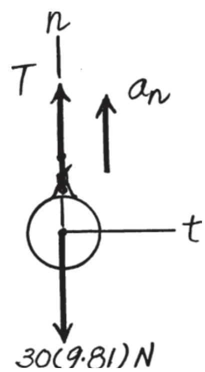
$a_t\, ds = v\, dv$ Since $ds = 4\, d\theta$, then

$$-9.81\int_0^\theta \sin\theta(4\, d\theta) = \int_4^0 v\, dv$$

$$\left[9.81(4)\cos\theta\right]_0^\theta = -\frac{1}{2}(4)^2$$

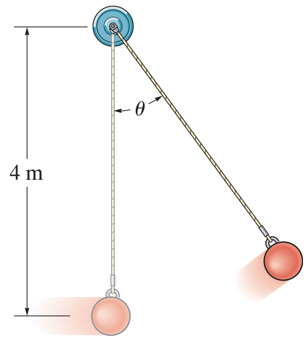
$$39.24(\cos\theta - 1) = -8$$

$\theta = 37.2°$ **Ans.**



***13–72.**

The ball has a mass of 30 kg and a speed $v = 4$ m/s at the instant it is at its lowest point, $\theta = 0°$. Determine the tension in the cord and the rate at which the ball's speed is decreasing at the instant $\theta = 20°$. Neglect the size of the ball.

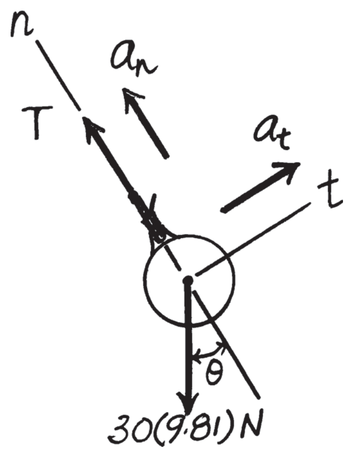


SOLUTION

$$+\nwarrow\Sigma F_n = ma_n; \qquad T - 30(9.81)\cos\theta = 30\left(\frac{v^2}{4}\right)$$

$$+\nearrow\Sigma F_t = ma_t; \qquad -30(9.81)\sin\theta = 30a_t$$

$$a_t = -9.81\sin\theta$$



$a_t\,ds = v\,dv$ Since $ds = 4\,d\theta$, then

$$-9.81\int_0^\theta \sin\theta\,(4\,d\theta) = \int_4^v v\,dv$$

$$9.81(4)\cos\theta\Big|_0^\theta = \frac{1}{2}(v)^2 - \frac{1}{2}(4)^2$$

$$39.24(\cos\theta - 1) + 8 = \frac{1}{2}v^2$$

At $\theta = 20°$

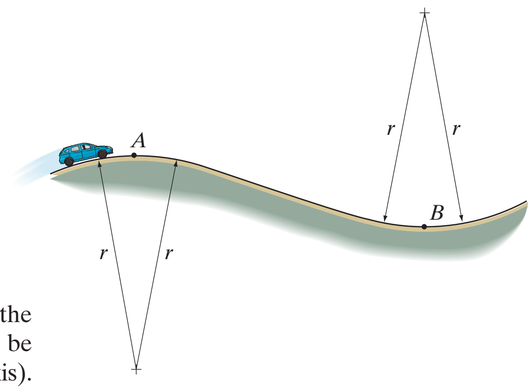
$v = 3.357$ m/s

$a_t = -3.36 \text{ m/s}^2 = 3.36 \text{ m/s}^2 \swarrow$ **Ans.**

$T = 361$ N **Ans.**

13–73.

Determine the maximum speed at which the car with mass m can pass over the top point A of the vertical curved road and still maintain contact with the road. If the car maintains this speed, what is the normal reaction the road exerts on the car when it passes the lowest point B on the road?



SOLUTION

Free-Body Diagram: The free-body diagram of the car at the top and bottom of the vertical curved road are shown in Figs. (a) and (b), respectively. Here, $\mathbf{a}_n$ must be directed towards the center of curvature of the vertical curved road (positive n axis).

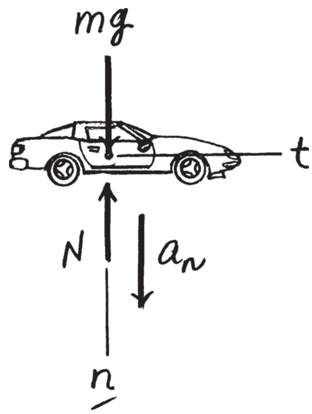
Equations of Motion: When the car is on top of the vertical curved road, it is required that its tires are about to lose contact with the road surface. Thus, $N = 0$. Realizing that $a_n = \dfrac{v^2}{\rho} = \dfrac{v^2}{r}$ and referring to Fig. (a),

$$+\downarrow \Sigma F_n = ma_n; \qquad mg = m\left(\frac{v^2}{r}\right) \qquad\qquad v = \sqrt{gr} \qquad \textbf{Ans.}$$

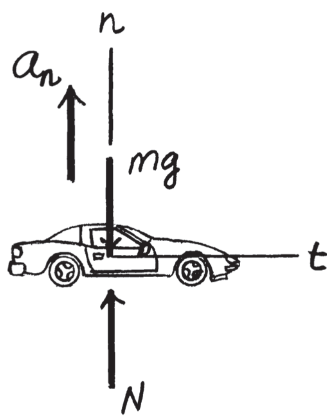
Using the result of v, the normal component of car acceleration is $a_n = \dfrac{v^2}{\rho} = \dfrac{gr}{r} = g$ when it is at the lowest point on the road. By referring to Fig. (b),

$$+\uparrow \Sigma F_n = ma_n; \qquad N - mg = mg$$

$$N = 2mg \qquad \textbf{Ans.}$$



(a)

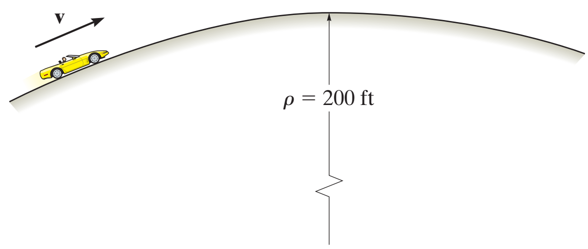


(b)

لجنة الميكانيك - الإتجاه الإسلامي

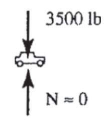
13–74.

If the crest of the hill has a radius of curvature $\rho = 200$ ft, determine the maximum constant speed at which the car can travel over it without leaving the surface of the road. Neglect the size of the car in the calculation. The car has a weight of 3500 lb.



SOLUTION

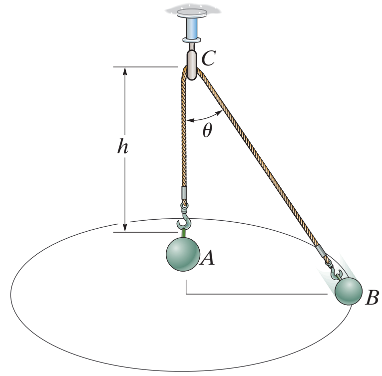
$$+\downarrow \Sigma F_n = ma_n; \qquad 3500 = \frac{3500}{32.2}\left(\frac{v^2}{200}\right)$$



$$v = 80.2 \text{ ft/s}$$ **Ans.**

13–75.

Bobs A and B of mass m_A and m_B $(m_A > m_B)$ are connected to an inextensible light string of length l that passes through the smooth ring at C. If bob B moves as a conical pendulum such that A is suspended a distance of h from C, determine the angle θ and the speed of bob B. Neglect the size of both bobs.



SOLUTION

Free-Body Diagram: The free-body diagram of bob B is shown in Fig. a. The tension developed in the string is equal to the weight of bob A, i.e., $T = m_A g$. Here, $\mathbf{a}_n$ must be directed towards the center of the horizontal circular path (positive n axis).

Equations of Motion: The radius of the horizontal circular path is $r = (l - h)\sin\theta$. Thus, $a_n = \dfrac{v^2}{\rho} = \dfrac{v_B{}^2}{(l-h)\sin\theta}$. By referring to Fig. a,

$+\uparrow \Sigma F_b = 0;$ $\quad m_A g\cos\theta - m_B g = 0$

$$\theta = \cos^{-1}\left(\frac{m_B}{m_A}\right) \qquad \textbf{Ans.}$$

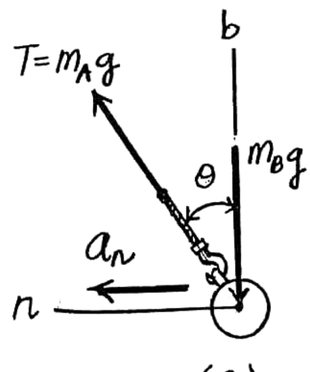
$\overset{+}{\leftarrow} \Sigma F_n = m a_n;$ $\quad m_A g\sin\theta = m_B\left[\dfrac{v_B{}^2}{(l-h)\sin\theta}\right]$

$$v_B = \sqrt{\frac{m_A g(l-h)}{m_B}}\sin\theta \qquad \textbf{(1)}$$

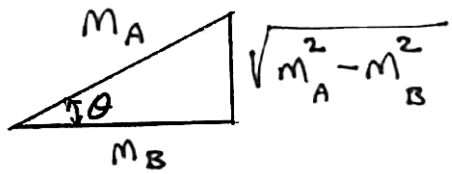
From Fig. b, $\sin\theta = \dfrac{\sqrt{m_A{}^2 - m_B{}^2}}{m_A}$. Substituting this value into Eq. (1),

$$v_B = \sqrt{\frac{m_A g(l-h)}{m_B}}\left(\frac{\sqrt{m_A{}^2 - m_B{}^2}}{m_A}\right)$$

$$= \sqrt{\frac{g(l-h)(m_A{}^2 - m_B{}^2)}{m_A m_B}} \qquad \textbf{Ans.}$$



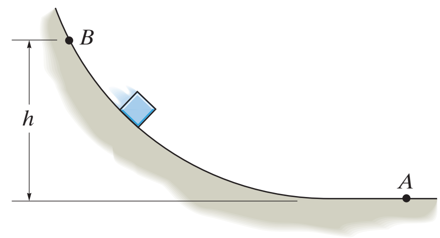
(a)



(b)

***13–76.**

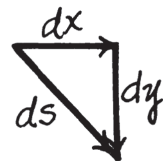
Prove that if the block is released from rest at point B of a smooth path of *arbitrary shape*, the speed it attains when it reaches point A is equal to the speed it attains when it falls freely through a distance h; i.e., $v = \sqrt{2gh}$.



SOLUTION

$+\searrow \Sigma F_t = ma_t;\qquad mg\sin\theta = ma_t \qquad a_t = g\sin\theta$

$v\,dv = a_t\,ds = g\sin\theta\,ds$ However $dy = ds\sin\theta$

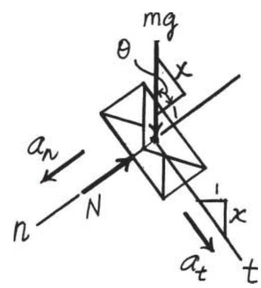


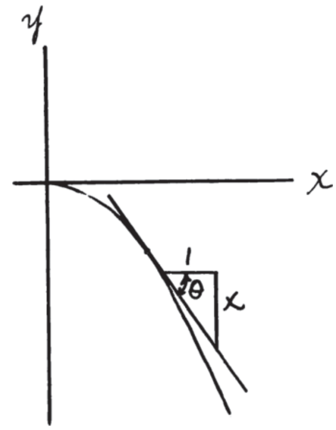
$$\int_0^v v\,dv = \int_0^h g\,dy$$

$$\frac{v^2}{2} = gh$$

$$v = \sqrt{2gh}$$

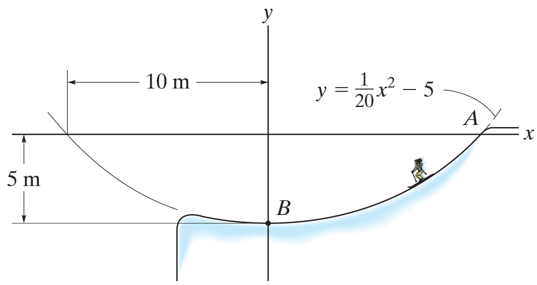
Q.E.D.





13–77.

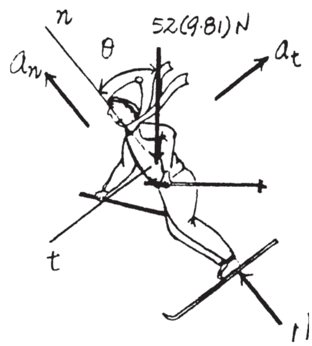
The skier starts from rest at $A(10 \text{ m}, 0)$ and descends the smooth slope, which may be approximated by a parabola. If she has a mass of 52 kg, determine the normal force the ground exerts on the skier at the instant she arrives at point B. Neglect the size of the skier. *Hint:* Use the result of Prob. 13–76.



SOLUTION

Geometry: Here, $\dfrac{dy}{dx} = \dfrac{1}{10}x$ and $\dfrac{d^2y}{dx^2} = \dfrac{1}{10}$. The slope angle θ at point B is given by

$$\tan\theta = \left.\frac{dy}{dx}\right|_{x=0\text{ m}} = 0 \qquad \theta = 0°$$



and the radius of curvature at point B is

$$\rho = \frac{\left[1 + (dy/dx)^2\right]^{3/2}}{|d^2y/dx^2|} = \left.\frac{\left[1 + \left(\frac{1}{10}x\right)^2\right]^{3/2}}{|1/10|}\right|_{x=0\text{ m}} = 10.0 \text{ m}$$

Equations of Motion:

$+\swarrow \Sigma F_t = ma_t; \qquad 52(9.81)\sin\theta = -52a_t \qquad a_t = -9.81\sin\theta$

$+\nwarrow \Sigma F_n = ma_n; \qquad N - 52(9.81)\cos\theta = m\left(\dfrac{v^2}{\rho}\right) \qquad$ **(1)**

Kinematics: The speed of the skier can be determined using $v\,dv = a_t\,ds$. Here, a_t must be in the direction of positive ds. Also, $ds = \sqrt{1 + (dy/dx)^2}dx = \sqrt{1 + \frac{1}{100}x^2}dx$

Here, $\tan\theta = \dfrac{1}{10}x$. Then, $\sin\theta = \dfrac{x}{10\sqrt{1 + \frac{1}{100}x^2}}$.

$$(+\ ) \qquad \int_0^v v\,dv = -9.81\int_{10\text{ m}}^{0}\left(\frac{x}{10\sqrt{1 + \frac{1}{100}x^2}}\right)\left(\sqrt{1 + \tfrac{1}{100}x^2}dx\right)$$

$$v^2 = 9.81 \text{ m}^2/\text{s}^2$$

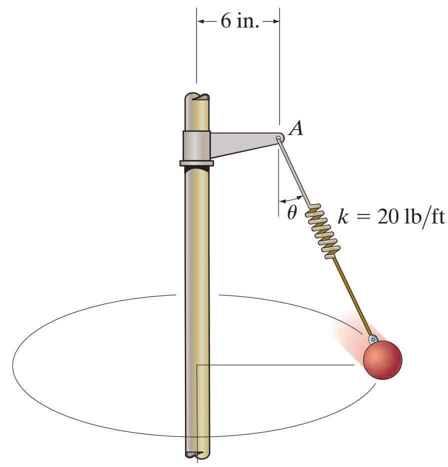
Substituting $v^2 = 98.1 \text{ m}^2/\text{s}^2$, $\theta = 0°$, and $\rho = 10.0$ m into Eq.(1) yields

$$N - 52(9.81)\cos 0° = 52\left(\frac{98.1}{10.0}\right)$$

$$N = 1020.24 \text{ N} = 1.02 \text{ kN}$$ **Ans.**

13–78.

A spring, having an unstretched length of 2 ft, has one end attached to the 10-lb ball. Determine the angle θ of the spring if the ball has a speed of 6 ft/s tangent to the horizontal circular path.



SOLUTION

Free-Body Diagram: The free-body diagram of the bob is shown in Fig. (a). If we denote the stretched length of the spring as l, then using the springforce formula, $F_{sp} = ks = 20(l - 2)$ lb. Here, $\mathbf{a}_n$ must be directed towards the center of the horizontal circular path (positive n axis).

Equations of Motion: The radius of the horizontal circular path is $r = 0.5 + l\sin\theta$. Since $a_n = \dfrac{v^2}{r} = \dfrac{6^2}{0.5 + l\sin\theta}$, by referring to Fig. (a),

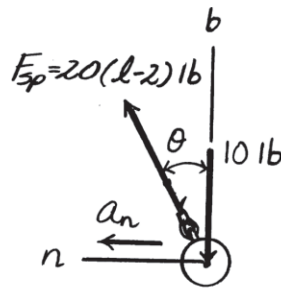
$$+\uparrow \Sigma F_b = 0; \qquad 20(l - 2)\cos\theta - 10 = 0 \qquad \textbf{(1)}$$

$$\overset{+}{\leftarrow} \Sigma F_n = ma_n; \qquad 20(l - 2)\sin\theta = \frac{10}{32.2}\left(\frac{6^2}{0.5 + l\sin\theta}\right) \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$\theta = 31.26° = 31.3° \qquad \textbf{Ans.}$$

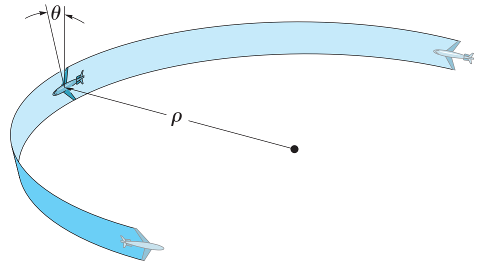
$$l = 2.585 \text{ ft} \qquad \textbf{Ans.}$$



(a)

13–79.

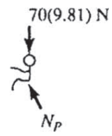
The airplane, traveling at a constant speed of 50 m/s, is executing a horizontal turn. If the plane is banked at $\theta = 15°$, when the pilot experiences only a normal force on the seat of the plane, determine the radius of curvature ρ of the turn. Also, what is the normal force of the seat on the pilot if he has a mass of 70 kg.



SOLUTION

$+\uparrow \sum F_b = ma_b;\qquad N_P \sin 15° - 70(9.81) = 0$

$N_P = 2.65\text{ kN}$ **Ans.**

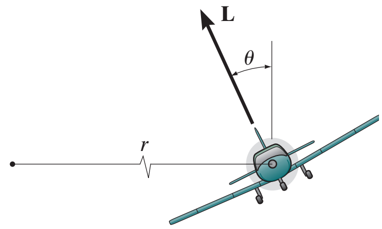


$\overset{+}{\leftarrow} \sum F_n = ma_n;\qquad N_P \cos 15° = 70\left(\frac{50^2}{\rho}\right)$

$\rho = 68.3\text{ m}$ **Ans.**

***13–80.**

A 5-Mg airplane is flying at a constant speed of 350 km/h along a horizontal circular path of radius $r = 3000\,\text{m}$. Determine the uplift force **L** acting on the airplane and the banking angle θ. Neglect the size of the airplane.



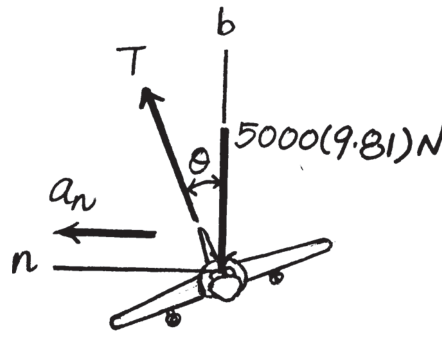
SOLUTION

Free-Body Diagram: The free-body diagram of the airplane is shown in Fig. (a). Here, $\mathbf{a}_n$ must be directed towards the center of curvature (positive n axis).

Equations of Motion: The speed of the airplane is $v = \left(350\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right)$ $= 97.22$ m/s. Realizing that $a_n = \frac{v^2}{\rho} = \frac{97.22^2}{3000} = 3.151\text{ m/s}^2$ and referring to Fig. (a),

$+\uparrow \Sigma F_b = 0;$ $\qquad T\cos\theta - 5000(9.81) = 0 \qquad$ **(1)**

$\overset{+}{\leftarrow} \Sigma F_n = ma_n;$ $\qquad T\sin\theta = 5000(3.151) \qquad$ **(2)**



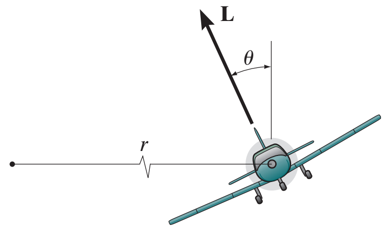
(a)

Solving Eqs. (1) and (2) yields

$\theta = 17.8°$ $\qquad T = 51517.75 = 51.5\text{ kN}$ $\qquad$ **Ans.**

13–81.

A 5-Mg airplane is flying at a constant speed of 350 km/h along a horizontal circular path. If the banking angle $\theta = 15^\circ$, determine the uplift force **L** acting on the airplane and the radius r of the circular path. Neglect the size of the airplane.



SOLUTION

Free-Body Diagram: The free-body diagram of the airplane is shown in Fig. (a). Here, $\mathbf{a}_n$ must be directed towards the center of curvature (positive n axis).

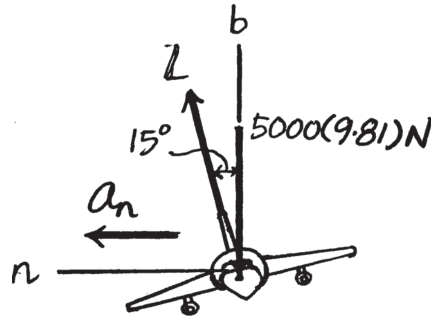
Equations of Motion: The speed of the airplane is $v = \left(350\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right)$ $= 97.22$ m/s. Realizing that $a_n = \frac{v^2}{\rho} = \frac{97.22^2}{r}$ and referring to Fig. (a),

$$+\uparrow \Sigma F_b = 0; \qquad L\cos 15^\circ - 5000(9.81) = 0$$

$$L = 50780.30\text{ N} = 50.8\text{ kN} \qquad \textbf{Ans.}$$

$$\overset{+}{\leftarrow} \Sigma F_n = ma_n; \qquad 50780.30\sin 15^\circ = 5000\left(\frac{97.22^2}{r}\right)$$

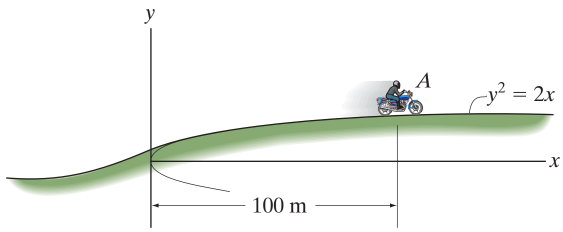
$$r = 3595.92\text{ m} = 3.60\text{ km} \qquad \textbf{Ans.}$$



(a)

13–82.

The 800-kg motorbike travels with a constant speed of 80 km/h up the hill. Determine the normal force the surface exerts on its wheels when it reaches point A. Neglect its size.



SOLUTION

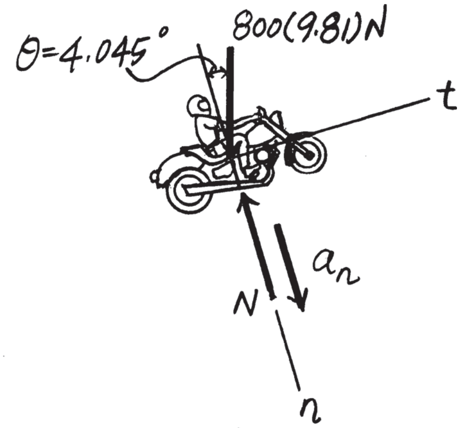
Geometry: Here, $y = \sqrt{2}x^{1/2}$. Thus, $\frac{dy}{dx} = \frac{\sqrt{2}}{2x^{1/2}}$ and $\frac{d^2y}{dx^2} = -\frac{\sqrt{2}}{4x^{3/2}}$. The angle that the hill slope at A makes with the horizontal is

$$\theta = \tan^{-1}\left(\frac{dy}{dx}\right)\bigg|_{x=100\text{ m}} = \tan^{-1}\left(\frac{\sqrt{2}}{2x^{1/2}}\right)\bigg|_{x=100\text{ m}} = 4.045°$$

The radius of curvature of the hill at A is given by

$$\rho_A = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}\Bigg|_{x=100\text{ m}} = \frac{\left[1 + \left(\frac{\sqrt{2}}{2(100^{1/2})}\right)^2\right]^{3/2}}{\left|-\frac{\sqrt{2}}{4(100^{3/2})}\right|} = 2849.67\text{ m}$$

Free-Body Diagram: The free-body diagram of the motorcycle is shown in Fig. (a). Here, $\mathbf{a}_n$ must be directed towards the center of curvature (positive n axis).



(a)

Equations of Motion: The speed of the motorcycle is

$$v = \left(80\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 22.22\text{ m/s}$$

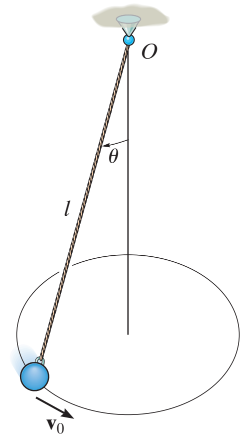
Thus, $a_n = \frac{v^2}{\rho_A} = \frac{22.22^2}{2849.67} = 0.1733\text{ m/s}^2$. By referring to Fig. (a),

$\searrow + \Sigma F_n = ma_n;$ $\qquad 800(9.81)\cos 4.045° - N = 800(0.1733)$

$N = 7689.82\text{ N} = 7.69\text{ kN}$ **Ans.**

13–83.

The ball has a mass m and is attached to the cord of length l. The cord is tied at the top to a swivel and the ball is given a velocity $\mathbf{v}_0$. Show that the angle θ which the cord makes with the vertical as the ball travels around the circular path must satisfy the equation $\tan\theta\sin\theta = v_0^2/gl$. Neglect air resistance and the size of the ball.



SOLUTION

$\overset{+}{\rightarrow}\Sigma F_n = ma_n; \qquad T\sin\theta = m\left(\frac{v_0^2}{r}\right)$

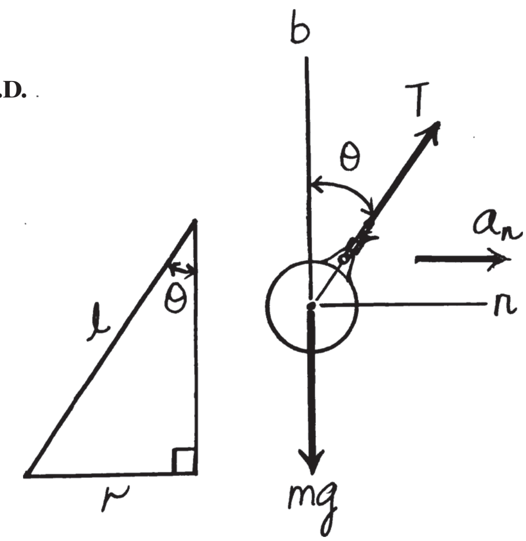
$+\uparrow\Sigma F_b = 0; \qquad T\cos\theta - mg = 0$

Since $r = l\sin\theta \qquad T = \dfrac{mv_0^2}{l\sin^2\theta}$

$$\left(\frac{mv_0^2}{l}\right)\left(\frac{\cos\theta}{\sin^2\theta}\right) = mg$$

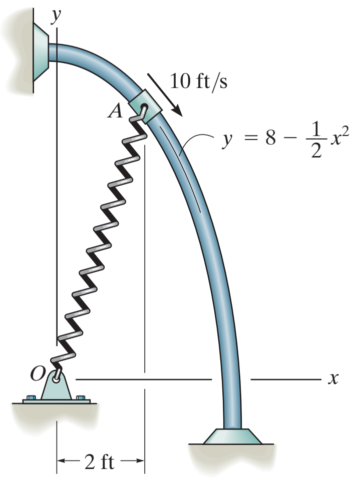
$$\tan\theta\sin\theta = \frac{v_0^2}{gl}$$

Q.E.D.



***13–84.**

The 5-lb collar slides on the smooth rod, so that when it is at A it has a speed of 10 ft/s. If the spring to which it is attached has an unstretched length of 3 ft and a stiffness of $k = 10$ lb/ft, determine the normal force on the collar and the acceleration of the collar at this instant.



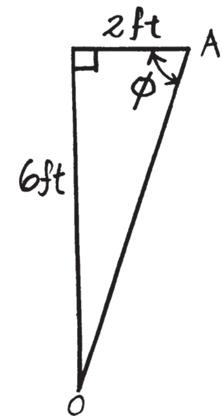
SOLUTION

$$y = 8 - \frac{1}{2}x^2$$

$$-\frac{dy}{dx} = \tan\theta = x\Big|_{x=2} = 2 \qquad \theta = 63.435^\circ$$

$$\frac{d^2y}{dx^2} = -1$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{(1 + (-2)^2)^{\frac{3}{2}}}{|-1|} = 11.18 \text{ ft}$$



$$y = 8 - \frac{1}{2}(2)^2 = 6$$

$$OA = \sqrt{(2)^2 + (6)^2} = 6.3246$$

$$F_s = kx = 10(6.3246 - 3) = 33.246 \text{ lb}$$

$$\tan\phi = \frac{6}{2};\ \phi = 71.565^\circ$$

$+\swarrow\Sigma F_n = ma_n;\qquad 5\cos 63.435^\circ - N + 33.246\cos 45.0^\circ = \left(\frac{5}{32.2}\right)\left(\frac{(10)^2}{11.18}\right)$

$N = 24.4$ lb **Ans.**

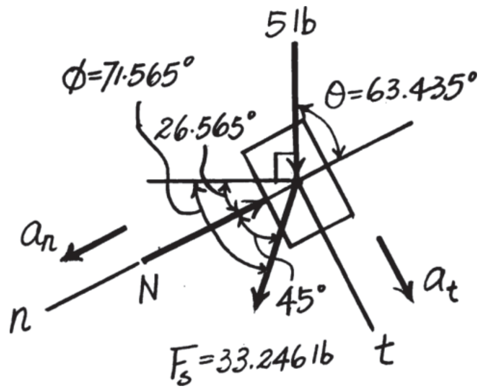
$+\searrow\Sigma F_t = ma_t;\qquad 5\sin 63.435^\circ + 33.246\sin 45.0^\circ = \left(\frac{5}{32.2}\right)a_t$

$$a_t = 180.2 \text{ ft/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{(10)^2}{11.18} = 8.9443 \text{ ft/s}^2$$

$$a = \sqrt{(180.2)^2 + (8.9443)^2}$$

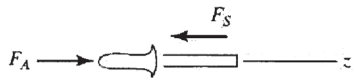
$a = 180 \text{ ft/s}^2$ **Ans.**



13–85.

The spring-held follower AB has a weight of 0.75 lb and moves back and forth as its end rolls on the contoured surface of the cam, where $r = 0.2$ ft and $z = (0.1 \sin \theta)$ ft. If the cam is rotating at a constant rate of 6 rad/s, determine the force at the end A of the follower when $\theta = 90°$. In this position the spring is compressed 0.4 ft. Neglect friction at the bearing C.

SOLUTION



$z = 0.1 \sin 2\theta$

$\dot{z} = 0.2 \cos 2\theta\dot{\theta}$

$\ddot{z} = -0.4 \sin 2\theta\dot{\theta}^2 + 0.2 \cos 2\theta\ddot{\theta}$

$\dot{\theta} = 6$ rad/s

$\ddot{\theta} = 0$

$\ddot{z} = -14.4 \sin 2\theta$

$\sum F_z = ma_z; \qquad F_A - 12(z + 0.3) = m\ddot{z}$

$$F_A - 12(0.1 \sin 2\theta + 0.3) = \frac{0.75}{32.2}(-14.4 \sin 2\theta)$$

For $\theta = 45°$,

$$F_A - 12(0.4) = \frac{0.75}{32.2}(-14.4)$$

$F_A = 4.46$ lb **Ans.**

13–86.

Determine the magnitude of the resultant force acting on a 5-kg particle at the instant $t = 2$ s, if the particle is moving along a horizontal path defined by the equations $r = (2t + 10)$ m and $\theta = (1.5t^2 - 6t)$ rad, where t is in seconds.

SOLUTION

$r = 2t + 10|_{t=2\text{ s}} = 14$

$\dot{r} = 2$

$\ddot{r} = 0$

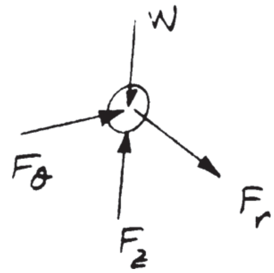
$\theta = 1.5t^2 - 6t$

$\dot{\theta} = 3t - 6\big|_{t=2\text{ s}} = 0$

$\ddot{\theta} = 3$

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 0 = 0$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 14(3) + 0 = 42$



Hence,

$\Sigma F_r = ma_r; \qquad F_r = 5(0) = 0$

$\Sigma F_\theta = ma_\theta; \qquad F_\theta = 5(42) = 210\text{ N}$

$F = \sqrt{(F_r)^2 + (F_\theta)^2} = 210\text{ N}$ **Ans.**

13–87.

The path of motion of a 5-lb particle in the horizontal plane is described in terms of polar coordinates as $r = (2t + 1)$ ft and $\theta = (0.5t^2 - t)$ rad, where t is in seconds. Determine the magnitude of the resultant force acting on the particle when $t = 2$ s.

SOLUTION

$$r = 2t + 1|_{t=2\,s} = 5 \text{ ft} \qquad \dot{r} = 2 \text{ ft/s} \qquad \ddot{r} = 0$$

$$\theta = 0.5t^2 - t|_{t=2\,s} = 0 \text{ rad} \qquad \dot{\theta} = t - 1|_{t=2\,s} = 1 \text{ rad/s} \qquad \ddot{\theta} = 1 \text{ rad/s}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 5(1)^2 = -5 \text{ ft/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 5(1) + 2(2)(1) = 9 \text{ ft/s}^2$$

$$\Sigma F_r = ma_r; \qquad F_r = \frac{5}{32.2}(-5) = -0.7764 \text{ lb}$$

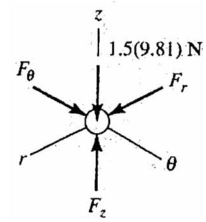
$$\Sigma F_\theta = ma_\theta; \qquad F_\theta = \frac{5}{32.2}(9) = 1.398 \text{ lb}$$

$$F = \sqrt{F_r^2 + F_\theta^2} = \sqrt{(-0.7764)^2 + (1.398)^2} = 1.60 \text{ lb}$$ **Ans.**

***13–88.**

A particle, having a mass of 1.5 kg, moves along a path defined by the equations $r = (4 + 3t)$ m, $\theta = (t^2 + 2)$ rad, and $z = (6 - t^3)$ m, where t is in seconds. Determine the r, θ, and z components of force which the path exerts on the particle when $t = 2$ s.

SOLUTION



$r = 4 + 3t|_{t=2\,s} = 10$ m $\qquad \dot{r} = 3$ m/s $\qquad \ddot{r} = 0$

$\theta = t^2 + 2 \qquad \dot{\theta} = 2t|_{t=2\,\text{s}} = 4$ rad/s $\qquad \ddot{\theta} = 2$ rad/s^2

$z = 6 - t^3 \qquad \dot{z} = -3t^2 \qquad \ddot{z} = -6t|_{t=2\,\text{s}} = -12$ m/s^2

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 10(4)^2 = -160$ m/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 10(2) + 2(3)(4) = 44$ m/s^2

$a_z = \ddot{z} = -12$ m/s^2

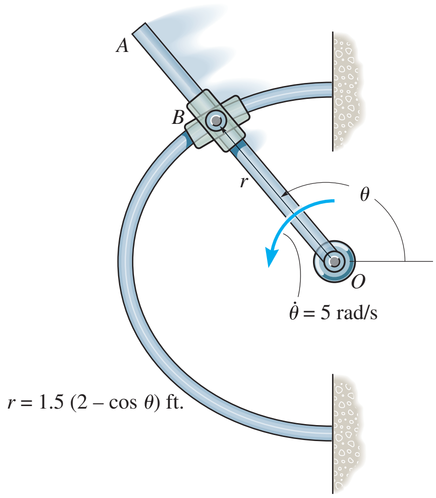
$\Sigma F_r = ma_r; \qquad F_r = 1.5(-160) = -240$ N **Ans.**

$\Sigma F_\theta = ma_\theta; \qquad F_\theta = 1.5(44) = 66$ N **Ans.**

$\Sigma F_z = ma_z; \qquad F_z - 1.5(9.81) = 1.5(-12) \qquad F_z = -3.28$ N **Ans.**

13–89.

Rod OA rotates counterclockwise with a constant angular velocity of $\dot{\theta} = 5$ rad/s. The double collar B is pin-connected together such that one collar slides over the rotating rod and the other slides over the *horizontal* curved rod, of which the shape is described by the equation $r = 1.5(2 - \cos\theta)$ ft. If both collars weigh 0.75 lb, determine the normal force which the curved rod exerts on one collar at the instant $\theta = 120°$. Neglect friction.



SOLUTION

Kinematic: Here, $\dot{\theta} = 5$ rad/s and $\ddot{\theta} = 0$. Taking the required time derivatives at $\theta = 120°$, we have

$$r = 1.5(2 - \cos\theta)|_{\theta=120°} = 3.75 \text{ ft}$$

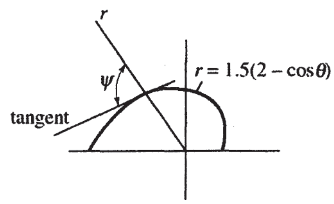
$$\dot{r} = 1.5 \sin\theta\dot{\theta}|_{\theta=120°} = 6.495 \text{ ft/s}$$

$$\ddot{r} = 1.5(\sin\theta\ddot{\theta} + \cos\theta\dot{\theta}^2)|_{\theta=120°} = -18.75 \text{ ft/s}^2$$

Applying Eqs. 12–29, we have

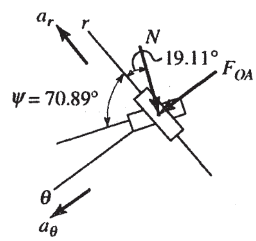
$$a_r = \ddot{r} - r\dot{\theta}^2 = -18.75 - 3.75(5^2) = -112.5 \text{ ft/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 3.75(0) + 2(6.495)(5) = 64.952 \text{ ft/s}^2$$



Equation of Motion: The angle ψ must be obtained first.

$$\tan\psi = \frac{r}{dr/d\theta} = \left.\frac{1.5(2 - \cos\theta)}{1.5\sin\theta}\right|_{\theta=120°} = 2.8867 \qquad \psi = 70.89°$$



Applying Eq. 13–9, we have

$$\sum F_r = ma_r; \qquad -N\cos 19.11° = \frac{0.75}{32.2}(-112.5)$$

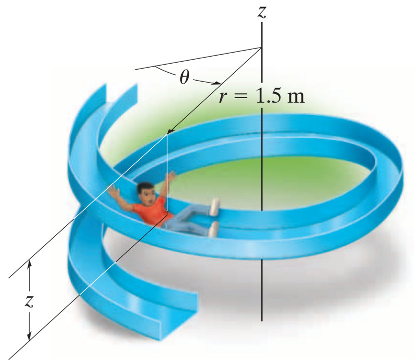
$$N = 2.773 \text{ lb} = 2.77 \text{ lb} \qquad \textbf{Ans.}$$

$$\sum F_\theta = ma_\theta; \qquad F_{OA} + 2.773\sin 19.11° = \frac{0.75}{32.2}(64.952)$$

$$F_{OA} = 0.605 \text{ lb}$$

13–90.

The boy of mass 40 kg is sliding down the spiral slide at a constant speed such that his position, measured from the top of the chute, has components $r = 1.5$ m, $\theta = (0.7t)$ rad, and $z = (-0.5t)$ m, where t is in seconds. Determine the components of force $\mathbf{F}_r$, $\mathbf{F}_\theta$, and $\mathbf{F}_z$ which the slide exerts on him at the instant $t = 2$ s. Neglect the size of the boy.



SOLUTION

$r = 1.5 \qquad \theta = 0.7t \qquad z = -0.5t$

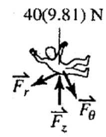
$\dot{r} = \ddot{r} = 0 \qquad \dot{\theta} = 0.7 \qquad \dot{z} = -0.5$

$\ddot{\theta} = 0 \qquad \ddot{z} = 0$

$a_r = \ddot{r} - r(\dot{\theta})^2 = 0 - 1.5(0.7)^2 = -0.735$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0$

$a_z = \ddot{z} = 0$



$\Sigma F_r = ma_r; \qquad F_r = 40(-0.735) = -29.4\text{ N}$ **Ans.**

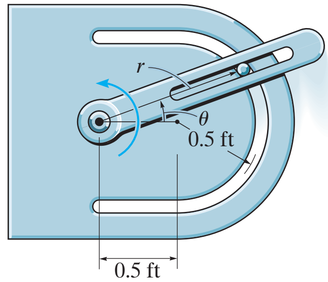
$\Sigma F_\theta = ma_\theta; \qquad F_\theta = 0$ **Ans.**

$\Sigma F_z = ma_z; \qquad F_z - 40(9.81) = 0$

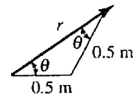
$F_z = 392\text{ N}$ **Ans.**

13–91.

The 0.5-lb particle is guided along the circular path using the slotted arm guide. If the arm has an angular velocity $\dot{\theta} = 4$ rad/s and an angular acceleration $\ddot{\theta} = 8$ rad/s^2 at the instant $\theta = 30°$, determine the force of the guide on the particle. Motion occurs in the *horizontal plane*.



SOLUTION

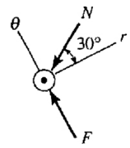


$$r = 2(0.5 \cos\theta) = 1 \cos\theta$$

$$\dot{r} = -\sin\theta\dot{\theta}$$

$$\bar{r} = -\cos\theta\dot{\theta}^2 - \sin\theta\ddot{\theta}$$

At $\theta = 30°$, $\dot{\theta} = 4$ rad/s and $\ddot{\theta} = 8$ rad/s^2



$$r = 1 \cos 30° = 0.8660 \text{ ft}$$

$$\dot{r} = -\sin 30°(4) = -2 \text{ ft/s}$$

$$\ddot{r} = -\cos 30°(4)^2 - \sin 30°(8) = -17.856 \text{ ft/s}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -17.856 - 0.8660(4)^2 = -31.713 \text{ ft/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.8660(8) + 2(-2)(4) = -9.072 \text{ ft/s}^2$$

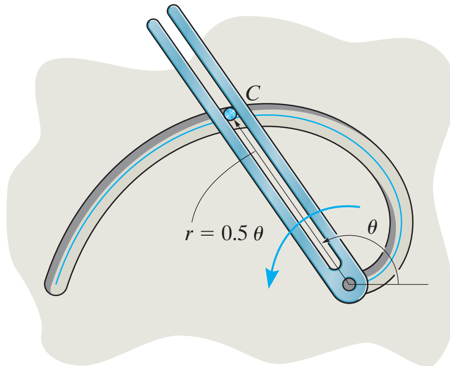
$$\nearrow + \Sigma F_r = ma_r; \qquad -N\cos 30° = \frac{0.5}{32.2}(-31.713) \qquad N = 0.5686 \text{ lb}$$

$$+\nwarrow \Sigma F_\theta = ma_\theta; \qquad F - 0.5686 \sin 30° = \frac{0.5}{32.2}(-9.072)$$

$$F = 0.143 \text{ lb}$$ **Ans.**

***13–92.**

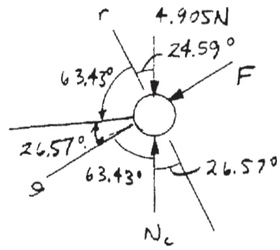
Using a forked rod, a smooth cylinder C having a mass of 0.5 kg is forced to move along the *vertical slotted* path $r = (0.5\theta)$ m, where θ is in radians. If the angular position of the arm is $\theta = (0.5t^2)$ rad, where t is in seconds, determine the force of the rod on the cylinder and the normal force of the slot on the cylinder at the instant $t = 2$ s. The cylinder is in contact with only *one edge* of the rod and slot at any instant.



SOLUTION

$r = 0.5\theta \qquad \mathrm{r} = 0.5\dot{\theta} \qquad \ddot{r} = 0.5\ddot{\theta}$

$\theta = 0.5t^2 \qquad \dot{\theta} = t \qquad \ddot{\theta} = 1$



At $t = 2$ s,

$\theta = 2 \text{ rad} = 114.59° \qquad \dot{\theta} = 2 \text{ rad/2} \qquad \ddot{\theta} = 1 \text{ rad/s}^2$

$r = 1 \text{ m} \qquad \dot{r} = 1 \text{ m/s} \qquad \ddot{r} = 0.5 \text{ m/s}^2$

$$\tan\psi = \frac{r}{dr/d\theta} = \frac{0.5(2)}{0.5} \qquad \psi = 63.43°$$

$a_r = \ddot{r} - r\dot{\theta}^2 = 0.5 - 1(2)^2 = -3.5$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 1(1) + 2(1)(2) = 5$

$+\nwarrow\Sigma F_r = ma_r; \qquad N_C \cos 26.57° - 4.905 \cos 24.59° = 0.5(-3.5)$

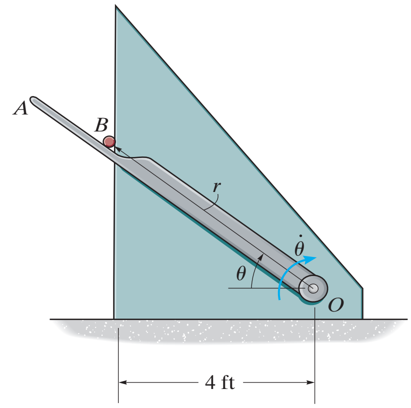
$N_C = 3.030 = 3.03 \text{ N}$ **Ans.**

$+\swarrow\Sigma F_\theta = ma_\theta; \qquad F - 3.030 \sin 26.57° + 4.905 \sin 24.59° = 0.5(5)$

$F = 1.81 \text{ N}$ **Ans.**

13–93.

If arm OA rotates with a constant clockwise angular velocity of $\dot{\theta} = 1.5$ rad/s. determine the force arm OA exerts on the smooth 4-lb cylinder B when $\theta = 45°$.



SOLUTION

Kinematics: Since the motion of cylinder B is known, $\mathbf{a}_r$ and $\mathbf{a}_\theta$ will be determined first. Here, $\frac{4}{r} = \cos\theta$ or $r = 4\sec\theta$ ft. The value of r and its time derivatives at the instant $\theta = 45°$ are

$$r = 4\sec\theta\big|_{\theta=45°} = 4\sec 45° = 5.657 \text{ ft}$$

$$\dot{r} = 4\sec\theta(\tan\theta)\dot{\theta}\big|_{\theta=45°} = 4\sec 45°\tan 45°(1.5) = 8.485 \text{ ft/s}$$

$$\ddot{r} = 4\left[\sec\theta(\tan\theta)\ddot{\theta} + \dot{\theta}\left(\sec\theta\ \sec^2\theta\dot{\theta} + \tan\theta\sec\theta\tan\theta\dot{\theta}\right)\right]$$

$$= 4\left[\sec\theta(\tan\theta)\ddot{\theta} + \sec^3\theta\dot{\theta}^2 + \sec\theta\tan^2\theta\dot{\theta}^2\right]\Big|_{\theta=45°}$$

$$= 4\left[\sec 45°\tan 45°(0) + \sec^3 45°(1.5)^2 + \sec 45°\tan^2 45°(1.5)^2\right]$$

$$= 38.18 \text{ ft/s}^2$$

Using the above time derivatives,

$$a_r = \ddot{r} - r\dot{\theta}^2 = 38.18 - 5.657\left(1.5^2\right) = 25.46 \text{ ft/s}^2$$

$$a_\theta = r\ddot{\theta} - 2\dot{r}\dot{\theta} = 5.657(0) + 2(8.485)(1.5) = 25.46 \text{ ft/s}^2$$

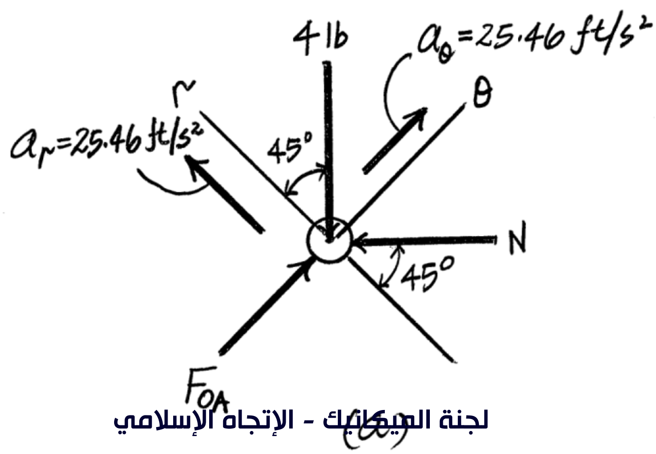
Equations of Motion: By referring to the free-body diagram of the cylinder shown in Fig. a,

$\Sigma F_r = ma_r;$ $\quad N\cos 45° - 4\cos 45° = \frac{4}{32.2}(25.46)$

$N = 8.472$ lb

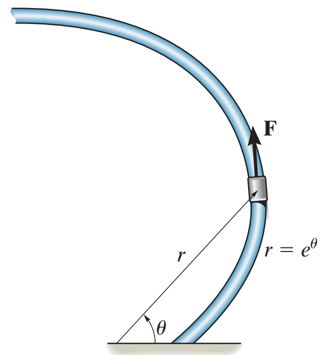
$\Sigma F_\theta = ma_\theta;$ $\quad F_{OA} - 8.472\sin 45° - 4\sin 45° = \frac{4}{32.2}(25.46)$

$F_{OA} = 12.0$ lb **Ans.**



13–94.

The collar has a mass of 2 kg and travels along the smooth horizontal rod defined by the equiangular spiral $r = (e^{\theta})$ m, where θ is in radians. Determine the tangential force F and the normal force N acting on the collar when $\theta = 90°$, if the force F maintains a constant angular motion $\dot{\theta} = 2$ rad/s.



SOLUTION

$r = e^{\theta}$

$\dot{r} = e^{\theta}\dot{\theta}$

$\ddot{r} = e^{\theta}(\dot{\theta})^2 + e^{\theta}\ddot{\theta}$

At $\theta = 90°$

$\dot{\theta} = 2$ rad/s

$\ddot{\theta} = 0$

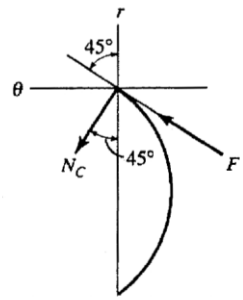
$r = 4.8105$

$\dot{r} = 9.6210$

$\ddot{r} = 19.242$

$a_r = \ddot{r} - r(\dot{\theta})^2 = 19.242 - 4.8105(2)^2 = 0$

$a_{\theta} = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(9.6210)(2) = 38.4838 \text{ m/s}^2$



$\tan\psi = \dfrac{r}{\left(\frac{dr}{d\theta}\right)} = e^{\theta}/e^{\theta} = 1$

$\psi = 45°$

$+\uparrow \sum F_r = ma_r; \qquad -N_C \cos 45° + F \cos 45° = 2(0)$

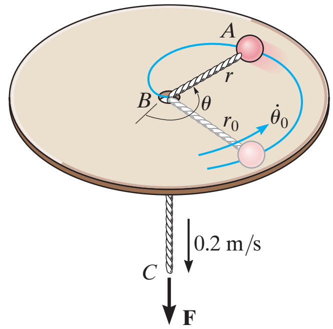
$\overset{+}{\leftarrow} \sum F_{\theta} = ma_{\theta}; \qquad F \sin 45° + N_C \sin 45° = 2(38.4838)$

$N_C = 54.4$ N **Ans.**

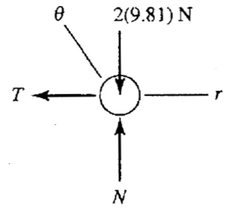
$F = 54.4$ N **Ans.**

13–95.

The ball has a mass of 2 kg and a negligible size. It is originally traveling around the horizontal circular path of radius $r_0 = 0.5$ m such that the angular rate of rotation is $\dot{\theta}_0 = 1$ rad/s. If the attached cord ABC is drawn down through the hole at a constant speed of 0.2 m/s, determine the tension the cord exerts on the ball at the instant $r = 0.25$ m. Also, compute the angular velocity of the ball at this instant. Neglect the effects of friction between the ball and horizontal plane. *Hint:* First show that the equation of motion in the θ direction yields $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (1/r)(d(r^2\dot{\theta})/dt) = 0$. When integrated, $r^2\dot{\theta} = c$, where the constant c is determined from the problem data.



SOLUTION



$$\sum F_\theta = ma_\theta; \qquad 0 = m[r\ddot{\theta} + 2\dot{r}\dot{\theta}] = m\left[\frac{1}{r}\frac{d}{dt}(r^2\dot{\theta})\right] = 0$$

Thus,

$$d(r^2\dot{\theta}) = 0$$

$$r^2\dot{\theta} = C$$

$$(0.5)^2(1) = C = (0.25)^2\dot{\theta}$$

$$\dot{\theta} = 4.00 \text{ rad/s}$$ **Ans.**

Since $\dot{r} = -0.2$ m/s, $\quad \ddot{r} = 0$

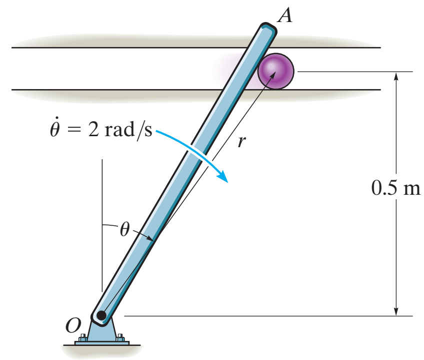
$$a_r = \ddot{r} - \dot{r}(\theta)^2 = 0 - 0.25(4.00)^2 = -4 \text{ m/s}^2$$

$$\sum F_r = ma_r; \quad -T = 2(-4)$$

$$T = 8 \text{ N}$$ **Ans.**

***13–96.**

The particle has a mass of 0.5 kg and is confined to move along the smooth horizontal slot due to the rotation of the arm OA. Determine the force of the rod on the particle and the normal force of the slot on the particle when $\theta = 30°$. The rod is rotating with a constant angular velocity $\dot{\theta} = 2$ rad/s. Assume the particle contacts only one side of the slot at any instant.



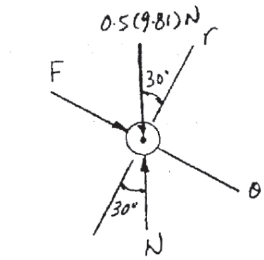
SOLUTION

$$r = \frac{0.5}{\cos\ \theta} = 0.5 \sec\theta$$

$$\dot{r} = 0.5 \sec\theta \tan\theta\dot{\theta}$$

$$\ddot{r} = 0.5\left\{\left[(\sec\theta\tan\theta\dot{\theta})\tan\theta + \sec\theta(\sec^2\theta\dot{\theta}\right]\dot{\theta} + \sec\theta\tan\theta\ddot{\theta}\right\}$$

$$= 0.5\left[\sec\theta\tan^2\theta\dot{\theta} + \sec^3\theta\dot{\theta}^2 + \sec\theta\tan\theta\ddot{\theta}\right]$$



When $\theta = 30°, \dot{\theta} = 2$ rad/s and $\ddot{\theta} = 0$

$$r = 0.5 \sec 30° = 0.5774 \text{ m}$$

$$\dot{r} = 0.5 \sec 30° \tan 30°(2) = 0.6667 \text{ m/s}$$

$$\ddot{r} = 0.5\left[\sec 30° \tan^2 30°(2)^2 + \sec^3 30°(2)^2 + \sec 30° \tan 30°(0)\right]$$

$$= 3.849 \text{ m/s}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 3.849 - 0.5774(2)^2 = 1.540 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.5774(0) + 2(0.6667)(2) = 2.667 \text{ m/s}^2$$

$\nearrow + \Sigma F_r = ma_r;\qquad N\ \cos 30° - 0.5(9.81)\ \cos 30° = 0.5(1.540)$

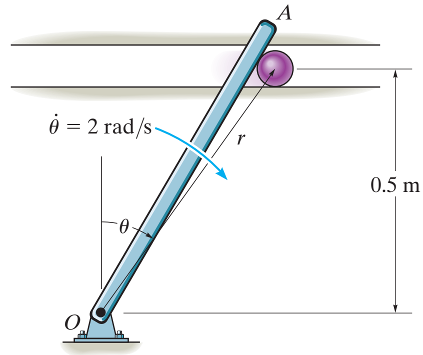
$$N = 5.79 \text{ N} \qquad \textbf{Ans.}$$

$+\searrow \Sigma F_\theta = ma_\theta;\qquad F + 0.5(9.81) \sin 30° - 5.79 \sin 30° = 0.5(2.667)$

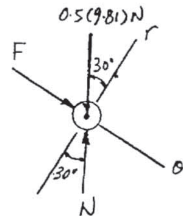
$$F = 1.78 \text{ N} \qquad \textbf{Ans.}$$

13–97.

Solve Problem 13–96 if the arm has an angular acceleration of $\ddot{\theta} = 3\ \text{rad/s}^2$ and $\dot{\theta} = 2$ rad/s at this instant. Assume the particle contacts only one side of the slot at any instant.



SOLUTION



$$r = \frac{0.5}{\cos\theta} = 0.5\sec\theta$$

$$\dot{r} = 0.5\sec\theta\tan\theta\dot{\theta}$$

$$\ddot{r} = 0.5\left\{\left[(\sec\theta\tan\theta\dot{\theta})\tan\theta + \sec\theta(\sec^2\theta\dot{\theta})\right]\dot{\theta} + \sec\theta\tan\theta\ddot{\theta}\right\}$$

$$= 0.5\left[\sec\theta\tan^2\theta\dot{\theta}^2 + \sec^3\theta\dot{\theta}^2 + \sec\theta\tan\theta\ddot{\theta}\right]$$

When $\theta = 30°$, $\dot{\theta} = 2$ rad/s and $\ddot{\theta} = 3\ \text{rad/s}^2$

$$r = 0.5\sec 30° = 0.5774\ \text{m}$$

$$\dot{r} = 0.5\sec 30°\tan 30°(2) = 0.6667\ \text{m/s}$$

$$\ddot{r} = 0.5\left[\sec 30°\tan^2 30°(2)^2 + \sec^3 30°(2)^2 + \sec 30°\tan 30°(3)\right]$$

$$= 4.849\ \text{m/s}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 4.849 - 0.5774(2)^2 = 2.5396\ \text{m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.5774(3) + 2(0.6667)(2) = 4.3987\ \text{m/s}^2$$

$\nearrow + \Sigma F_r = ma_r;$ $\quad N\cos 30° - 0.5(9.81)\cos 30° = 0.5(2.5396)$

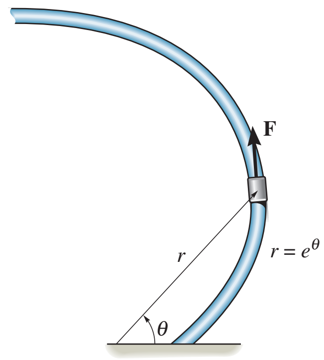
$N = 6.3712 = 6.37\ \text{N}$ **Ans.**

$+\searrow \Sigma F_\theta = ma_\theta;$ $\quad F + 0.5(9.81)\sin 30° - 6.3712\sin 30° = 0.5(4.3987)$

$F = 2.93\ \text{N}$ **Ans.**

13–98.

The collar has a mass of 2 kg and travels along the smooth horizontal rod defined by the equiangular spiral $r = (e^{\theta})$ m, where θ is in radians. Determine the tangential force F and the normal force N acting on the collar when $\theta = 45°$, if the force F maintains a constant angular motion $\dot{\theta} = 2$ rad/s.



SOLUTION

$r = e^{\theta}$

$\dot{r} = e^{\theta}\,\dot{\theta}$

$\ddot{r} = e^{\theta}(\dot{\theta})^2 + e^{\theta}\ddot{\theta}$

At $\theta = 45°$

$\dot{\theta} = 2$ rad/s

$\ddot{\theta} = 0$

$r = 2.1933$

$\dot{r} = 4.38656$

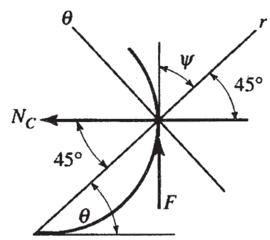
$\ddot{r} = 8.7731$

$a_r = \ddot{r} - r(\dot{\theta})^2 = 8.7731 - 2.1933(2)^2 = 0$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(4.38656)(2) = 17.5462 \text{ m/s}^2$

$$\tan\psi = \frac{r}{\left(\dfrac{dr}{d\theta}\right)} = e^{\theta}/e^{\theta} = 1$$

$\psi = \theta = 45°$



$\nearrow\!+ \sum F_r = ma_r; \qquad -N_C \cos 45° + F\cos 45° = 2(0)$

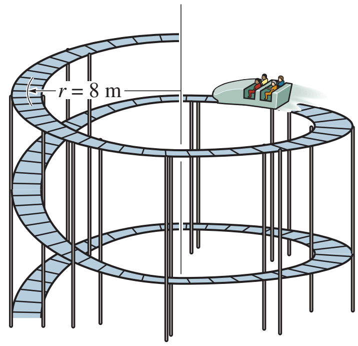
$+\nwarrow \sum F_\theta = ma_\theta; \qquad F\sin 45° + N_C \sin 45° = 2(17.5462)$

$N = 24.8$ N **Ans.**

$F = 24.8$ N **Ans.**

13–99.

For a short time, the 250-kg roller coaster car is traveling along the spiral track such that its position measured from the top of the track has components $r = 8$ m, $\theta = (0.1t + 0.5)$ rad, and $z = (-0.2t)$ m, where t is in seconds. Determine the magnitudes of the components of force which the track exerts on the car in the r, θ, and z directions at the instant $t = 2$ s. Neglect the size of the car.



SOLUTION

Kinematic: Here, $r = 8$ m, $\dot{r} = \ddot{r} = 0$. Taking the required time derivatives at $t = 2$s, we have

$$\theta = 0.1t + 0.5|_{t=2s} = 0.700 \text{ rad} \qquad \dot{\theta} = 0.100 \text{ rad/s} \qquad \ddot{\theta} = 0$$

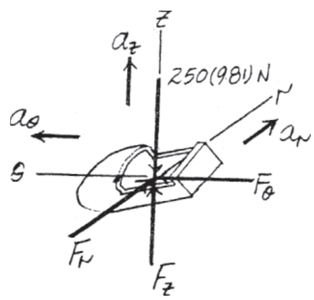
$$z = -0.2t|_{t=2s} = -0.400 \text{ m} \qquad \dot{z} = -0.200 \text{ m/s} \qquad \ddot{z} = 0$$

Applying Eqs. 12–29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 8(0.100^2) = -0.0800 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 8(0) + 2(0)(0.200) = 0$$

$$a_z = \ddot{z} = 0$$



Equation of Motion:

$\Sigma F_r = ma_r; \qquad F_r = 250(-0.0800) = -20.0 \text{ N}$ **Ans.**

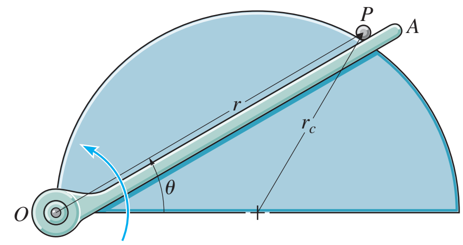
$\Sigma F_\theta = ma_\theta; \qquad F_\theta = 250(0) = 0$ **Ans.**

$\Sigma F_z = ma_z; \qquad F_z - 250(9.81) = 250(0)$

$F_z = 2452.5 \text{ N} = 2.45 \text{ kN}$ **Ans.**

***13–100.**

The 0.5-lb ball is guided along the vertical circular path $r = 2r_c \cos\theta$ using the arm OA. If the arm has an angular velocity $\dot{\theta} = 0.4$ rad/s and an angular acceleration $\ddot{\theta} = 0.8\ \text{rad/s}^2$ at the instant $\theta = 30°$, determine the force of the arm on the ball. Neglect friction and the size of the ball. Set $r_c = 0.4$ ft.



SOLUTION

$r = 2(0.4)\cos\theta = 0.8\cos\theta$

$\dot{r} = -0.8\sin\theta\dot{\theta}$

$\ddot{r} = -0.8\cos\theta\dot{\theta}^2 - 0.8\sin\theta\ddot{\theta}$

At $\theta = 30°$, $\dot{\theta} = 0.4$ rad/s, and $\ddot{\theta} = 0.8\ \text{rad/s}^2$

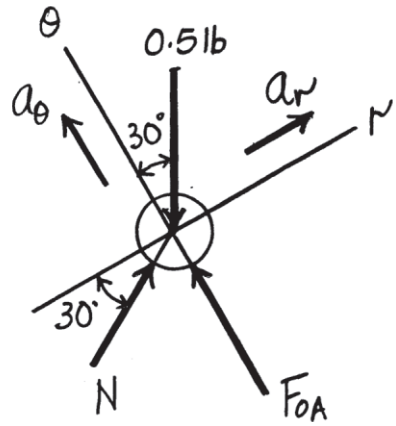
$r = 0.8\cos 30° = 0.6928$ ft

$\dot{r} = -0.8\sin 30°(0.4) = -0.16$ ft/s

$\ddot{r} = -0.8\cos 30°(0.4)^2 - 0.8\sin 30°(0.8) = -0.4309\ \text{ft/s}^2$

$a_r = \ddot{r} - r\dot{\theta}^2 = -0.4309 - 0.6928(0.4)^2 = -0.5417\ \text{ft/s}^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.6928(0.8) + 2(-0.16)(0.4) = 0.4263\ \text{ft/s}^2$



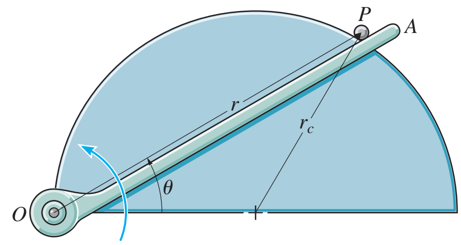
$+\nearrow\Sigma F_r = ma_r;\qquad N\cos 30° - 0.5\sin 30° = \dfrac{0.5}{32.2}(-0.5417)\qquad N = 0.2790$ lb

$\nwarrow+\Sigma F_\theta = ma_\theta;\qquad F_{OA} + 0.2790\sin 30° - 0.5\cos 30° = \dfrac{0.5}{32.2}(0.4263)$

$F_{OA} = 0.300$ lb **Ans.**

13–101.

The ball of mass m is guided along the vertical circular path $r = 2r_c \cos\theta$ using the arm OA. If the arm has a constant angular velocity $\dot{\theta}_0$, determine the angle $\theta \leq 45°$ at which the ball starts to leave the surface of the semicylinder. Neglect friction and the size of the ball.



SOLUTION

$r = 2r_c \cos\theta$

$\dot{r} = -2r_c \sin\theta\dot{\theta}$

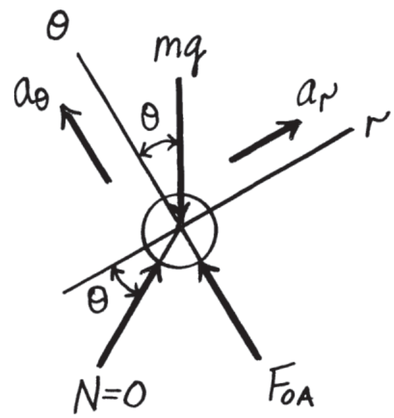
$\ddot{r} = -2r_c \cos\theta\dot{\theta}^2 - 2r_c \sin\theta\ddot{\theta}$

Since $\dot{\theta}$ is constant, $\ddot{\theta} = 0$.

$$a_r = \ddot{r} - r\dot{\theta}^2 = -2r_c \cos\theta\dot{\theta}_0^2 - 2r_c \cos\theta\dot{\theta}_0^2 = -4r_c \cos\theta\dot{\theta}_0^2$$

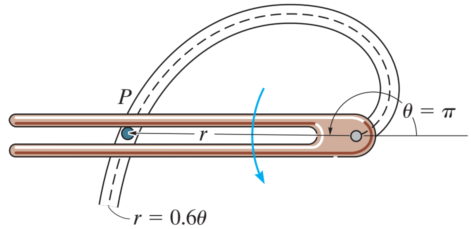
$+\nearrow\Sigma F_r = ma_r;\qquad -mg\sin\theta = m(-4r_c \cos\theta\dot{\theta}_0^2)$

$$\tan\theta = \frac{4r_c\,\dot{\theta}_0^2}{g} \qquad \theta = \tan^{-1}\left(\frac{4r_c\,\dot{\theta}_0^2}{g}\right) \qquad \textbf{Ans.}$$



13–102.

Using a forked rod, a smooth cylinder P, having a mass of 0.4 kg, is forced to move along the *vertical slotted* path $r = (0.6\theta)$ m, where θ is in radians. If the cylinder has a constant speed of $v_C = 2$ m/s, determine the force of the rod and the normal force of the slot on the cylinder at the instant $\theta = \pi$ rad. Assume the cylinder is in contact with only *one edge* of the rod and slot at any instant. *Hint:* To obtain the time derivatives necessary to compute the cylinder's acceleration components a_r and a_θ, take the first and second time derivatives of $r = 0.6\theta$. Then, for further information, use Eq. 12–26 to determine $\dot{\theta}$. Also, take the time derivative of Eq. 12–26, noting that $\dot{v}_C = 0$, to determine $\ddot{\theta}$.



SOLUTION

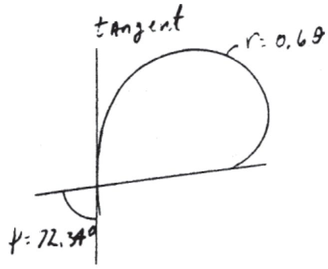
$$r = 0.6\theta \qquad \dot{r} = 0.6\dot{\theta} \qquad \ddot{r} = 0.6\ddot{\theta}$$

$$v_r = \dot{r} = 0.6\dot{\theta} \qquad v_\theta = r\dot{\theta} = 0.6\theta\dot{\theta}$$

$$v^2 = \dot{r}^2 + \left(r\theta\right)^2$$

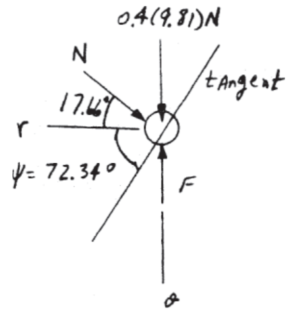
$$2^2 = \left(0.6\dot{\theta}\right)^2 + \left(0.6\theta\dot{\theta}\right)^2 \qquad \dot{\theta} = \frac{2}{0.6\sqrt{1+\theta^2}}$$

$$0 = 0.72\dot{\theta}\ddot{\theta} + 0.36\left(2\theta\dot{\theta}^3 + 2\theta^2\dot{\theta}\ddot{\theta}\right) \qquad \ddot{\theta} = -\frac{\theta\dot{\theta}^2}{1+\theta^2}$$



At $\theta = \pi$ rad, $\quad \dot{\theta} = \dfrac{2}{0.6\sqrt{1+\pi^2}} = 1.011$ rad/s

$$\ddot{\theta} = -\frac{(\pi)(1.011)^2}{1+\pi^2} = -0.2954 \text{ rad/s}^2$$



$$r = 0.6(\pi) = 0.6\,\pi \text{ m} \qquad \dot{r} = 0.6(1.011) = 0.6066 \text{ m/s}$$

$$\ddot{r} = 0.6(-0.2954) = -0.1772 \text{ m/s}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -0.1772 - 0.6\pi(1.011)^2 = -2.104 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.6\pi(-0.2954) + 2(0.6066)(1.011) = 0.6698 \text{ m/s}^2$$

$$\tan\psi = \frac{r}{dr/d\theta} = \frac{0.6\theta}{0.6} = \theta = \pi \qquad \psi = 72.34^\circ$$

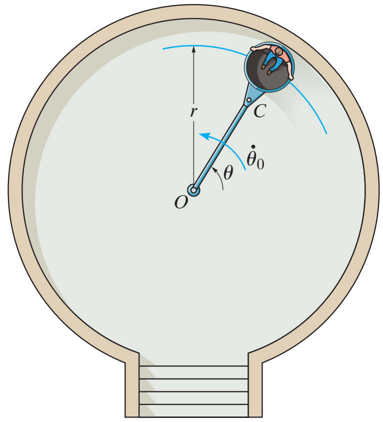
$\overset{+}{\leftarrow} \Sigma F_r = ma_r; \qquad -N\cos 17.66^\circ = 0.4(-2.104) \qquad N = 0.883$ N **Ans.**

$+\downarrow \Sigma F_\theta = ma_\theta; \qquad -F + 0.4(9.81) + 0.883\sin 17.66^\circ = 0.4(0.6698)$

$F = 3.92$ N **Ans.**

13–103.

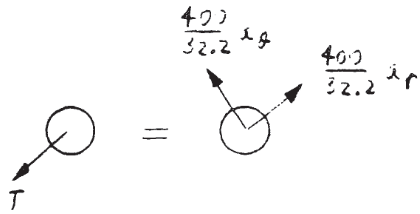
A ride in an amusement park consists of a cart which is supported by small wheels. Initially the cart is traveling in a circular path of radius $r_0 = 16$ ft such that the angular rate of rotation is $\dot{\theta}_0 = 0.2$ rad/s. If the attached cable OC is drawn inward at a constant speed of $\dot{r} = -0.5$ ft/s, determine the tension it exerts on the cart at the instant $r = 4$ ft. The cart and its passengers have a total weight of 400 lb. Neglect the effects of friction. *Hint:* First show that the equation of motion in the θ direction yields $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (1/r)\,d(r^2\dot{\theta})/dt = 0$. When integrated, $r^2\dot{\theta} = c$, where the constant c is determined from the problem data.



SOLUTION

$$+\nearrow\Sigma F_r = ma_r; \qquad -T = \left(\frac{400}{32.2}\right)\left(\ddot{r} - r\dot{\theta}^2\right) \qquad \textbf{(1)}$$

$$+\nwarrow\Sigma F_\theta = ma_\theta; \qquad 0 = \left(\frac{400}{32.2}\right)\left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right) \qquad \textbf{(2)}$$



From Eq. (2), $\left(\frac{1}{r}\right)\frac{d}{dt}\left(r^2\dot{\theta}\right) = 0 \qquad r^2\dot{\theta} = c$

Since $\dot{\theta}_0 = 0.2$ rad/s when $r_0 = 16$ ft, $c = 51.2$.

Hence, when $r = 4$ ft,

$$\dot{\theta} = \left(\frac{51.2}{(4)^2}\right) = 3.2 \text{ rad/s}$$

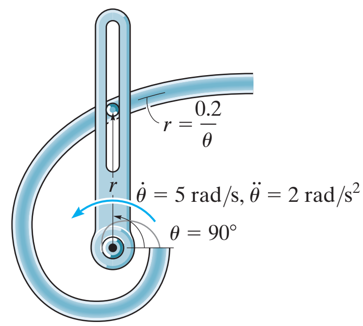
Since $r = -0.5$ ft/s, $\ddot{r} = 0$, Eq. (1) becomes

$$-T = \left(\frac{400}{32.2}\right)\left(0 - (4)(3.2)^2\right)$$

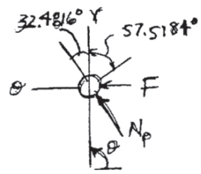
$T = 509$ lb **Ans.**

***13–104.**

The arm is rotating at a rate of $\dot{\theta} = 5\ \text{rad/s}$ when $\ddot{\theta} = 2\ \text{rad/s}^2$ and $\theta = 90^\circ$. Determine the normal force it must exert on the 0.5-kg particle if the particle is confined to move along the slotted path defined by the *horizontal* hyperbolic spiral $r\theta = 0.2$ m.



SOLUTION



$$\theta = \frac{\pi}{2} = 90^\circ$$

$$\dot{\theta} = 5\ \text{rad/s}$$

$$\ddot{\theta} = 2\ \text{rad/s}^2$$

$$r = 0.2/\theta = 0.12732\ \text{m}$$

$$\dot{r} = -0.2\,\theta^{-2}\dot{\theta} = -0.40528\ \text{m/s}$$

$$\ddot{r} = -0.2[-2\theta^{-3}\,(\dot{\theta})^2 + \theta^{-2}\ddot{\theta}] = 2.41801$$

$$a_r = \ddot{r} - r(\dot{\theta})^2 = 2.41801 - 0.12732(5)^2 = -0.7651\ \text{m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\,\dot{r}\dot{\theta} = 0.12732(2) + 2(-0.40528)(5) = -3.7982\ \text{m/s}^2$$

$$\tan\psi = \frac{r}{\left(\frac{dr}{d\theta}\right)} = \frac{0.2/\theta}{-0.2\theta^{-2}}$$

$$\psi = \tan^{-1}\left(-\frac{\pi}{2}\right) = -57.5184^\circ$$

$+\uparrow \Sigma F_r = m\,a_r;\qquad N_p \cos 32.4816^\circ = 0.5(-0.7651)$

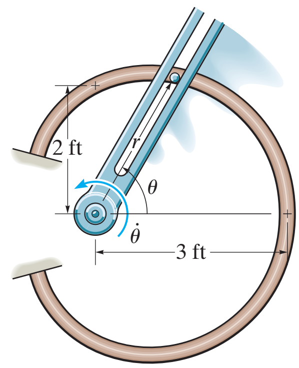
$\overset{+}{\leftarrow} \Sigma F_\theta = ma_\theta;\qquad F + N_p \sin 32.4816^\circ = 0.5(-3.7982)$

$N_P = -0.453\ \text{N}$

$F = -1.66\ \text{N}$ **Ans.**

13–105.

The forked rod is used to move the smooth 2-lb particle around the horizontal path in the shape of a limaçon, $r = (2 + \cos\theta)$ ft. If at all times $\dot{\theta} = 0.5$ rad/s, determine the force which the rod exerts on the particle at the instant $\theta = 90°$. The fork and path contact the particle on only one side.



SOLUTION

$r = 2 + \cos\theta$

$\dot{r} = -\sin\theta\dot{\theta}$

$\ddot{r} = -\cos\theta\dot{\theta}^2 - \sin\theta\ddot{\theta}$

At $\theta = 90°$, $\dot{\theta} = 0.5$ rad/s, and $\ddot{\theta} = 0$

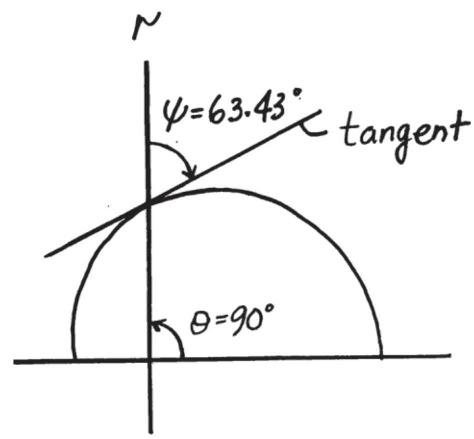
$r = 2 + \cos 90° = 2$ ft

$\dot{r} = -\sin 90°(0.5) = -0.5$ ft/s

$\ddot{r} = -\cos 90°(0.5)^2 - \sin 90°(0) = 0$

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 2(0.5)^2 = -0.5 \text{ ft/s}^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2(0) + 2(-0.5)(0.5) = -0.5 \text{ ft/s}^2$

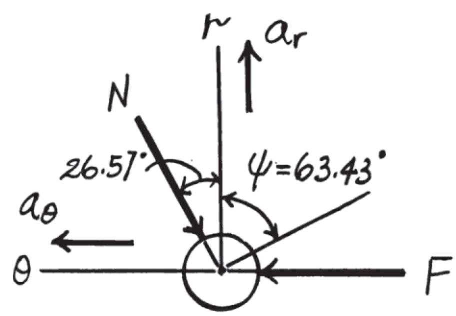


$$\tan\psi = \frac{r}{dr/d\theta} = \frac{2 + \cos\theta}{-\sin\theta}\bigg|_{\theta=90°} = -2 \qquad \psi = -63.43°$$

$$+\uparrow \Sigma F_r = ma_r; \qquad -N\cos 26.57° = \frac{2}{32.2}(-0.5) \qquad N = 0.03472 \text{ lb}$$

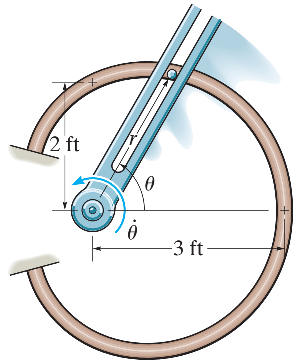
$$\overset{+}{\leftarrow} \Sigma F_\theta = ma_\theta; \qquad F - 0.03472\ \sin 26.57° = \frac{2}{32.2}(-0.5)$$

$$F = -0.0155 \text{ lb}$$ **Ans.**



13–106.

The forked rod is used to move the smooth 2-lb particle around the horizontal path in the shape of a limaçon, $r = (2 + \cos\theta)$ ft. If at all times $\dot{\theta} = 0.5$ rad/s, determine the force which the rod exerts on the particle at the instant $\theta = 60°$. The fork and path contact the particle on only one side.



SOLUTION

$r = 2 + \cos\theta$

$\dot{r} = -\sin\theta\dot{\theta}$

$\ddot{r} = -\cos\theta\dot{\theta}^2 - \sin\theta\ddot{\theta}$

At $\theta = 60°, \dot{\theta} = 0.5$ rad/s, and $\ddot{\theta} = 0$

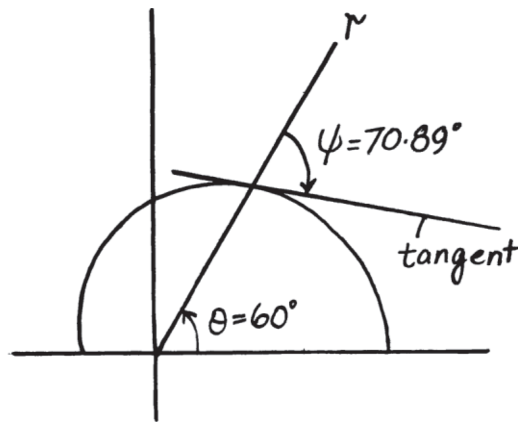
$r = 2 + \cos 60° = 2.5$ ft

$\dot{r} = -\sin 60°(0.5) = -0.4330$ ft/s

$\ddot{r} = -\cos 60°(0.5)^2 - \sin 60°(0) = -0.125$ ft/s^2

$a_r = \ddot{r} - r\dot{\theta}^2 = -0.125 - 2.5(0.5)^2 = -0.75$ ft/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2.5(0) + 2(-0.4330)(0.5) = -0.4330$ ft/s^2

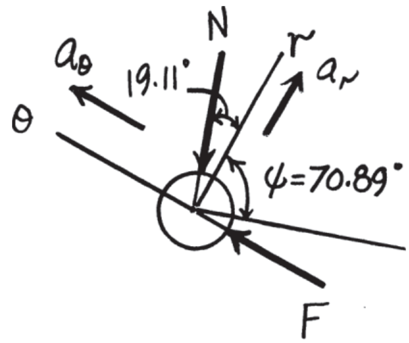


$$\tan\psi = \frac{r}{dr/d\theta} = \frac{2 + \cos\theta}{-\sin\theta}\bigg|_{\theta=60°} = -2.887 \qquad \psi = -70.89°$$

$+\nearrow\Sigma F_r = ma_r; \qquad -N\cos 19.11° = \dfrac{2}{32.2}(-0.75) \qquad N = 0.04930$ lb

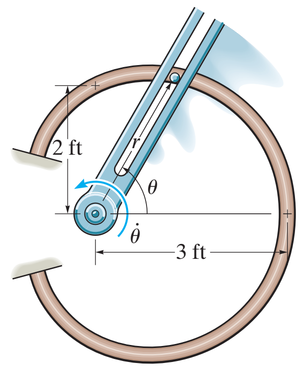
$_{+}\nwarrow\Sigma F_\theta = ma_\theta; \qquad F - 0.04930\sin 19.11° = \dfrac{2}{32.2}(-0.4330)$

$F = -0.0108$ lb **Ans.**



13–107.

The forked rod is used to move the smooth 2-lb particle around the horizontal path in the shape of a limaçon, $r = (2 + \cos\theta)$ ft. If $\theta = (0.5t^2)$ rad, where t is in seconds, determine the force which the rod exerts on the particle at the instant $t = 1$ s. The fork and path contact the particle on only one side.



SOLUTION

$r = 2 + \cos\theta \qquad \theta = 0.5t^2$

$\dot{r} = -\sin\theta\theta \qquad \dot{\theta} = t$

$\ddot{r} = -\cos\theta\dot{\theta}^2 - \sin\theta\ddot{\theta} \qquad \ddot{\theta} = 1 \text{ rad/s}^2$

At $t = 1$ s, $\theta = 0.5$ rad, $\theta = 1$ rad/s, and $\ddot{\theta} = 1$ rad/s^2

$r = 2 + \cos 0.5 = 2.8776$ ft

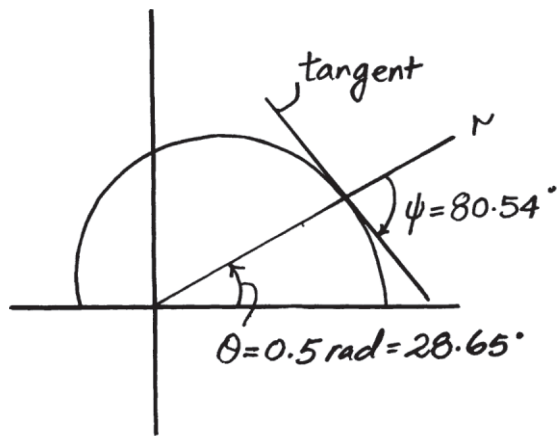
$\dot{r} = -\sin 0.5(1) = -0.4974 \text{ ft/s}^2$

$\ddot{r} = -\cos 0.5(1)^2 - \sin 0.5(1) = -1.357 \text{ ft/s}^2$

$a_r = \ddot{r} - r\dot{\theta}^2 = -1.375 - 2.8776(1)^2 = -4.2346 \text{ ft/s}^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2.8776(1) + 2(-0.4794)(1) = 1.9187 \text{ ft/s}^2$

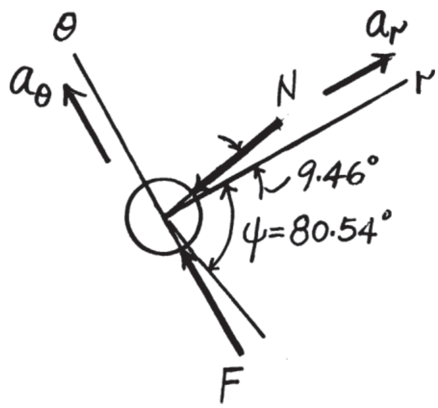
$$\tan\psi = \frac{r}{dr/d\theta} = \frac{2 + \cos\theta}{-\sin\theta}\bigg|_{\theta = 0.5 \text{ rad}} = -6.002 \qquad \psi = -80.54°$$



$+\nearrow\Sigma F_r = ma_r; \qquad -N\cos 9.46° = \frac{2}{32.2}(-4.2346) \qquad N = 0.2666 \text{ lb}$

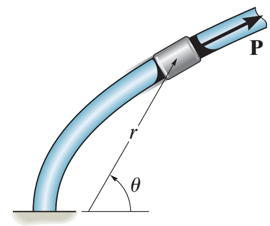
$+\nwarrow\Sigma F_\theta = ma_\theta; \qquad F - 0.2666\sin 9.46° = \frac{2}{32.2}(1.9187)$

$F = 0.163$ lb **Ans.**



***13–108.**

The collar, which has a weight of 3 lb, slides along the smooth rod lying in the *horizontal plane* and having the shape of a parabola $r = 4/(1 - \cos\theta)$, where θ is in radians and r is in feet. If the collar's angular rate is constant and equals $\dot{\theta} = 4$ rad/s, determine the tangential retarding force P needed to cause the motion and the normal force that the collar exerts on the rod at the instant $\theta = 90°$.



SOLUTION

$$r = \frac{4}{1 - \cos\theta}$$

$$\dot{r} = \frac{-4\sin\theta\,\dot{\theta}}{(1 - \cos\theta)^2}$$

$$\ddot{r} = \frac{-4\sin\theta\,\ddot{\theta}}{(1 - \cos\theta)^2} + \frac{-4\cos\theta\,(\dot{\theta})^2}{(1 - \cos\theta)^2} + \frac{8\sin^2\theta\,\dot{\theta}^2}{(1 - \cos\theta)^3}$$

At $\theta = 90°$, $\quad \dot{\theta} = 4, \quad \ddot{\theta} = 0$

$$r = 4$$

$$\dot{r} = -16$$

$$\ddot{r} = 128$$

$$a_r = \ddot{r} - r(\theta)^2 = 128 - 4(4)^2 = 64$$

$$a_\theta = r\ddot{\theta} + 2\,\dot{r}\dot{\theta} = 0 + 2(-16)(4) = -128$$

$$r = \frac{4}{1 - \cos\theta}$$

$$\frac{dr}{d\theta} = \frac{-4\sin\theta}{(1 - \cos\theta)^2}$$

$$\tan\psi = \frac{r}{\left(\frac{dr}{d\theta}\right)} = \left.\frac{\frac{4}{1 - \cos\theta)}}{\frac{-4\sin\theta}{(1 - \cos\theta)^2}}\right|_{\theta = 90°} = \frac{4}{-4} = -1$$

$$\psi = -45° = 135°$$

$$+\uparrow \Sigma F_r = m\,a_r; \qquad P\sin 45° - N\cos 45° = \frac{3}{32.2}(64)$$

$$\overset{+}{\leftarrow} \Sigma F_\theta = ma_\theta; \qquad -P\cos 45° - N\sin 45° = \frac{3}{32.2}(-128)$$

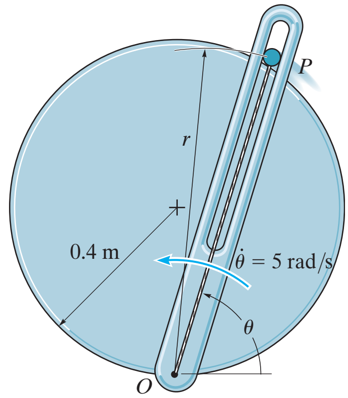
Solving,

$$P = 12.6 \text{ lb}$$ **Ans.**

$$N = 4.22 \text{ lb}$$ **Ans.**

13–109.

The smooth particle has a mass of 80 g. It is attached to an elastic cord extending from O to P and due to the slotted arm guide moves along the *horizontal* circular path $r = (0.8 \sin \theta)$ m. If the cord has a stiffness $k = 30$ N/m and an unstretched length of 0.25 m, determine the force of the guide on the particle when $\theta = 60°$. The guide has a constant angular velocity $\dot{\theta} = 5$ rad/s.



SOLUTION

$r = 0.8 \sin \theta$

$\dot{r} = 0.8 \cos \theta\, \dot{\theta}$

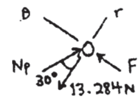
$\ddot{r} = -0.8 \sin \theta\, (\dot{\theta})^2 + 0.8 \cos \theta \ddot{\theta}$

$\dot{\theta} = 5, \qquad \ddot{\theta} = 0$

At $\theta = 60°$, $\qquad r = 0.6928$

$\dot{r} = 2$

$\ddot{r} = -17.321$



$a_r = \ddot{r} - r(\dot{\theta})^2 = -17.321 - 0.6928(5)^2 = -34.641$

$a_\theta = r\ddot{\theta} + 2\,\dot{r}\dot{\theta} = 0 + 2(2)(5) = 20$

$F_s = ks; \qquad F_s = 30(0.6928 - 0.25) = 13.284$ N

$\nearrow + \Sigma F_r = m a_r; \qquad -13.284 + N_P \cos 30° = 0.08(-34.641)$

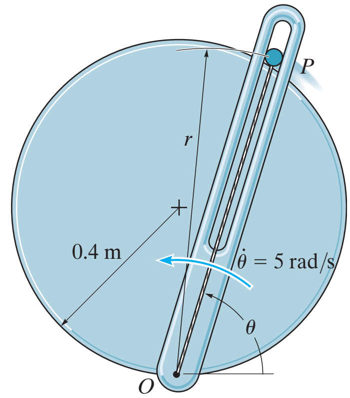
$\nwarrow + \Sigma F_\theta = m a_\theta; \qquad F - N_P \sin 30° = 0.08(20)$

$F = 7.67$ N **Ans.**

$N_P = 12.1$ N

13–110.

The smooth particle has a mass of 80 g. It is attached to an elastic cord extending from O to P and due to the slotted arm guide moves along the horizontal circular path $r = (0.8 \sin \theta)$ m. If the cord has a stiffness $k = 30$ N/m and an unstretched length of 0.25 m, determine the force of the guide on the particle when $\ddot{\theta} = 2$ rad/s^2, $\dot{\theta} = 5$ rad/s, and $\theta = 60°$.



SOLUTION

$$r = 0.8 \sin \theta$$

$$\dot{r} = 0.8 \cos \theta \, \dot{\theta}$$

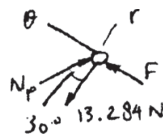
$$\ddot{r} = -0.8 \sin \theta \, (\dot{\theta})^2 + 0.8 \cos \theta \ddot{\theta}$$

$$\dot{\theta} = 5, \qquad \ddot{\theta} = 2$$

At $\theta = 60°$, $\quad r = 0.6928$

$$\dot{r} = 2$$

$$\ddot{r} = -16.521$$



$$a_r = \ddot{r} - r(\dot{\theta})^2 = -16.521 - 0.6928(5)^2 = -33.841$$

$$a_\theta = r\,\ddot{\theta} + 2\,\dot{r}\dot{\theta} = 0.6925(2) + 2(2)(5) = 21.386$$

$F_s = ks;$ $\qquad F_s = 30(0.6928 - 0.25) = 13.284$ N

$\nearrow + \Sigma F_r = m\,a_r;$ $\qquad -13.284 + N_P \cos 30° = 0.08(-33.841)$

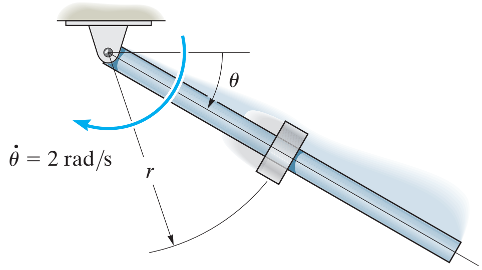
$+\nwarrow \Sigma F_\theta = m a_\theta;$ $\qquad F - N_P \sin 30° = 0.08(21.386)$

$F = 7.82$ N **Ans.**

$N_P = 12.2$ N

13–111.

A 0.2-kg spool slides down along a smooth rod. If the rod has a constant angular rate of rotation $\dot{\theta} = 2$ rad/s in the vertical plane, show that the equations of motion for the spool are $\ddot{r} - 4r - 9.81 \sin\theta = 0$ and $0.8\dot{r} + N_s - 1.962\cos\theta = 0$, where N_s is the magnitude of the normal force of the rod on the spool. Using the methods of differential equations, it can be shown that the solution of the first of these equations is $r = C_1 e^{-2t} + C_2 e^{2t} - (9.81/8)\sin 2t$. If r, $\dot{r}$, and θ are zero when $t = 0$, evaluate the constants C_1 and C_2 to determine r at the instant $\theta = \pi/4$ rad.

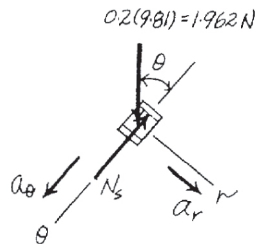


SOLUTION

Kinematic: Here, $\dot{\theta}. = 2$ rad/s and $\dot{\theta} = 0$. Applying Eqs. 12–29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = \ddot{r} - r(2^2) = \ddot{r} - 4r$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = r(0) + 2\dot{r}(2) = 4\dot{r}$$



Equation of Motion: Applying Eq. 13–9, we have

$\Sigma F_r = ma_r;$ $\quad 1.962 \sin\theta = 0.2(\ddot{r} - 4r)$

$$\ddot{r} - 4r - 9.81 \sin\theta = 0 \qquad \textbf{(Q.E.D.)} \qquad \textbf{(1)}$$

$\Sigma F_\theta = ma_\theta;$ $\quad 1.962 \cos\theta - N_s = 0.2(4\dot{r})$

$$0.8\dot{r} + N_s - 1.962\cos\theta = 0 \qquad \textbf{(Q.E.D.)} \qquad \textbf{(2)}$$

Since $\theta. = 2$ rad/s, then $\int_0^\theta \dot{\theta} = \int_0^1 2dt, \theta = 2t$. The solution of the differential equation (Eq.(1)) is given by

$$r = C_1\, e^{-2t} + C_2\, e^{2t} - \frac{9.81}{8} \sin 2t \qquad \textbf{(3)}$$

Thus,

$$\dot{r} = -2\, C_1 e^{-2t} + 2C_2\, e^{2t} - \frac{9.81}{4} \cos 2t \qquad \textbf{(4)}$$

At $t = 0, r = 0$. From Eq.(3) $0 = C_1(1) + C_2(1) - 0$ **(5)**

At $t = 0, \dot{r} = 0$. From Eq.(4) $0 = -2\,C_1(1) + 2C_2(1) - \frac{9.81}{4}$ **(6)**

Solving Eqs. (5) and (6) yields

$$C_1 = -\frac{9.81}{16} \qquad C_2 = \frac{9.81}{16}$$

Thus,

$$r = -\frac{9.81}{16} e^{-2t} + \frac{9.81}{16} e^{2t} - \frac{9.81}{8} \sin 2t$$

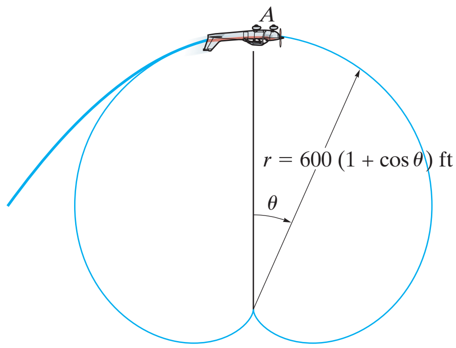
$$= \frac{9.81}{8}\left(\frac{-e^{-2t} + e^{2t}}{2} - \sin 2t\right)$$

$$= \frac{9.81}{8}(\sin\text{h}\, 2t - \sin 2t)$$

At $\theta = 2t = \frac{\pi}{4}$, $\qquad r = \frac{9.81}{8}\left(\sin \text{h}\frac{\pi}{4} - \sin\frac{\pi}{4}3\right) = 0.198$ m **Ans.**

***13–112.**

The pilot of an airplane executes a vertical loop which in part follows the path of a cardioid, $r = 600(1 + \cos\theta)$ ft. If his speed at A $(\theta = 0°)$ is a constant $v_P = 80$ ft/s, determine the vertical force the seat belt must exert on him to hold him to his seat when the plane is upside down at A. He weighs 150 lb.



SOLUTION

$$r = 600(1 + \cos\theta)\big|_{\theta=0°} = 1200 \text{ ft}$$

$$\dot{r} = -600\sin\theta\dot{\theta}\big|_{\theta=0°} = 0$$

$$\ddot{r} = -600\sin\theta\ddot{\theta} - 600\cos\theta\dot{\theta}^2\big|_{\theta=0°} = -600\dot{\theta}^2$$

$$v_p^2 = \dot{r}^2 + \left(r\dot{\theta}\right)^2$$

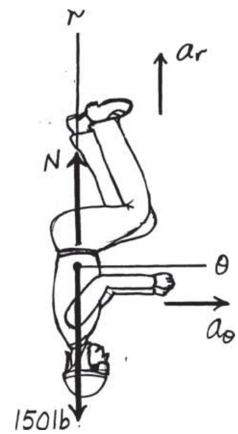
$$(80)^2 = 0 + \left(1200\dot{\theta}\right)^2 \qquad \dot{\theta} = 0.06667$$

$$2v_p\dot{v}_p = 2\dot{r}\ddot{r} + 2\left(r\dot{\theta}\right)\left(\dot{r}\dot{\theta} + r\ddot{\theta}\right)$$

$$0 = 0 + 0 + 2r^2\dot{\theta}\ddot{\theta} \qquad \ddot{\theta} = 0$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -600(0.06667)^2 - 1200(0.06667)^2 = -8 \text{ ft/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 0 = 0$$



$$+\uparrow\Sigma F_r = ma_r; \qquad N - 150 = \left(\frac{150}{32.2}\right)(-8) \qquad N = 113 \text{ lb}$$ **Ans.**

13–113.

The earth has an orbit with eccentricity $e = 0.0821$ around the sun. Knowing that the earth's minimum distance from the sun is $151.3(10^6)$ km, find the speed at which the earth travels when it is at this distance. Determine the equation in polar coordinates which describes the earth's orbit about the sun.

SOLUTION

$$e = \frac{Ch^2}{GM_S} \qquad \text{where } C = \frac{1}{r_0}\left(1 - \frac{GM_S}{r_0 v_0^2}\right) \text{ and } h = r_0 v_0$$

$$e = \frac{1}{GM_S r_0}\left(1 - \frac{GM_S}{r_0 v_0^2}\right)(r_0 v_0)^2 \qquad e = \left(\frac{r_0 v_0^2}{GM_S} - 1\right) \qquad \frac{r_0 v_0^2}{GM_S} = e + 1$$

$$v_0 = \sqrt{\frac{GM_S(e+1)}{r_0}}$$

$$= \sqrt{\frac{66.73(10^{-12})(1.99)(10^{30})(0.0821 + 1)}{151.3(10^9)}} = 30818 \text{ m/s} = 30.8 \text{ km/s} \qquad \textbf{Ans.}$$

$$\frac{1}{r} = \frac{1}{r_0}\left(1 - \frac{GM_S}{r_0 v_0^2}\right)\cos\theta + \frac{GM_S}{r_0^2 v_0^2}$$

$$\frac{1}{r} = \frac{1}{151.3(10^9)}\left(1 - \frac{66.73(10^{-12})(1.99)(10^{30})}{151.3(10^9)(30818)^2}\right)\cos\theta + \frac{66.73(10^{-12})(1.99)(10^{30})}{\left[151.3(10^9)\right]^2 (30818)^2}$$

$$\frac{1}{r} = 0.502(10^{-12})\cos\theta + 6.11(10^{-12}) \qquad \textbf{Ans.}$$

13–114.

A communications satellite is in a circular orbit above the earth such that it always remains directly over a point on the earth's surface. As a result, the period of the satellite must equal the rotation of the earth, which is approximately 24 hours. Determine the satellite's altitude h above the earth's surface and its orbital speed.

SOLUTION

The period of the satellite around the circular orbit of radius $r_0 = h + r_e = \left[h + 6.378(10^6)\right]$ m is given by

$$T = \frac{2\pi r_0}{v_s}$$

$$24(3600) = \frac{2\pi\left[h + 6.378(10^6)\right]}{v_s}$$

$$v_s = \frac{2\pi\left[h + 6.378(10^6)\right)}{86.4(10^3)} \qquad \textbf{(1)}$$

The velocity of the satellite orbiting around the circular orbit of radius $r_0 = h + r_e = \left[h + 6.378(10^6)\right]$ m is given by

$$v_S = \sqrt{\frac{GM_e}{r_0}}$$

$$v_S = \sqrt{\frac{66.73(10^{-12})(5.976)(10^{24})}{h + 6.378(10^6)}} \qquad \textbf{(2)}$$

Solving Eqs.(1) and (2),

$h = 35.87(10^6)\text{ m} = 35.9\text{ Mm}$ $\qquad$ $v_S = 3072.32\text{ m/s} = 3.07\text{ km/s}$ **Ans.**

13–115.

The speed of a satellite launched into a circular orbit about the earth is given by Eq. 13–25. Determine the speed of a satellite launched parallel to the surface of the earth so that it travels in a circular orbit 800 km from the earth's surface.

SOLUTION

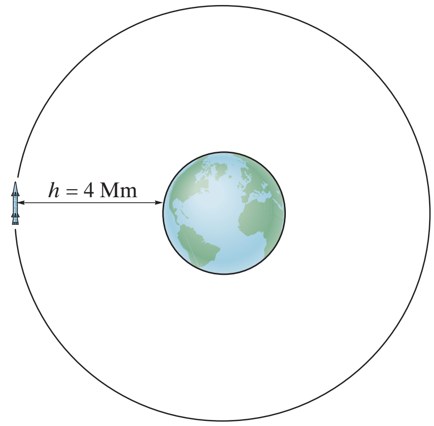
For a 800-km orbit

$$v_0 = \sqrt{\frac{66.73(10^{-12})(5.976)(10^{24})}{(800 + 6378)(10^3)}}$$

$$= 7453.6 \text{ m/s} = 7.45 \text{ km/s} \qquad \textbf{Ans.}$$

***13–116.**

A rocket is in circular orbit about the earth at an altitude of $h = 4$ Mm. Determine the minimum increment in speed it must have in order to escape the earth's gravitational field.



SOLUTION

Circular Orbit:

$$v_C = \sqrt{\frac{GM_e}{r_0}} = \sqrt{\frac{66.73(10^{-12})5.976(10^{24})}{4000(10^3) + 6378(10^3)}} = 6198.8 \text{ m/s}$$

Parabolic Orbit:

$$v_e = \sqrt{\frac{2GM_e}{r_0}} = \sqrt{\frac{2(66.73)(10^{-12})5.976(10^{24})}{4000(10^3) + 6378(10^3)}} = 8766.4 \text{ m/s}$$

$$\Delta v = v_e - v_C = 8766.4 - 6198.8 = 2567.6 \text{ m/s}$$

$$\Delta v = 2.57 \text{ km/s}$$ **Ans.**

13–117.

Prove Kepler's third law of motion. *Hint:* Use Eqs. 13–19, 13–28, 13–29, and 13–31.

SOLUTION

From Eq. 13–19,

$$\frac{1}{r} = C\cos\theta + \frac{GM_s}{h^2}$$

For $\theta = 0°$ and $\theta = 180°$,

$$\frac{1}{r_p} = C + \frac{GM_s}{h^2}$$

$$\frac{1}{r_a} = -C + \frac{GM_s}{h^2}$$

Eliminating C, from Eqs. 13–28 and 13–29,

$$\frac{2a}{b^2} = \frac{2GM_s}{h^2}$$

From Eq. 13–31,

$$T = \frac{\pi}{h}(2a)(b)$$

Thus,

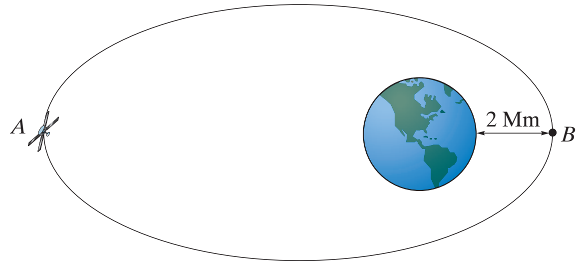
$$b^2 = \frac{T^2h^2}{4\pi^2a^2}$$

$$\frac{4\pi^2a^3}{T^2h^2} = \frac{GM_s}{h^2}$$

$$T^2 = \left(\frac{4\pi^2}{GM_s}\right)a^3 \qquad \textbf{Q.E.D.}$$

13–118.

The satellite is moving in an elliptical orbit with an eccentricity $e = 0.25$. Determine its speed when it is at its maximum distance A and minimum distance B from the earth.



SOLUTION

$$e = \frac{Ch^2}{GM_e}$$

where $C = \frac{1}{r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right)$ and $h = r_0\, v_0$.

$$e = \frac{1}{GM_e\, r_0}\left(1 - \frac{GM_e}{r_0\, v_0^2}\right)(r_0\, v_0)^2$$

$$e = \left(\frac{r_0\, v_0^2}{GM_e} - 1\right)$$

$$\frac{r_0\, v_0^2}{GM_e} = e + 1 \qquad v_0 = \sqrt{\frac{GM_e\,(e + 1)}{r_0}}$$

where $r_0 = r_p = 2\left(10^6\right) + 6378\left(10^3\right) = 8.378\left(10^6\right)$ m.

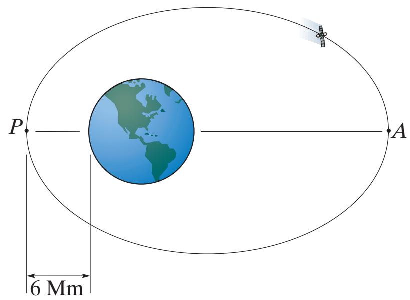
$$v_B = v_0 = \sqrt{\frac{66.73(10^{-12})(5.976)(10^{24})(0.25 + 1)}{8.378(10^6)}} = 7713 \text{ m/s} = 7.71 \text{ km/s} \qquad \textbf{Ans.}$$

$$r_a = \frac{r_0}{\frac{2GM_e}{r_0 v_0} - 1} = \frac{8.378(10^6)}{\frac{2(66.73)(10^{-12})(5.976)(10^{24})}{8.378(10^6)(7713)^2} - 1} = 13.96\left(10^6\right) \text{ m}$$

$$v_A = \frac{r_p}{r_a} v_B = \frac{8.378(10^6)}{13.96(10^6)}(7713) = 4628 \text{ m/s} = 4.63 \text{ km/s} \qquad \textbf{Ans.}$$

13–119.

The elliptical orbit of a satellite orbiting the earth has an eccentricity of $e = 0.45$. If the satellite has an altitude of 6 Mm at perigee, determine the velocity of the satellite at apogee and the period.



SOLUTION

Here, $r_O = r_P = 6(10^6) + 6378(10^3) = 12.378(10^6)$ m.

$$h = r_P v_P$$

$$h = 12.378(10^6) v_P \qquad \textbf{(1)}$$

and

$$C = \frac{1}{r_P}\left(1 - \frac{GM_e}{r_P {v_P}^2}\right)$$

$$C = \frac{1}{12.378(10^6)}\left[1 - \frac{66.73(10^{-12})(5.976)(10^{24})}{12.378(10^6){v_P}^2}\right]$$

$$C = 80.788(10^{-9}) - \frac{2.6027}{{v_P}^2} \qquad \textbf{(2)}$$

Using Eqs. (1) and (2),

$$e = \frac{Ch^2}{GM_e}$$

$$0.45 = \frac{\left[80.788(10^{-9}) - \dfrac{2.6027}{{v_P}^2}\right]\left[12.378(10^6)v_P\right]^2}{66.73(10^{-12})(5.976)(10^{24})}$$

$$v_P = 6834.78 \text{ m/s}$$

Using the result of v_P,

$$r_a = \frac{r_P}{\dfrac{2GM_e}{r_P\, {v_P}^2} - 1}$$

$$= \frac{12.378(10^6)}{\dfrac{2(66.73)(10^{-12})(5.976)(10^{24})}{12.378(10^6)(6834.78^2)} - 1}$$

$$= 32.633(10^6) \text{ m}$$

Since $h = r_P v_P = 12.378(10^6)(6834.78) = 84.601(10^9)\ \text{m}^2/\text{s}$ is constant,

$$r_a v_a = h$$

$$32.633(10^6) v_a = 84.601(10^9)$$

$$v_a = 2592.50 \text{ m/s} = 2.59 \text{ km/s} \qquad \textbf{Ans.}$$

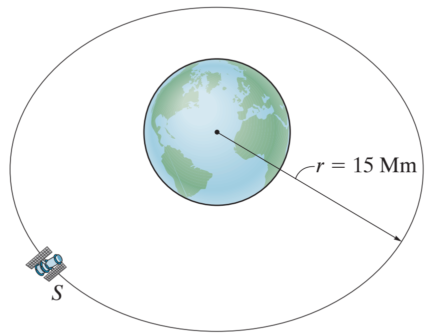
Using the result of h,

$$T = \frac{\pi}{h}(r_P + r_a)\sqrt{r_P r_a}$$

$$= \frac{\pi}{84.601(10^9)}\left[12.378(10^6) + 32.633(10^6)\right]\sqrt{12.378(10^6)(32.633)(10^6)}$$

***13–120.**

Determine the constant speed of satellite S so that it circles the earth with an orbit of radius $r = 15$ Mm. *Hint:* Use Eq. 13–1.



SOLUTION

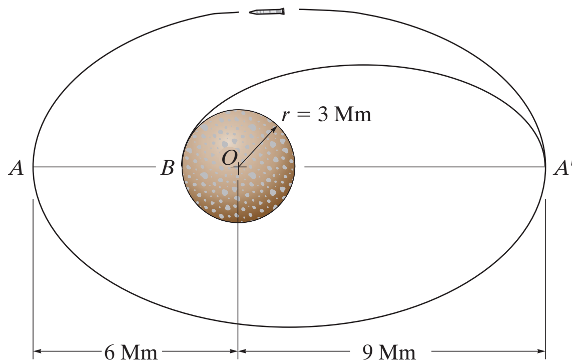
$$F = G\frac{m_s m_e}{r^2} \qquad \text{Also} \qquad F = m_s\left(\frac{v_s^2}{r}\right) \qquad \text{Hence}$$

$$m_s\left(\frac{v_0^2}{r}\right) = G\frac{m_s m_e}{r^2}$$

$$v = \sqrt{G\frac{m_e}{r}} = \sqrt{66.73(10^{-12})\left(\frac{5.976(10^{24})}{15(10^6)}\right)} = 5156 \text{ m/s} = 5.16 \text{ km/s} \qquad \textbf{Ans.}$$

13–121.

The rocket is in free flight along an elliptical trajectory $A'A$. The planet has no atmosphere, and its mass is 0.70 times that of the earth. If the rocket has an apoapsis and periapsis as shown in the figure, determine the speed of the rocket when it is at point A.



SOLUTION

Central-Force Motion: Use $r_a = \dfrac{r_0}{\left(2GM/r_0 v_0^2\right)-1}$, with $r_0 = r_p = 6\left(10^6\right)$ m and $M = 0.70M_e$, we have

$$9\left(10^6\right) = \frac{6(10)^6}{\left(\dfrac{2(66.73)\,(10^{-12})\,(0.7)\,[5.976(10^{24})]}{6(10^6)v_P^2}\right) - 1}$$

$$v_A = 7471.89 \text{ m/s} = 7.47 \text{ km/s} \qquad \textbf{Ans.}$$

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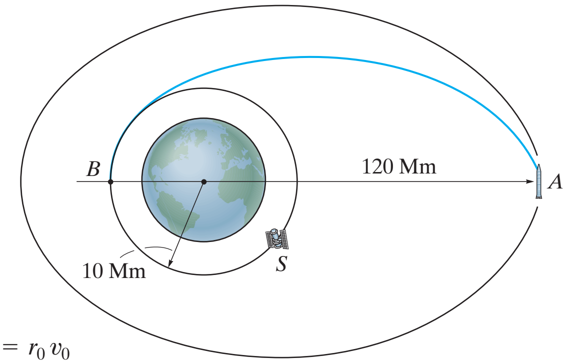
13–122.

A satellite S travels in a circular orbit around the earth. A rocket is located at the apogee of its elliptical orbit for which $e = 0.58$. Determine the sudden change in speed that must occur at A so that the rocket can enter the satellite's orbit while in free flight along the blue elliptical trajectory. When it arrives at B, determine the sudden adjustment in speed that must be given to the rocket in order to maintain the circular orbit.



SOLUTION

Central-Force Motion: Here, $C = \frac{1}{r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right)$ [Eq. 13–21] and $h = r_0 v_0$ [Eq. 13–20]. Substitute these values into Eq. 13–17 gives

$$e = \frac{ch^2}{GM_e} = \frac{\frac{1}{r_0}\left(1 - \frac{GM_e}{r_0^2 v_0^2}\right)\left(r_0^2 v_0^2\right)}{GM_e} = \frac{r_0 v_0^2}{GM_e} - 1 \qquad \textbf{(1)}$$

Rearrange Eq. (1) gives

$$\frac{1}{1 + e} = \frac{GM_e}{r_0 v_0^2} \qquad \textbf{(2)}$$

Rearrange Eq. (2), we have

$$v_0 = \sqrt{\frac{(1 + e)GM_e}{r_0}} \qquad \textbf{(3)}$$

Substitute Eq. (2) into Eq. 13–27, $r_a = \dfrac{r_0}{\left(2\,GM_e/r_0 v_0^2\right) - 1}$, we have

$$r_a = \frac{r_0}{2\left(\frac{1}{1+e}\right) - 1} \qquad \text{or} \qquad r_0 = \left(\frac{1 - e}{1 + e}\right) r_a \qquad \textbf{(4)}$$

or the first elliptical orbit $e = 0.58$, from Eq. (4)

$$(r_p)_1 = r_0 = \left(\frac{1 - 0.58}{1 + 0.58}\right)\left[120(10^6)\right] = 31.899(10^6)\text{ m}$$

Substitute $r_0 = (r_p)_1 = 31.899(10^6)$ m into Eq. (3) yields

$$(v_p)_1 = \sqrt{\frac{(1 + 0.58)(66.73)(10^{-12})(5.976)(10^{24})}{31.899(10^6)}} = 4444.34\text{ m/s}$$

Applying Eq. 13–20, we have

$$(v_a)_1 = \left(\frac{r_p}{r_a}\right)(v_p)_1 = \left[\frac{31.899(10^6)}{120(10^6)}\right](4444.34) = 1181.41\text{ m/s}$$

When the rocket travels along the second elliptical orbit, from Eq. (4), we have

$$10(10^6) = \left(\frac{1 - e}{1 + e}\right)\left[120(10^6)\right] \qquad e = 0.8462$$

Substitute $r_0 = (r_p)_2 = 10(10^6)$ m into Eq. (3) yields

$$(v_p) = \sqrt{\frac{(1 + 0.8462)(66.73)(10^{-12})(5.967)(10^{24})}{10(10^6)}} = 8580.25\text{ m/s}$$

13–122. continued

And in Eq. 13–20, we have

$$(v_a)_2 = \left[\frac{(r_p)_2}{(r_a)_2}\right](v_p)_2 = \left[\frac{10(10^6)}{120(10^6)}\right](8580.25) = 715.02 \text{ m/s}$$

For the rocket to enter into orbit two from orbit one at A, its speed must be decreased by

$$\Delta v = (v_a)_1 - (v_a)_2 = 1184.41 - 715.02 = 466 \text{ m/s} \qquad \textbf{Ans.}$$

If the rocket travels in a circular free-flight trajectory, its speed is given by Eq. 13–25.

$$v_c = \sqrt{\frac{GM_e}{r_0}} = \sqrt{\frac{66.73(10^{-12})(5.976)(10^{24})}{10(10^6)}} = 6314.89 \text{ m/s}$$

The speed for which the rocket must be decreased in order to have a circular orbit is

$$\Delta v = (v_p)_2 - v_e = 8580.25 - 6314.89 = 2265.36 \text{ m/s} = 2.27 \text{ km/s} \qquad \textbf{Ans.}$$

13–123.

An asteroid is in an elliptical orbit about the sun such that its perihelion distance is $9.30(10^9)$ km. If the eccentricity of the orbit is $e = 0.073$, determine the aphelion distance of the orbit.

SOLUTION

$$r_p = r_0 = 9.30(10^9)\text{ km}$$

$$e = \frac{ch^2}{GM_s} = \frac{1}{r_0}\left(1 - \frac{GM_s}{r_0 v_0^2}\right)\left(\frac{r_0 v_0^2}{GM_s}\right)$$

$$e = \left(\frac{r_0\, v_0^2}{GM_s} - 1\right)$$

$$\frac{r_0\, v_0^2}{GM_s} = e + 1 \qquad \textbf{(1)}$$

$$\frac{GM_s}{r_0 v_0^2} = \left(\frac{1}{e + 1}\right)$$

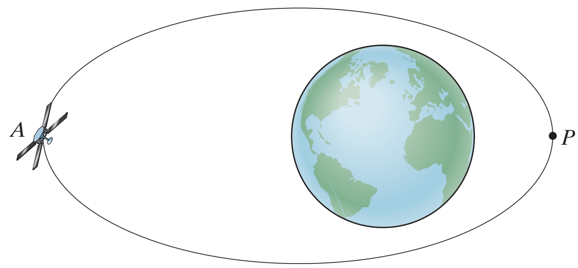
$$r_a = \frac{r_0}{\frac{2GM_s}{r_0\, v_0^2} - 1} = \frac{r_0}{\left(\frac{2}{e\, +\, 1}\right) - 1} \qquad \textbf{(2)}$$

$$r_a = \frac{r_0(e + 1)}{(1 - e)} = \frac{9.30(10^9)(1.073)}{0.927}$$

$$r_a = 10.8(10^9)\text{ km} \qquad \textbf{Ans.}$$

***13–124.**

An elliptical path of a satellite has an eccentricity $e = 0.130$. If it has a speed of 15 Mm/h when it is at perigee, P, determine its speed when it arrives at apogee, A. Also, how far is it from the earth's surface when it is at A?



SOLUTION

$$e = 0.130$$

$$v_p = v_0 = 15 \text{ Mm/h} = 4.167 \text{ km/s}$$

$$e = \frac{Ch^2}{GM_e} = \frac{1}{r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right)\left(\frac{r_0^2 v_0^2}{GM_e}\right)$$

$$e = \left(\frac{r_0 v_0^2}{GM_e} - 1\right)$$

$$\frac{r_0 v_0^2}{GM_e} = e + 1$$

$$r_0 = \frac{(e+1)GM_e}{v_0^2}$$

$$= \frac{1.130(66.73)(10^{-12})(5.976)(10^{24})}{\left[4.167(10^3)\right]^2}$$

$$= 25.96 \text{ Mm}$$

$$\frac{GM_e}{r_0 v_0^2} = \frac{1}{e+1}$$

$$r_A = \frac{r_0}{\frac{2GM_e}{r_0 v_0^2} - 1} = \frac{r_0}{\left(\frac{2}{e+1}\right) - 1}$$

$$r_A = \frac{r_0(e+1)}{1-e}$$

$$= \frac{25.96(10^6)(1.130)}{0.870}$$

$$= 33.71(10^6) \text{ m} = 33.7 \text{ Mm}$$

$$v_A = \frac{v_0 r_0}{r_A}$$

$$= \frac{15(25.96)(10^6)}{33.71(10^6)}$$

$$= 11.5 \text{ Mm/h}$$ **Ans.**

$$d = 33.71(10^6) - 6.378(10^6)$$

$$= 27.3 \text{ Mm}$$ **Ans.**

13–125.

A satellite is launched with an initial velocity $v_0 = 2500$ mi/h parallel to the surface of the earth. Determine the required altitude (or range of altitudes) above the earth's surface for launching if the free-flight trajectory is to be (a) circular, (b) parabolic, (c) elliptical, and (d) hyperbolic. Take $G = 34.4(10^{-9})(\text{lb}\cdot\text{ft}^2)/\text{slug}^2$, $M_e = 409(10^{21})$ slug, the earth's radius $r_e = 3960$ mi, and 1 mi = 5280 ft.

SOLUTION

$$v_0 = 2500 \text{ mi/h} = 3.67(10^3) \text{ ft/s}$$

(a) $$e = \frac{C^2 h}{GM_e} = 0 \qquad \text{or } C = 0$$

$$1 = \frac{GM_e}{r_0 v_0^2}$$

$$GM_e = 34.4(10^{-9})(409)(10^{21})$$

$$= 14.07(10^{15})$$

$$r_0 = \frac{GM_e}{v_0^2} = \frac{14.07(10^{15})}{[3.67(10^{13})]^2} = 1.046(10^9) \text{ ft}$$

$$r = \frac{1.047(10^9)}{5280} - 3960 = 194(10^3) \text{ mi}$$ **Ans.**

(b) $$e = \frac{C^2 h}{GM_e} = 1$$

$$\frac{1}{GM_e}(r_0^2 v_0^2)\left(\frac{1}{r_0}\right)\left(1 - \frac{GM_e}{r_0 v_0^2}\right) = 1$$

$$r_0 = \frac{2GM_e}{v_0^2} = \frac{2(14.07)(10^{15})}{[3.67(10^3)]^2} = 2.09(10^9) \text{ ft} = 396(10^3) \text{ mi}$$

$$r = 396(10^3) - 3960 = 392(10^3) \text{ mi}$$ **Ans.**

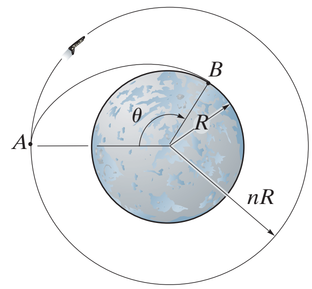
(c) $$e < 1$$

$$194(10^3) \text{ mi} < r < 392(10^3) \text{ mi}$$ **Ans.**

(d) $$e > 1$$

$$r > 392(10^3) \text{ mi}$$ **Ans.**

13–126.



A probe has a circular orbit around a planet of radius R and mass M. If the radius of the orbit is nR and the explorer is traveling with a constant speed v_0, determine the angle θ at which it lands on the surface of the planet B when its speed is reduced to kv_0, where $k < 1$ at point A.

SOLUTION

When the probe is orbiting the planet in a circular orbit of radius $r_O = nR$, its speed is given by

$$v_O = \sqrt{\frac{GM}{r_O}} = \sqrt{\frac{GM}{nR}}$$

The probe will enter the elliptical trajectory with its apoapsis at point A if its speed is decreased to $v_a = kv_O = k\sqrt{\frac{GM}{nR}}$ at this point. When it lands on the surface of the planet, $r = r_B = R$.

$$\frac{1}{r} = \frac{1}{r_P}\left(1 - \frac{GM}{r_P v_P{}^2}\right)\cos\theta + \frac{GM}{r_P{}^2 v_P{}^2}$$

$$\frac{1}{R} = \left(\frac{1}{r_P} - \frac{GM}{r_P{}^2 v_P{}^2}\right)\cos\theta + \frac{GM}{r_P{}^2 v_P{}^2} \qquad \textbf{(1)}$$

Since $h = r_a v_a = nR\left(k\sqrt{\frac{GM}{nR}}\right) = k\sqrt{nGMR}$ is constant,

$$r_P v_P = h$$

$$v_P = \frac{k\sqrt{nGMR}}{r_P} \qquad \textbf{(2)}$$

Also,

$$r_a = \frac{r_P}{\dfrac{2GM}{r_P v_P{}^2} - 1}$$

$$nR = \frac{r_P}{\dfrac{2GM}{r_P v_P{}^2} - 1}$$

$$v_P{}^2 = \frac{2nGMR}{r_p(r_p + nR)} \qquad \textbf{(3)}$$

Solving Eqs.(2) and (3),

$$r_p = \frac{k^2 n}{2 - k^2}R \qquad\qquad v_p = \frac{2 - k^2}{k}\sqrt{\frac{GM}{nR}}$$

Substituting the result of r_p and v_p into Eq. (1),

$$\frac{1}{R} = \left(\frac{2 - k^2}{k^2 nR} - \frac{1}{k^2 nR}\right)\cos\theta + \frac{1}{k^2 nR}$$

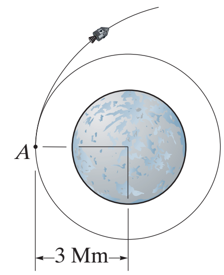
$$\theta = \cos^{-1}\left(\frac{k^2 n - 1}{1 - k^2}\right)$$

Here θ was measured from periapsis. When measured from apoapsis, as in the figure then

$$\theta = \pi - \cos^{-1}\left(\frac{k^2 n - 1}{1 - k^2}\right) \qquad \textbf{Ans.}$$

13–127.

Upon completion of the moon exploration mission, the command module, which was originally in a circular orbit as shown, is given a boost so that it escapes from the moon's gravitational field. Determine the necessary increase in velocity so that the command module follows a parabolic trajectory. The mass of the moon is 0.01230 M_e.



SOLUTION

When the command module is moving around the circular orbit of radius $r_0 = 3(10^6)$ m, its velocity is

$$v_c = \sqrt{\frac{GM_m}{r_0}} = \sqrt{\frac{66.73(10^{-12})(0.0123)(5.976)(10^{24})}{3(10^6)}}$$
$$= 1278.67 \text{ m/s}$$

The escape velocity of the command module entering into the parabolic trajectory is

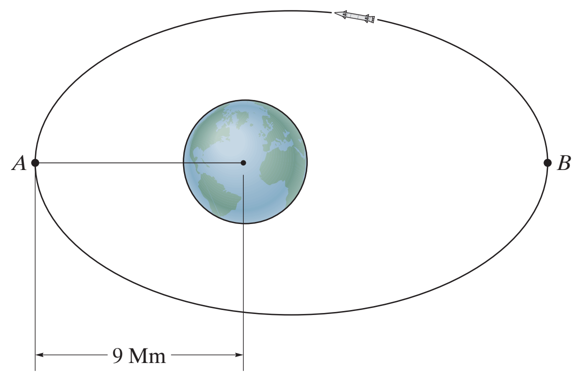
$$v_e = \sqrt{\frac{2GM_m}{r_0}} = \sqrt{\frac{2(66.73)(10^{-12})(0.0123)(5.976)(10^{24})}{3(10^6)}}$$
$$= 1808.31 \text{ m/s}$$

Thus, the required increase in the command module is

$$\Delta v = v_e - v_c = 1808.31 - 1278.67 = 529.64 \text{ m/s} = 530 \text{ m/s} \qquad \textbf{Ans.}$$

***13–128.**

The rocket is traveling in a free-flight elliptical orbit about the earth such that $e = 0.76$ and its perigee is 9 Mm as shown. Determine its speed when it is at point B. Also determine the sudden decrease in speed the rocket must experience at A in order to travel in a circular orbit about the earth.



SOLUTION

Central-Force Motion: Here $C = \dfrac{1}{r_0}\left(1 - \dfrac{GM_e}{r_0 v_0^2}\right)$ [Eq. 13–21] and $h = r_0 v_0$ [Eq. 13–20] Substitute these values into Eq. 13–17 gives

$$e = \frac{ch^2}{GM_e} = \frac{\frac{1}{r_0}\left(1 - \frac{GM_e}{r_0^2 v_0^2}\right)\left(r_0^2 v_0^2\right)}{GM_e} = \frac{r_0 v_0^2}{GM_e} - 1 \qquad \textbf{(1)}$$

Rearrange Eq.(1) gives

$$\frac{1}{1+e} = \frac{GM_e}{r_0 v_0^2} \qquad \textbf{(2)}$$

Rearrange Eq.(2), we have

$$v_0 = \sqrt{\frac{(1+e)\,GM_e}{r_0}} \qquad \textbf{(3)}$$

Substitute Eq.(2) into Eq. 13–27, $r_a = \dfrac{r_0}{(2GM_e/r_0 v_0^2) - 1}$, we have

$$r_a = \frac{r_0}{2\left(\frac{1}{1+e}\right) - 1} \qquad \textbf{(4)}$$

Rearrange Eq.(4), we have

$$r_a = \left(\frac{1+e}{1-e}\right) r_0 = \left(\frac{1+0.76}{1-0.76}\right)\left[9(10^6)\right] = 66.0(10^6)\text{ m}$$

Substitute $r_0 = r_p = 9\left(10^6\right)$ m into Eq.(3) yields

$$v_p = \sqrt{\frac{(1+0.76)(66.73)(10^{-12})(5.976)\,(10^{24})}{9(10^6)}} = 8830.82\text{ m/s}$$

Applying Eq. 13–20, we have

$$v_a = \left(\frac{r_p}{r_a}\right) v_p = \left[\frac{9(10^6)}{66.0(10^6)}\right](8830.82) = 1204.2\text{ m/s} = 1.20\ \text{km/s} \qquad \textbf{Ans.}$$

If the rocket travels in a circular free-flight trajectory, its speed is given by Eq. 13–25.

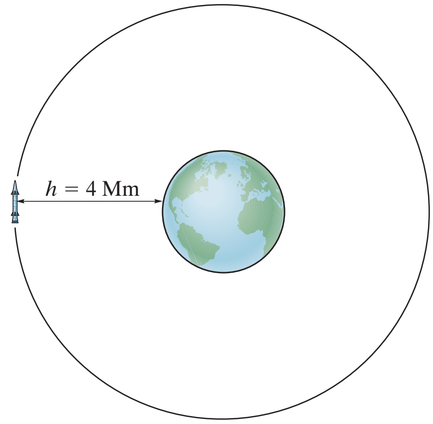
$$v_e = \sqrt{\frac{GM_e}{r_0}} = \sqrt{\frac{66.73(10^{-12})(5.976)(10^{24})}{9(10^6)}} = 6656.48\ \text{m/s}$$

The speed for which the rocket must be decreased in order to have a circular orbit is

$$\Delta v = v_p - v_c = 8830.82 - 6656.48 = 2174.34\text{ m/s} = 2.17\text{ km/s} \qquad \textbf{Ans.}$$

13–129.

A rocket is in circular orbit about the earth at an altitude above the earth's surface of $h = 4$ Mm. Determine the minimum increment in speed it must have in order to escape the earth's gravitational field.



SOLUTION

Circular orbit:

$$v_C = \sqrt{\frac{GM_e}{r_0}} = \sqrt{\frac{66.73(10^{-12})5.976(10^{24})}{4000(10^3) + 6378(10^3)}} = 6198.8 \text{ m/s}$$

Parabolic orbit:

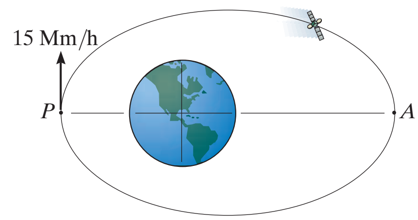
$$v_e = \sqrt{\frac{2GM_e}{r_0}} = \sqrt{\frac{2(66.73)(10^{-12})5.976(10^{24})}{4000(10^3) + 6378(10^3)}} = 8766.4 \text{ m/s}$$

$$\Delta v = v_e - v_C = 8766.4 - 6198.8 = 2567.6 \text{ m/s}$$

$$\Delta v = 2.57 \text{ km/s}$$ **Ans.**

13–130.

The satellite is in an elliptical orbit having an eccentricity of $e = 0.15$. If its velocity at perigee is $v_P = 15\,\text{Mm/h}$, determine its velocity at apogee A and the period of the satellite.



SOLUTION

Here, $v_P = \left[15(10^6)\,\frac{\text{m}}{\text{h}}\right]\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 4166.67\text{ m/s}.$

$$h = r_P v_P$$

$$h = r_P\,(4166.67) = 4166.67 r_p \qquad \textbf{(1)}$$

and

$$C = \frac{1}{r_P}\left(1 - \frac{GM_e}{r_P\,v_P{}^2}\right)$$

$$C = \frac{1}{r_p}\left[1 - \frac{66.73(10^{-12})(5.976)(10^{24})}{r_p(4166.67^2)}\right]$$

$$C = \frac{1}{r_P}\left[1 - \frac{22.97(10^6)}{r_P}\right] \qquad \textbf{(2)}$$

$$e = \frac{Ch^2}{GM_e}$$

$$0.15 = \frac{\frac{1}{r_P}\left[1 - \frac{22.97(10^6)}{r_P}\right](4166.67\,r_P)^2}{66.73(10^{-12})(5.976)(10^{24})}$$

$$r_P = 26.415(10^6)\text{ m}$$

Using the result of r_p

$$r_A = \frac{r_P}{\frac{2GM_e}{r_P\,v_P{}^2} - 1}$$

$$= \frac{26.415(10^6)}{\frac{2(66.73)(10^{-12})(5.976)(10^{24})}{26.415(10^6)(4166.67^2)} - 1}$$

$$= 35.738(10^6)\text{ m}$$

Since $h = r_P\,v_P = 26.415(10^6)(4166.67^2) = 110.06(10^9)\text{ m}^2\text{/s}$ is constant,

$$r_A\,v_A = h$$

$$35.738(10^6)v_A = 110.06(10^9)$$

$$v_A = 3079.71\text{ m/s} = 3.08\text{ km/s} \qquad \textbf{Ans.}$$

Using the results of h, r_A, and r_P,

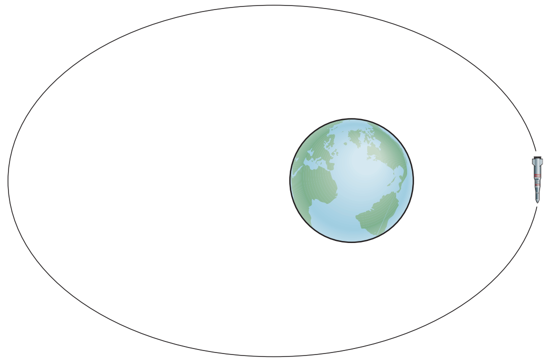
$$T = \frac{\pi}{6}\left(r_P + r_A\right)\sqrt{r_P\,r_A}$$

$$= \frac{\pi}{110.06(10^9)}\left[26.415(10^6) + 35.738(10^6)\right]\sqrt{26.415(10^6)(35.738)(10^6)}$$

$$= 54{,}508.43\text{ s} = 15.1\text{ hr} \qquad \textbf{Ans.}$$

13–131.

A rocket is in a free-flight elliptical orbit about the earth such that the eccentricity of its orbit is e and its perigee is r_0. Determine the minimum increment of speed it should have in order to escape the earth's gravitational field when it is at this point along its orbit.



SOLUTION

To escape the earth's gravitational field, the rocket has to make a parabolic trajectory.

ParabolicTrajectory:

$$v_e = \sqrt{\frac{2GM_e}{r_0}}$$

Elliptical Orbit:

$$e = \frac{Ch^2}{GM_e} \qquad \text{where } C = \frac{1}{r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right) \text{ and } h = r_0 v_0$$

$$e = \frac{1}{GM_e r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right)(r_0 v_0)^2$$

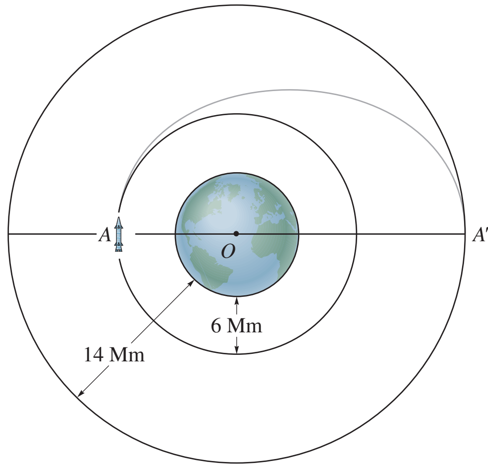
$$e = \left(\frac{r_0 v_0^2}{GM_e} - 1\right)$$

$$\frac{r_0 v_0^2}{GM_e} = e + 1 \qquad v_0 = \sqrt{\frac{GM_e(e+1)}{r_0}}$$

$$\Delta v = \sqrt{\frac{2GM_e}{r_0}} - \sqrt{\frac{GM_e(e+1)}{r_0}} = \sqrt{\frac{GM_e}{r_0}}\left(\sqrt{2} - \sqrt{1+e}\right) \qquad \textbf{Ans.}$$

*13–132.

The rocket shown is originally in a circular orbit 6 Mm above the surface of the earth. It is required that it travel in another circular orbit having an altitude of 14 Mm. To do this, the rocket is given a short pulse of power at A so that it travels in free flight along the gray elliptical path from the first orbit to the second orbit. Determine the necessary speed it must have at A just after the power pulse, and at the time required to get to the outer orbit along the path AA'. What adjustment in speed must be made at A' to maintain the second circular orbit?



SOLUTION

Central-Force Motion: Substitute Eq. 13–27, $r_a = \dfrac{r_0}{(2GM/r_0v_0^2) - 1}$, with $r_a = (14 + 6.378)(10^6) = 20.378(10^6)$ m and $r_0 = r_p = (6 + 6.378)(10^6) = 12.378(10^6)$ m, we have

$$20.378(10^6) = \frac{12.378(10^6)}{\left(\dfrac{2(66.73)(10^{-12})[5.976(10^{24})]}{12.378(10^6)v_p^2}\right) - 1}$$

$$v_p = 6331.27 \text{ m/s}$$

Applying Eq. 13–20. we have

$$v_a = \left(\frac{r_p}{r_a}\right)v_p = \left[\frac{12.378(10^6)}{20.378(10^6)}\right](6331.27) = 3845.74 \text{ m/s}$$

Eq. 13–20 gives $h = r_p v_p = 12.378(10^6)(6331.27) = 78.368(10^9)\ \text{m}^2/\text{s}$. Thus, applying Eq. 13–31, we have

$$T = \frac{\pi}{h}(r_p + r_a)\sqrt{r_p r_a}$$

$$= \frac{\pi}{78.368(10^9)}[(12.378 + 20.378)(10^6)]\sqrt{12.378(20.378)}(10^6)$$

$$= 20854.54 \text{ s}$$

The time required for the rocket to go from A to A' (half the orbit) is given by

$$t = \frac{T}{2} = 10427.38 \text{ s} = 2.90 \text{ hr} \qquad \textbf{Ans.}$$

In order for the satellite to stay in the second circular orbit, it must achieve a speed of (Eq. 13–25)

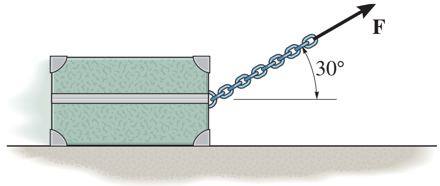
$$v_c = \sqrt{\frac{GM_e}{r_0}} = \sqrt{\frac{66.73(10^{-12})(5.976)(10^{24})}{20.378(10^6)}} = 4423.69 \text{ m/s} = 4.42 \text{ km/s} \qquad \textbf{Ans.}$$

The speed for which the rocket must be increased in order to enter the second circular orbit at A' is

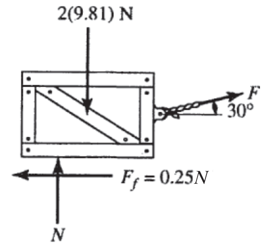
$$\Delta v = v_c - v_a = 4423.69 - 3845.74 = 578 \text{ m/s} \qquad \textbf{Ans.}$$

14–1.

The 20-kg crate is subjected to a force having a constant direction and a magnitude $F = 100$ N. When $s = 15$ m, the crate is moving to the right with a speed of 8 m/s. Determine its speed when $s = 25$ m. The coefficient of kinetic friction between the crate and the ground is $\mu_k = 0.25$.



SOLUTION



Equation of Motion: Since the crate slides, the friction force developed between the crate and its contact surface is $F_f = \mu_k N = 0.25N$. Applying Eq. 13–7, we have

$$+\uparrow \sum F_y = ma_y; \qquad N + 100 \sin 30° - 20(9.81) = 20(0)$$

$$N = 146.2 \text{ N}$$

Principle of Work and Energy: The horizontal component of force F which acts in the direction of displacement does *positive* work, whereas the friction force $F_f = 0.25(146.2) = 36.55$ N does *negative* work since it acts in the opposite direction to that of displacement. The normal reaction N, the vertical component of force F and the weight of the crate do not displace hence do no work. Applying Eq.14–7, we have

$$T_1 + \sum U_{1-2} = T_2$$

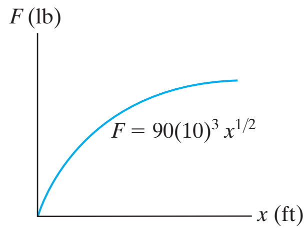
$$\frac{1}{2}(20)(8^2) + \int_{15\text{ m}}^{25\text{ m}} 100 \cos 30° \, ds$$

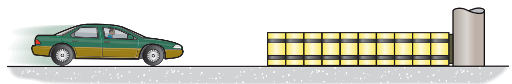
$$- \int_{15\text{ m}}^{25\text{ m}} 36.55 \, ds = \frac{1}{2}(20)v^2$$

$v = 10.7$ m/s **Ans.**

14–2.

For protection, the barrel barrier is placed in front of the bridge pier. If the relation between the force and deflection of the barrier is $F = (90(10^3)x^{1/2})$ lb, where x is in ft, determine the car's maximum penetration in the barrier. The car has a weight of 4000 lb and it is traveling with a speed of 75 ft/s just before it hits the barrier.





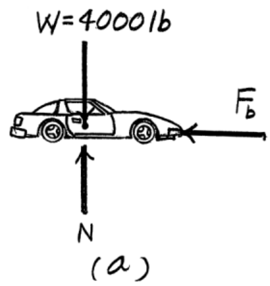
SOLUTION

Principle of Work and Energy: The speed of the car just before it crashes into the barrier is $v_1 = 75$ ft/s. The maximum penetration occurs when the car is brought to a stop, i.e., $v_2 = 0$. Referring to the free-body diagram of the car, Fig. a, **W** and **N** do no work; however, $\mathbf{F}_b$ does negative work.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}\left(\frac{4000}{32.2}\right)(75^2) + \left[-\int_0^{x_{max}} 90(10^3)x^{1/2}dx\right] = 0$$

$$x_{max} = 3.24 \text{ ft}$$ **Ans.**



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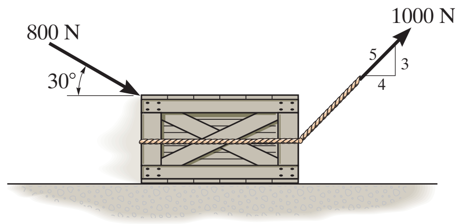
14–3.

The crate, which has a mass of 100 kg, is subjected to the action of the two forces. If it is originally at rest, determine the distance it slides in order to attain a speed of 6 m/s. The coefficient of kinetic friction between the crate and the surface is $\mu_k = 0.2$.

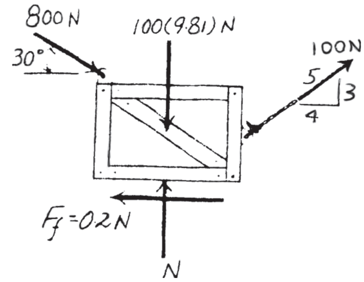


SOLUTION

Equations of Motion: Since the crate slides, the friction force developed between the crate and its contact surface is $F_f = \mu_k N = 0.2N$. Applying Eq. 13–7, we have

$$+\uparrow \Sigma F_y = ma_y; \qquad N + 1000\left(\frac{3}{5}\right) - 800 \sin 30° - 100(9.81) = 100(0)$$

$$N = 781 \text{ N}$$



Principle of Work and Energy: The horizontal components of force 800 N and 1000 N which act in the direction of displacement do *positive* work, whereas the friction force $F_f = 0.2(781) = 156.2$ N does *negative* work since it acts in the opposite direction to that of displacement. The normal reaction N, the vertical component of 800 N and 1000 N force and the weight of the crate do not displace, hence they do no work. Since the crate is originally at rest, $T_1 = 0$. Applying Eq. 14–7, we have

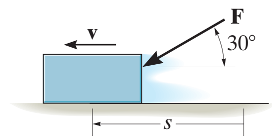
$$T_1 + \sum U_{1-2} = T_2$$

$$0 + 800 \cos 30°(s) + 1000\left(\frac{4}{5}\right)s - 156.2s = \frac{1}{2}(100)\left(6^2\right)$$

$$s = 1.35\text{m} \qquad \textbf{Ans.}$$

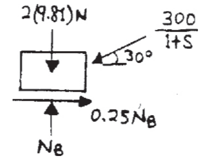
***14–4.**

The 2-kg block is subjected to a force having a constant direction and a magnitude $F = (300/(1 + s))$ N, where s is in meters. When $s = 4$ m, the block is moving to the left with a speed of 8 m/s. Determine its speed when $s = 12$ m. The coefficient of kinetic friction between the block and the ground is $\mu_k = 0.25$.



SOLUTION

$+\uparrow \Sigma F_y = 0; \qquad N_B = 2(9.81) + \dfrac{150}{1 + s}$



$T_1 + \Sigma U_{1-2} = T_2$

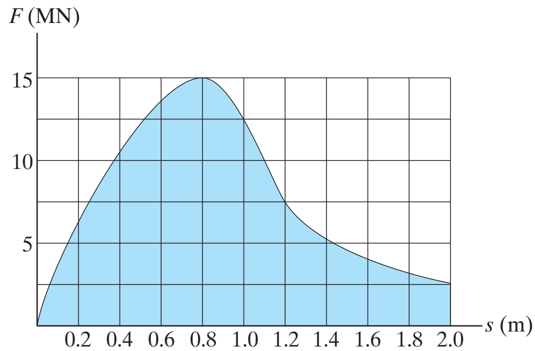
$$\frac{1}{2}(2)(8)^2 - 0.25[2(9.81)(12 - 4)] - 0.25\int_4^{12} \frac{150}{1 + s}\,ds + \int_4^{12}\left(\frac{300}{1 + s}\right) ds\,\cos 30° = \frac{1}{2}(2)(v_2^2)$$

$$v_2^2 = 24.76 - 37.5\ln\left(\frac{1 + 12}{1 + 4}\right) + 259.81\ln\left(\frac{1 + 12}{1 + 4}\right)$$

$v_2 = 15.4$ m/s **Ans.**

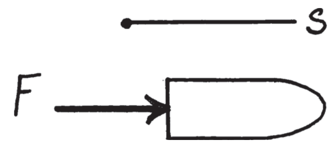
14–5.

When a 7-kg projectile is fired from a cannon barrel that has a length of 2 m, the explosive force exerted on the projectile, while it is in the barrel, varies in the manner shown. Determine the approximate muzzle velocity of the projectile at the instant it leaves the barrel. Neglect the effects of friction inside the barrel and assume the barrel is horizontal.



SOLUTION

The work done is measured as the area under the force–displacement curve. This area is approximately 31.5 squares. Since each square has an area of $2.5\left(10^6\right)(0.2)$,



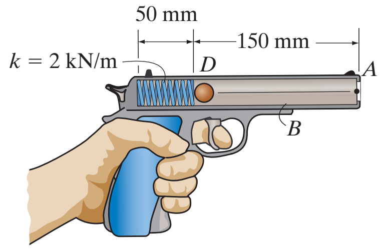
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + \left[(31.5)(2.5)\left(10^6\right)(0.2)\right] = \frac{1}{2}(7)(v_2)^2$$

$v_2 = 2121 \text{ m/s} = 2.12 \text{ km/s} \qquad \text{(approx.)}$ **Ans.**

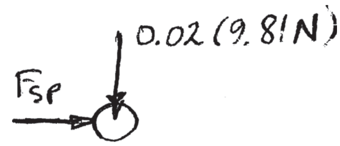
14–6.

The spring in the toy gun has an unstretched length of 100 mm. It is compressed and locked in the position shown. When the trigger is pulled, the spring unstretches 12.5 mm, and the 20-g ball moves along the barrel. Determine the speed of the ball when it leaves the gun. Neglect friction.



SOLUTION

Principle of Work and Energy: Referring to the free-body diagram of the ball bearing shown in Fig. *a*, notice that $\mathbf{F}_{sp}$ does positive work. The spring has an initial and final compression of $s_1 = 0.1 - 0.05 = 0.05$ m and $s_2 = 0.1 - (0.05 + 0.0125) = 0.0375$ m.



(a)

$$T_1 + \Sigma U_{1-2} = T_2$$

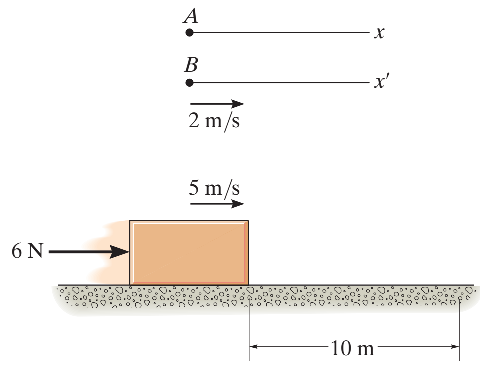
$$0 + \left[\frac{1}{2}ks_1^{\,2} - \frac{1}{2}ks_2^{\,2}\right] = \frac{1}{2}mv_A^{\,2}$$

$$0 + \left[\frac{1}{2}(2000)(0.05)^2 - \frac{1}{2}(2000)(0.0375^2)\right] = \frac{1}{2}(0.02)v_A^{\,2}$$

$$v_A = 10.5 \text{ m/s}$$ **Ans.**

14–7.

As indicated by the derivation, the principle of work and energy is valid for observers in *any* inertial reference frame. Show that this is so, by considering the 10-kg block which rests on the smooth surface and is subjected to a horizontal force of 6 N. If observer A is in a *fixed* frame x, determine the final speed of the block if it has an initial speed of 5 m/s and travels 10 m, both directed to the right and measured from the fixed frame. Compare the result with that obtained by an observer B, attached to the x' axis and moving at a constant velocity of 2 m/s relative to A. *Hint:* The distance the block travels will first have to be computed for observer B before applying the principle of work and energy.



SOLUTION

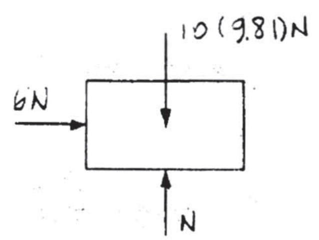
Observer A:

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(10)(5)^2 + 6(10) = \frac{1}{2}(10)v_2^2$$

$$v_2 = 6.08 \text{ m/s}$$ **Ans.**

Observer B:



$$F = ma$$

$$6 = 10a \qquad a = 0.6 \text{ m/s}^2$$

$$(\xrightarrow{+}) \qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$10 = 0 + 5t + \frac{1}{2}(0.6)t^2$$

$$t^2 + 16.67t - 33.33 = 0$$

$$t = 1.805 \text{ s}$$

At $v = 2$ m/s, $s' = 2(1.805) = 3.609$ m

Block moves $10 - 3.609 = 6.391$ m

Thus

$$T_1 + \Sigma U_{1-2} = T_2$$

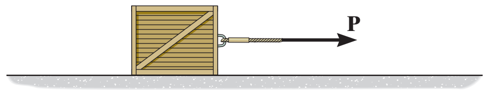
$$\frac{1}{2}(10)(3)^2 + 6(6.391) = \frac{1}{2}(10)v_2^2$$

$$v_2 = 4.08 \text{ m/s}$$ **Ans.**

Note that this result is 2 m/s less than that observed by A.

***14–8.**

If the 50-kg crate is subjected to a force of $P = 200$ N, determine its speed when it has traveled 15 m starting from rest. The coefficient of kinetic friction between the crate and the ground is $\mu_k = 0.3$.



SOLUTION

Free-Body Diagram: Referring to the free-body diagram of the crate, Fig. a,

$+\uparrow F_y = ma_y;$ $\qquad N - 50(9.81) = 50(0) \qquad N = 490.5\text{ N}$

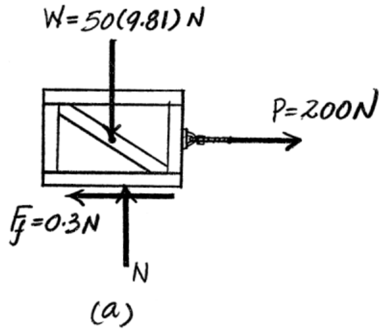
Thus, the frictional force acting on the crate is $F_f = \mu_k N = 0.3(490.5) = 147.15\text{ N}$.

Principle of Work and Energy: Referring to Fig. a, only **P** and $\mathbf{F}_f$ do work. The work of **P** will be positive, whereas $\mathbf{F}_f$ does negative work.

$$T_1 + \Sigma U_{1-2} = T_2$$

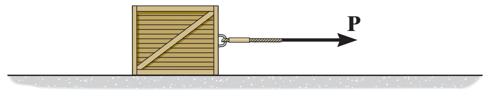
$$0 + 200(15) - 147.15(15) = \frac{1}{2}(50)v^2$$

$$v = 5.63\text{ m/s}$$ **Ans.**



14–9.

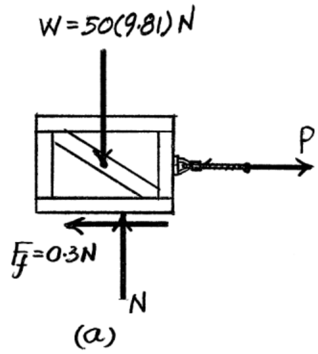
If the 50-kg crate starts from rest and attains a speed of 6 m/s when it has traveled a distance of 15 m, determine the force **P** acting on the crate. The coefficient of kinetic friction between the crate and the ground is $\mu_k = 0.3$.



SOLUTION

Free-Body Diagram: Referring to the free-body diagram of the crate, Fig. *a*,

$$+\uparrow F_y = ma_y; \qquad N - 50(9.81) = 50(0) \qquad N = 490.5\text{ N}$$



Thus, the frictional force acting on the crate is $F_f = \mu_k N = 0.3(490.5) = 147.15\text{ N}$.

Principle of Work and Energy: Referring to Fig. *a*, only **P** and $\mathbf{F}_f$ do work. The work of **P** will be positive, whereas $\mathbf{F}_f$ does negative work.

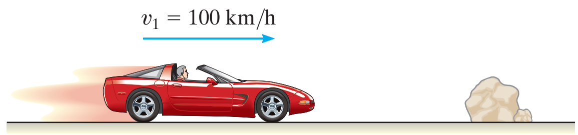
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + P(15) - 147.15(15) = \frac{1}{2}(50)(6^2)$$

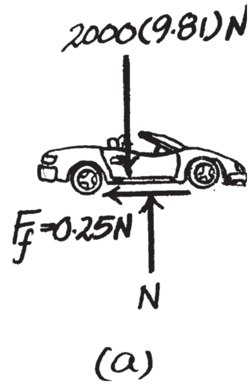
$$P = 207\text{ N}$$ **Ans.**

14–10.

The 2-Mg car has a velocity of $v_1 = 100\,\text{km/h}$ when the driver sees an obstacle in front of the car. If it takes 0.75 s for him to react and lock the brakes, causing the car to skid, determine the distance the car travels before it stops. The coefficient of kinetic friction between the tires and the road is $\mu_k = 0.25$.



SOLUTION



Free-Body Diagram: The normal reaction **N** on the car can be determined by writing the equation of motion along the y axis. By referring to the free-body diagram of the car, Fig. a,

$$+\uparrow \Sigma F_y = ma_y; \qquad N - 2000(9.81) = 2000(0) \qquad N = 19\,620\text{ N}$$

Since the car skids, the frictional force acting on the car is $F_f = \mu_k N = 0.25(19620) = 4905N$.

Principle of Work and Energy: By referring to Fig. a, notice that only $\mathbf{F}_f$ does work, which is negative. The initial speed of the car is $v_1 = \left[100(10^3)\dfrac{\text{m}}{\text{h}}\right]\left(\dfrac{1\text{ h}}{3600\text{ s}}\right) =$ 27.78 m/s. Here, the skidding distance of the car is denoted as s'.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(2000)(27.78^2) + (-4905s') = 0$$

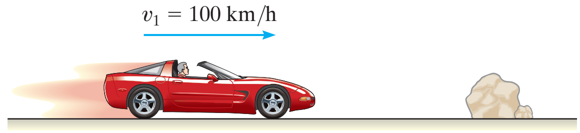
$$s' = 157.31\text{ m}$$

The distance traveled by the car during the reaction time is $s'' = v_1 t = 27.78(0.75) = 20.83$ m. Thus, the total distance traveled by the car before it stops is

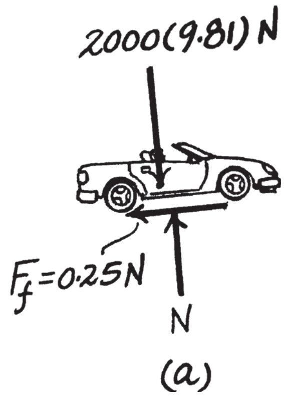
$$s = s' + s'' = 157.31 + 20.83 = 178.14\text{ m} = 178\text{ m} \qquad \textbf{Ans.}$$

14–11.

The 2-Mg car has a velocity of $v_1 = 100\,\text{km/h}$ when the driver sees an obstacle in front of the car. It takes 0.75 s for him to react and lock the brakes, causing the car to skid. If the car stops when it has traveled a distance of 175 m, determine the coefficient of kinetic friction between the tires and the road.



SOLUTION



Free-Body Diagram: The normal reaction **N** on the car can be determined by writing the equation of motion along the y axis and referring to the free-body diagram of the car, Fig. a,

$$+\uparrow \Sigma F_y = ma_y; \qquad N - 2000(9.81) = 2000(0) \qquad N = 19\,620\text{ N}$$

Since the car skids, the frictional force acting on the car can be computed from $F_f = \mu_k N = \mu_k(19\,620)$.

Principle of Work and Energy: By referring to Fig. a, notice that only $\mathbf{F}_f$ does work, which is negative. The initial speed of the car is $v_1 = \left[100(10^3)\,\dfrac{\text{m}}{\text{h}}\right]\left(\dfrac{1\text{ h}}{3600\text{ s}}\right) =$ 27.78 m/s. Here, the skidding distance of the car is s'.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(2000)(27.78^2) + \left[-\mu_k(19\,620)s'\right] = 0$$

$$s' = \frac{39.327}{\mu_k}$$

The distance traveled by the car during the reaction time is $s'' = v_1 t = 27.78(0.75) = 20.83$ m. Thus, the total distance traveled by the car before it stops is

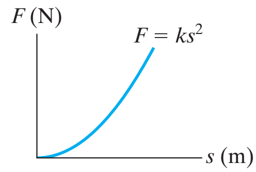
$$s = s' + s''$$

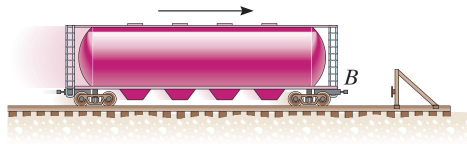
$$175 = \frac{39.327}{\mu_k} + 20.83$$

$$\mu_k = 0.255 \qquad \textbf{Ans.}$$

***14–12.**

Design considerations for the bumper B on the 5-Mg train car require use of a nonlinear spring having the load-deflection characteristics shown in the graph. Select the proper value of k so that the maximum deflection of the spring is limited to 0.2 m when the car, traveling at 4 m/s, strikes the rigid stop. Neglect the mass of the car wheels.



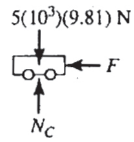


SOLUTION

$$\frac{1}{2}(5000)(4)^2 - \int_0^{0.2} ks^2\,ds = 0$$

$$40\,000 - k\frac{(0.2)^3}{3} = 0$$

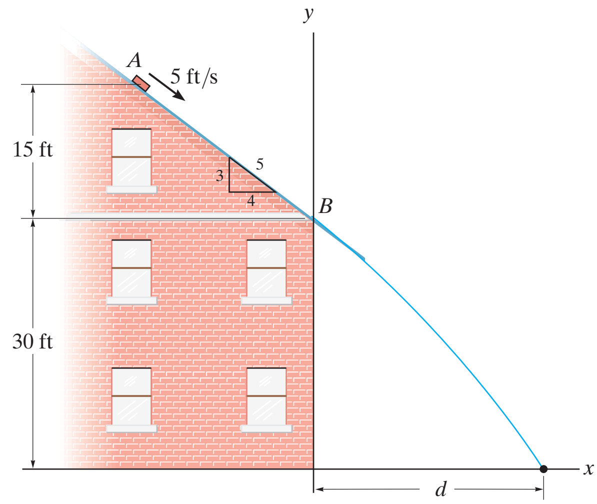
$$k = 15.0\ \text{MN/m}^2$$ **Ans.**



لجنة الميكانيك - الإتجاه الإسلامي

14–13.

The 2-lb brick slides down a smooth roof, such that when it is at A it has a velocity of 5 ft/s. Determine the speed of the brick just before it leaves the surface at B, the distance d from the wall to where it strikes the ground, and the speed at which it hits the ground.



SOLUTION

$$T_A + \Sigma U_{A-B} = T_B$$

$$\frac{1}{2}\left(\frac{2}{32.2}\right)(5)^2 + 2(15) = \frac{1}{2}\left(\frac{2}{32.2}\right)v_B^2$$

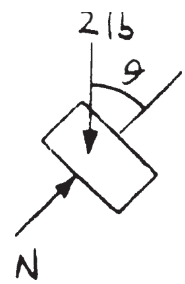
$$v_B = 31.48 \text{ ft/s} = 31.5 \text{ ft/s}$$

$$\left(\xrightarrow{+}\right) \qquad s = s_0 + v_0 t$$

$$d = 0 + 31.48\left(\frac{4}{5}\right)t$$

$$(+\downarrow) \qquad s = s_0 + v_0 t - \frac{1}{2}a_c t^2$$

$$30 = 0 + 31.48\left(\frac{3}{5}\right)t + \frac{1}{2}(32.2)t^2$$



$$16.1t^2 + 18.888t - 30 = 0$$

Solving for the positive root,

$$t = 0.89916 \text{ s}$$

$$d = 31.48\left(\frac{4}{5}\right)(0.89916) = 22.6 \text{ ft}$$ **Ans.**

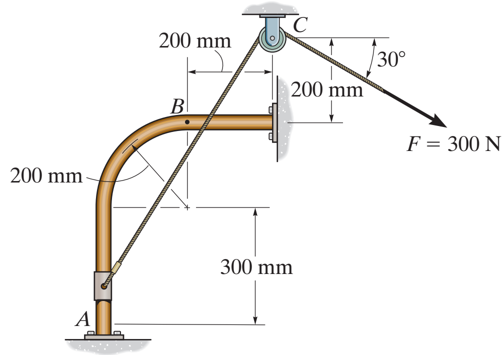
$$T_A + \Sigma U_{A-C} = T_C$$

$$\frac{1}{2}\left(\frac{2}{32.2}\right)(5)^2 + 2(45) = \frac{1}{2}\left(\frac{2}{32.2}\right)v_C^2$$

$$v_C = 54.1 \text{ ft/s}$$ **Ans.**

14–14.

If the cord is subjected to a constant force of $F = 300$ N and the 15-kg smooth collar starts from rest at A, determine the velocity of the collar when it reaches point B. Neglect the size of the pulley.



SOLUTION

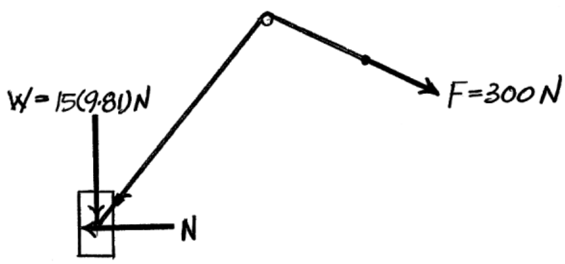
Free-Body Diagram: The free-body diagram of the collar and cord system at an arbitrary position is shown in Fig. *a*.

Principle of Work and Energy: Referring to Fig. *a*, only **N** does no work since it always acts perpendicular to the motion. When the collar moves from position A to position B, **W** displaces vertically upward a distance $h = (0.3 + 0.2)$ m $= 0.5$ m, while force F displaces a distance of $s = AC - BC = \sqrt{0.7^2 + 0.4^2} - \sqrt{0.2^2 + 0.2^2} = 0.5234$ m. Here, the work of **F** is positive, whereas **W** does negative work.

$$T_A + \Sigma U_{A-B} = T_B$$

$$0 + 300(0.5234) + [-15(9.81)(0.5)] = \frac{1}{2}(15)v_B^2$$

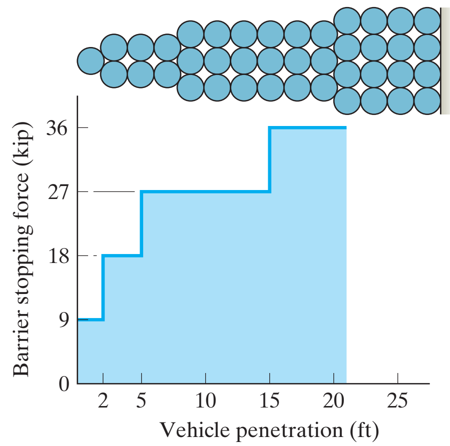
$$v_B = 3.335 \text{ m/s} = 3.34 \text{ m/s}$$ **Ans.**



(a)

14–15.

The crash cushion for a highway barrier consists of a nest of barrels filled with an impact-absorbing material. The barrier stopping force is measured versus the vehicle penetration into the barrier. Determine the distance a car having a weight of 4000 lb will penetrate the barrier if it is originally traveling at 55 ft/s when it strikes the first barrel.



SOLUTION

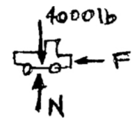
$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}\left(\frac{4000}{32.2}\right)(55)^2 - Area = 0$$

$$Area = 187.89 \text{ kip} \cdot \text{ft}$$

$$2(9) + (5 - 2)(18) + x(27) = 187.89$$

$$x = 4.29 \text{ ft} < (15 - 5) \text{ ft}$$ **(O.K!)**

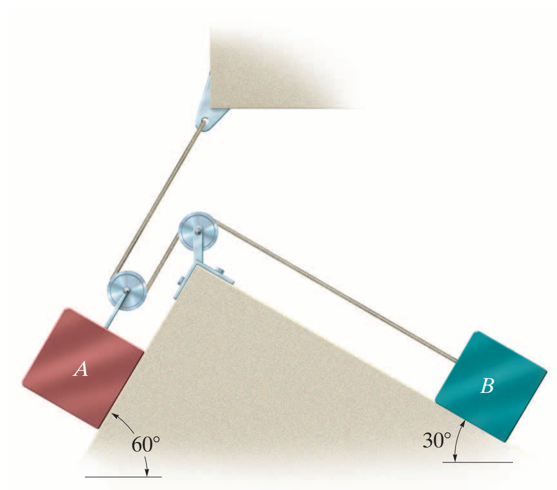


Thus

$$s = 5 \text{ ft} + 4.29 \text{ ft} = 9.29 \text{ ft}$$ **Ans.**

***14–16.**

Determine the velocity of the 60-lb block A if the two blocks are released from rest and the 40-lb block B moves 2 ft up the incline. The coefficient of kinetic friction between both blocks and the inclined planes is $\mu_k = 0.10$.



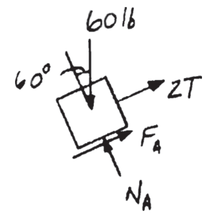
SOLUTION

Block A:

$+\nwarrow\Sigma F_y = ma_y;\qquad N_A - 60\cos 60° = 0$

$N_A = 30\text{ lb}$

$F_A = 0.1(30) = 3\text{ lb}$



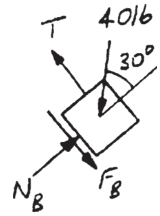
Block B:

$+\nearrow\Sigma F_y = ma_y;\qquad N_B - 40\cos 30° = 0$

$N_B = 34.64\text{ lb}$

$F_B = 0.1(34.64) = 3.464\text{ lb}$

Use the system of both blocks. N_A, N_B, T, and R do no work.



$T_1 + \Sigma U_{1-2} = T_2$

$$(0+0) + 60\sin 60°|\Delta s_A| - 40\sin 30°|\Delta s_B| - 3|\Delta s_A| - 3.464|\Delta s_B| = \frac{1}{2}\left(\frac{60}{32.2}\right)v_A^2 + \frac{1}{2}\left(\frac{40}{32.2}\right)v_B^2$$

$2s_A + s_B = l$

$2\Delta s_A = -\Delta s_B$

When $|\Delta s_B| = 2$ ft, $|\Delta s_A| = 1$ ft

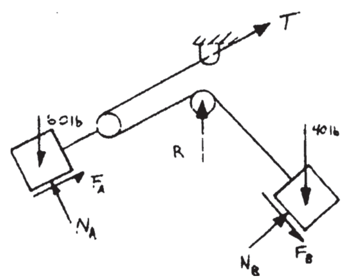
Also,

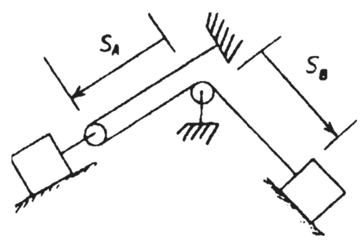
$2v_A = -v_B$

Substituting and solving,

$v_A = 0.771\text{ ft/s}$ **Ans.**

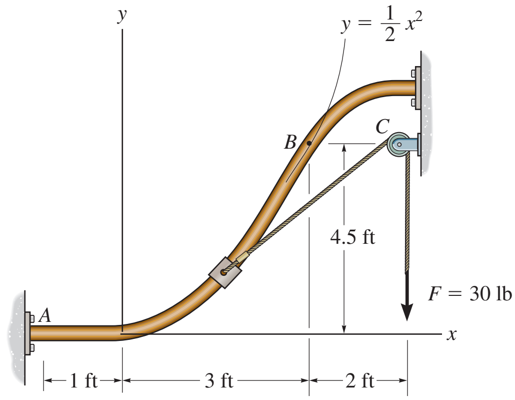
$v_B = -1.54\text{ ft/s}$





14–17.

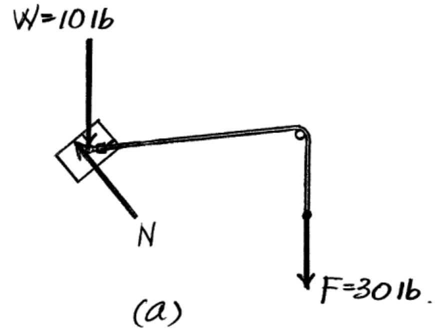
If the cord is subjected to a constant force of $F = 30$ lb and the smooth 10-lb collar starts from rest at A, determine its speed when it passes point B. Neglect the size of pulley C.



SOLUTION

Free-Body Diagram: The free-body diagram of the collar and cord system at an arbitrary position is shown in Fig. *a*.

Principle of Work and Energy: By referring to Fig. *a*, only **N** does no work since it always acts perpendicular to the motion. When the collar moves from position A to position B, **W** displaces upward through a distance $h = 4.5$ ft, while force **F** displaces a distance of $s = AC - BC = \sqrt{6^2 + 4.5^2} - 2 = 5.5$ ft. The work of **F** is positive, whereas **W** does negative work.



(a)

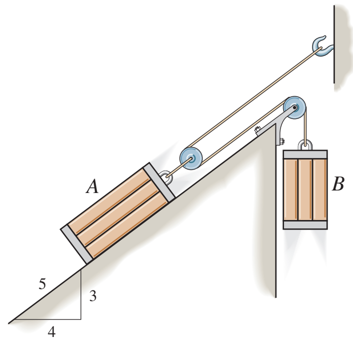
$$T_A + \Sigma U_{A-B} = T_B$$

$$0 + 30(5.5) + [-10(4.5)] = \frac{1}{2}\left(\frac{10}{32.2}\right)v_B^2$$

$$v_B = 27.8 \text{ ft/s}$$ **Ans.**

14–18.

The two blocks A and B have weights $W_A = 60$ lb and $W_B = 10$ lb. If the kinetic coefficient of friction between the incline and block A is $\mu_k = 0.2$, determine the speed of A after it moves 3 ft down the plane starting from rest. Neglect the mass of the cord and pulleys.



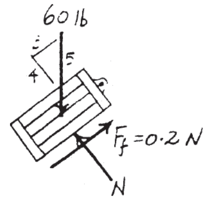
SOLUTION

Kinematics: The speed of the block A and B can be related by using position coordinate equation.

$$s_A + (s_A - s_B) = l \qquad 2s_A - s_B = l$$

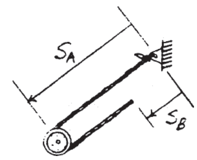
$$2\Delta s_A - \Delta s_B = 0 \qquad \Delta s_B = 2\Delta s_A = 2(3) = 6 \text{ ft}$$

$$2v_A - v_B = 0 \qquad \textbf{(1)}$$



Equation of Motion: Applying Eq. 13–7, we have

$$+\Sigma F_{y'} = ma_{y'}; \qquad N - 60\left(\frac{4}{5}\right) = \frac{60}{32.2}(0) \qquad N = 48.0 \text{ lb}$$



Principle of Work and Energy: By considering the whole system, W_A which acts in the direction of the displacement does *positive* work. W_B and the friction force $F_f = \mu_k N = 0.2(48.0) = 9.60$ lb does *negative* work since they act in the opposite direction to that of displacement Here, W_A is being displaced vertically (downward) $\frac{3}{5}\Delta s_A$ and W_B is being displaced vertically (upward) Δs_B. Since blocks A and B are at rest initially, $T_1 = 0$. Applying Eq. 14–7, we have

$$T_1 + \sum U_{1-2} = T_2$$

$$0 + W_A\left(\frac{3}{5}\Delta s_A\right) - F_f\Delta s_A - W_B\Delta s_B = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B\ v_B^2$$

$$60\left[\frac{3}{5}(3)\right] - 9.60(3) - 10(6) = \frac{1}{2}\left(\frac{60}{32.2}\right)v_A^2 + \frac{1}{2}\left(\frac{10}{32.2}\right)v_B^2$$

$$1236.48 = 60v_A^2 + 10v_B^2 \qquad \textbf{(2)}$$

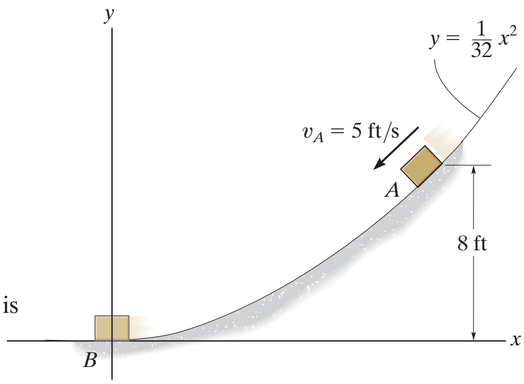
Eqs. (1) and (2) yields

$$v_A = 3.52 \text{ ft/s} \qquad \textbf{Ans.}$$

$$v_B = 7.033 \text{ ft/s}$$

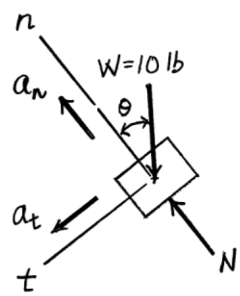
14–19.

If the 10-lb block passes point A on the smooth track with a speed of $v_A = 5$ ft/s, determine the normal reaction on the block when it reaches point B.



SOLUTION

Free-Body Diagram: The free-body diagram of the block at an arbitrary position is shown in Fig. *a*.



(a)

Principle of Work and Energy: Referring to Fig. *a*, **N** does no work since it always acts perpendicular to the motion. When the block slides down the track from position A to position B, **W** displaces vertically downward $h = 8$ ft and does positive work.

$$T_A + \Sigma U_{A-B} = T_B$$

$$\frac{1}{2}\left(\frac{10}{32.2}\right)(5^2) + 10(8) = \frac{1}{2}\left(\frac{10}{32.2}\right)v_B^{\,2}$$

$$v_B = 23.24 \text{ ft/s}$$

Equation of Motion: Here, $a_n = \dfrac{v^2}{\rho}$. By referring to Fig. *a*,

$$\Sigma F_n = ma_n; \qquad N - 10\cos\theta = \left(\frac{10}{32.2}\right)\left(\frac{v^2}{\rho}\right)$$

$$N = \frac{10}{32.2}\left(\frac{v^2}{\rho}\right) + 10\cos\theta \qquad \textbf{(1)}$$

Geometry: Here, $\dfrac{dy}{dx} = \dfrac{1}{16}x$ and $\dfrac{d^2y}{dx^2} = \dfrac{1}{16}$. The slope that the track at position B makes with the horizontal is $\theta_B = \tan^{-1}\left(\dfrac{dx}{dy}\right)\Big|_{x=0} = \tan^{-1}(0) = 0°$. The radius of curvature of the track at position B

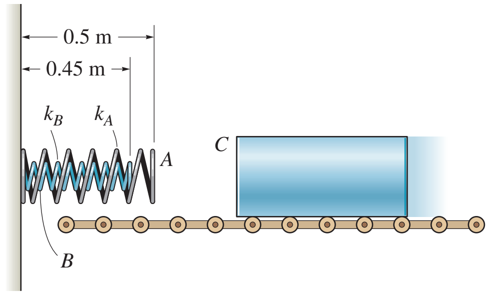
$$\rho_B = \frac{\left[1 + \left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}{\left|\dfrac{d^2y}{dx^2}\right|} = \frac{\left[1 + \left(\dfrac{1}{16}x\right)^2\right]^{3/2}}{\left|\dfrac{1}{16}\right|}\Bigg|_{x=0} = 16 \text{ ft}$$

Substituting $\theta = \theta_B = 0°$, $v = v_B = 23.24$ ft/s, and $\rho = \rho_B = 16$ ft into Eq. (1),

$$N_B = \frac{10}{32.2}\left[\frac{23.24^2}{16}\right] + 10\cos 0° = 20.5 \text{ lb} \qquad \textbf{Ans.}$$

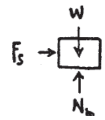
***14–20.**

The steel ingot has a mass of 1800 kg. It travels along the conveyor at a speed $v = 0.5$ m/s when it collides with the "nested" spring assembly. Determine the maximum deflection in each spring needed to stop the motion of the ingot. Take $k_A = 5$ kN/m, $k_B = 3$ kN/m.



SOLUTION

Assume both springs compress



$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(1800)(0.5)^2 - \frac{1}{2}(5000)s^2 - \frac{1}{2}(3000)(s - 0.05)^2 = 0$$

$$225 - 2500\,s^2 - 1500(s^2 - 0.1\,s + 0.0025) = 0$$

$$s^2 - 0.0375\,s - 0.05531 = 0$$

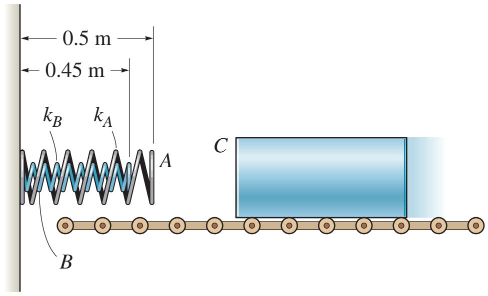
$s = 0.2547 \text{ m} > 0.05 \text{ m}$ **(O.K!)**

$s_A = 0.255 \text{ m}$ **Ans.**

$s_B = 0.205 \text{ m}$ **Ans.**

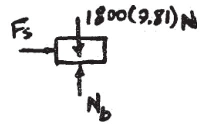
14–21.

The steel ingot has a mass of 1800 kg. It travels along the conveyor at a speed $v = 0.5$ m/s when it collides with the "nested" spring assembly. If the stiffness of the outer spring is $k_A = 5$ kN/m, determine the required stiffness k_B of the inner spring so that the motion of the ingot is stopped at the moment the front, C, of the ingot is 0.3 m from the wall.



SOLUTION

$T_1 + \Sigma U_{1-2} = T_2$

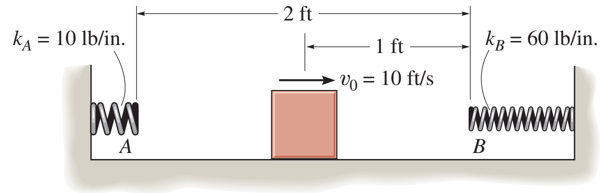


$$\frac{1}{2}(1800)(0.5)^2 - \frac{1}{2}(5000)(0.5 - 0.3)^2 - \frac{1}{2}(k_B)(0.45 - 0.3)^2 = 0$$

$k_B = 11.1$ kN/m **Ans.**

14–22.

The 25-lb block has an initial speed of $v_0 = 10$ ft/s when it is midway between springs A and B. After striking spring B, it rebounds and slides across the horizontal plane toward spring A, etc. If the coefficient of kinetic friction between the plane and the block is $\mu_k = 0.4$, determine the total distance traveled by the block before it comes to rest.



SOLUTION

Principle of Work and Energy: Here, the friction force $F_f = \mu_k N = 0.4(25) = 10.0$ lb. Since the friction force is always opposite the motion, it does negative work. When the block strikes spring B and stops momentarily, the spring force does *negative* work since it acts in the opposite direction to that of displacement. Applying Eq. 14–7, we have

$$T_l + \sum U_{1-2} = T_2$$

$$\frac{1}{2}\left(\frac{25}{32.2}\right)(10)^2 - 10(1 + s_1) - \frac{1}{2}(60)s_1^2 = 0$$

$$s_1 = 0.8275 \text{ ft}$$

Assume the block bounces back and stops without striking spring A. The spring force does *positive* work since it acts in the direction of displacement. Applying Eq. 14–7, we have

$$T_2 + \sum U_{2-3} = T_3$$

$$0 + \frac{1}{2}(60)(0.8275^2) - 10(0.8275 + s_2) = 0$$

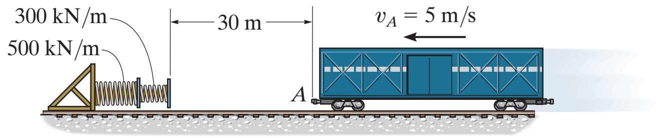
$$s_2 = 1.227 \text{ ft}$$

Since $s_2 = 1.227$ ft < 2 ft, the block stops before it strikes spring A. Therefore, the above assumption was correct. Thus, the total distance traveled by the block before it stops is

$$s_{\text{Tot}} = 2s_1 + s_2 + 1 = 2(0.8275) + 1.227 + 1 = 3.88 \text{ ft} \qquad \textbf{Ans.}$$

14–23.

The train car has a mass of 10 Mg and is traveling at 5 m/s when it reaches A. If the rolling resistance is 1/100 of the weight of the car, determine the compression of each spring when the car is momentarily brought to rest.



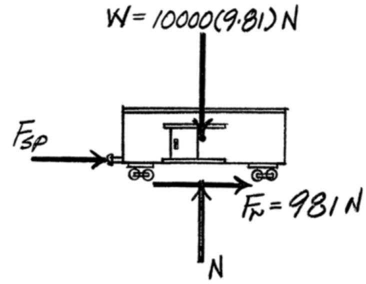
SOLUTION

Free-Body Diagram: The free-body diagram of the train in contact with the spring is shown in Fig. a. Here, the rolling resistance is $F_r = \frac{1}{100}[10\,000(9.81)] = 981$ N. The compression of springs 1 and 2 at the instant the train is momentarily at rest will be denoted as s_1 and s_2. Thus, the force developed in springs 1 and 2 are $(F_{sp})_1 = k_1 s_1 = 300(10^3)s_1$ and $(F_{sp})_2 = 500(10^3)s_2$. Since action is equal to reaction,

$$(F_{sp})_1 = (F_{sp})_2$$

$$300(10^3)s_1 = 500(10^3)s_2$$

$$s_1 = 1.6667s_2$$



(a)

Principle of Work and Energy: Referring to Fig. a, **W** and **N** do no work, and $\mathbf{F}_{sp}$ and $\mathbf{F}_r$ do negative work.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(10\,000)(5^2) + [-981(30 + s_1 + s_2)] + \left\{-\frac{1}{2}\left[300(10^3)\right]s_1^2\right\} + \left\{-\frac{1}{2}\left[500(10^3)\right]s_2^2\right\} = 0$$

$$150(10^3)s_1^2 + 250(10^3)s_2^2 + 981(s_1 + s_2) - 95570 = 0$$

Substituting Eq. (1) into Eq. (2),

$$666.67(10^3)s_2^2 + 2616s_2 - 95570 = 0$$

Solving for the positive root of the above equation,

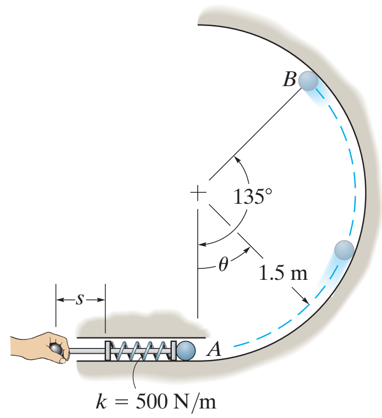
$$s_2 = 0.3767 \text{ m} = 0.377 \text{ m}$$

Substituting the result of s_2 into Eq. (1),

$$s_1 = 0.6278 \text{ m} = 0.628 \text{ m}$$ **Ans.**

***14–24.**

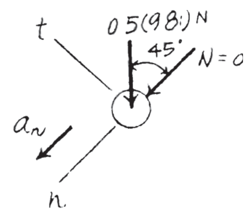
The 0.5-kg ball is fired up the smooth vertical circular track using the spring plunger. The plunger keeps the spring compressed 0.08 m when $s = 0$. Determine how far s it must be pulled back and released so that the ball will begin to leave the track when $\theta = 135°$.



SOLUTION

Equations of Motion:

$$\Sigma F_n = ma_n; \qquad 0.5(9.81)\cos 45° = 0.5\left(\frac{v_B^2}{1.5}\right) \qquad v_B^2 = 10.41 \text{ m}^2/\text{s}^2$$



Principle of Work and Energy: Here, the weight of the ball is being displaced vertically by $s = 1.5 + 1.5 \sin 45° = 2.561$ m and so it does *negative* work. The spring force, given by $F_{sp} = 500(s + 0.08)$, does positive work. Since the ball is at rest initially, $T_1 = 0$. Applying Eq. 14–7, we have

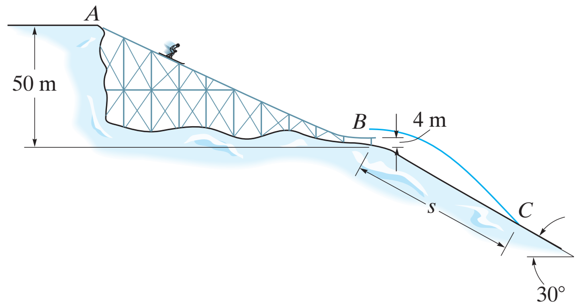
$$T_A + \sum U_{A-B} = T_B$$

$$0 + \int_0^s 500(s + 0.08)\, ds - 0.5(9.81)(2.561) = \frac{1}{2}(0.5)(10.41)$$

$$s = 0.1789 \text{ m} = 179 \text{ mm} \qquad \textbf{Ans.}$$

14–25.

The skier starts from rest at A and travels down the ramp. If friction and air resistance can be neglected, determine his speed v_B when he reaches B. Also, find the distance s to where he strikes the ground at C, if he makes the jump traveling horizontally at B. Neglect the skier's size. He has a mass of 70 kg.

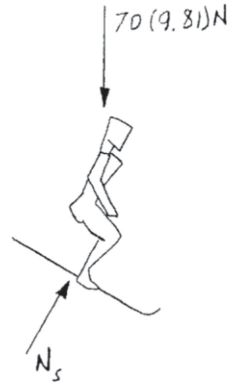


SOLUTION

$$T_A + \Sigma U_{A-B} = T_B$$

$$0 + 70(9.81)(46) = \frac{1}{2}(70)(v_B)^2$$

$$v_B = 30.04 \text{ m/s} = 30.0 \text{ m/s}$$ **Ans.**



$(\overset{+}{\rightarrow})$ $\quad s = s_0 + v_0 t$

$$s \cos 30° = 0 + 30.04t$$

$(+\downarrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$s \sin 30° + 4 = 0 + 0 + \frac{1}{2}(9.81)t^2$$

Eliminating t,

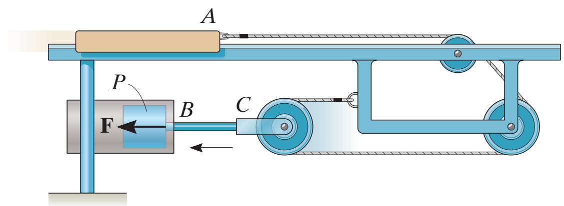
$$s^2 - 122.67s - 981.33 = 0$$

Solving for the positive root

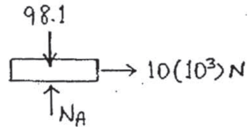
$$s = 130 \text{ m}$$ **Ans.**

14–26.

The catapulting mechanism is used to propel the 10-kg slider A to the right along the smooth track. The propelling action is obtained by drawing the pulley attached to rod BC rapidly to the left by means of a piston P. If the piston applies a constant force $F = 20$ kN to rod BC such that it moves it 0.2 m, determine the speed attained by the slider if it was originally at rest. Neglect the mass of the pulleys, cable, piston, and rod BC.



SOLUTION

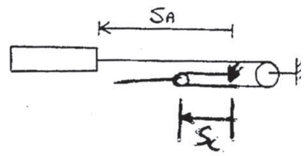


$2\,s_C + s_A = l$

$2\,\Delta\,s_C + \Delta\,s_A = 0$

$2(0.2) = -\,\Delta\,s_A$

$-0.4 = \Delta\,s_A$



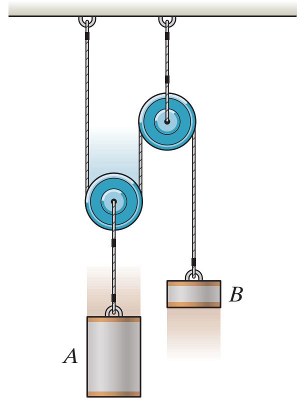
$T_1 + \Sigma U_{1-2} = T_2$

$$0 + (10\,000)(0.4) = \frac{1}{2}(10)(v_A)^2$$

$v_A = 28.3\text{ m/s}$ **Ans.**

14–27.

Block A has a weight of 60 lb and B has a weight of 10 lb. Determine the distance A must descend from rest before it obtains a speed of 8 ft/s. Also, what is the tension in the cord supporting A? Neglect the mass of the cord and pulleys.



SOLUTION

$2\,s_A + s_B = l$

$2\,\Delta\,s_A = -\,\Delta\,s_B$

$2\,v_A = -v_B$

For $v_A = 8$ ft/s, $\quad v_B = -16$ ft/s

For the system:

$T_1 + \Sigma U_{1-2} = T_2$

$$[0 + 0] + [60(s_A) - 10(2s_A)] = \frac{1}{2}\left(\frac{60}{32.2}\right)(8)^2 + \frac{1}{2}\left(\frac{10}{32.2}\right)(-16)^2$$

$s_A = 2.484 = 2.48$ ft **Ans.**

For block A:

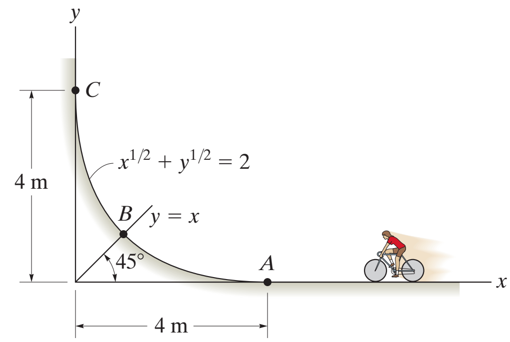
$T_1 + \Sigma U_{1-2} = T_2$

$$0 + 60(2.484) - T_A(2.484) = \frac{1}{2}\left(\frac{60}{32.2}\right)(8)^2$$

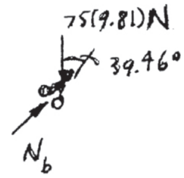
$T_A = 36.0$ lb **Ans.**

***14–28.**

The cyclist travels to point A, pedaling until he reaches a speed $v_A = 4$ m/s. He then coasts freely up the curved surface. Determine how high he reaches up the surface before he comes to a stop. Also, what are the resultant normal force on the surface at this point and his acceleration? The total mass of the bike and man is 75 kg. Neglect friction, the mass of the wheels, and the size of the bicycle.



SOLUTION



$$x^{\frac{1}{2}} + y^{\frac{1}{2}} = 2$$

$$\frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}y^{-\frac{1}{2}}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-x^{-\frac{1}{2}}}{y^{-\frac{1}{2}}}$$

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(75)(4)^2 - 75(9.81)(y) = 0$$

$$y = 0.81549\text{ m} = 0.815\text{ m}$$ **Ans.**

$$x^{1/2} + (0.81549)^{1/2} = 2$$

$$x = 1.2033\text{ m}$$

$$\tan\theta = \frac{dy}{dx} = \frac{-(1.2033)^{-1/2}}{(0.81549)^{-1/2}} = -0.82323$$

$$\theta = -39.46°$$

$\nearrow + \Sigma F_n = m\,a_n;$ $\quad N_b - 9.81(75)\cos 39.46° = 0$

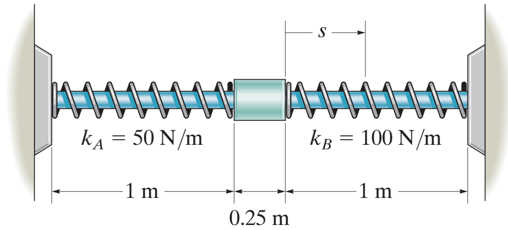
$$N_b = 568\text{ N}$$ **Ans.**

$+\searrow \Sigma F_t = m\,a_t;$ $\quad 75(9.81)\sin 39.46° = 75\,a_t$

$$a = a_t = 6.23\text{ m/s}^2$$ **Ans.**

14–29.

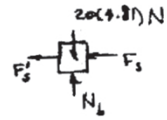
The collar has a mass of 20 kg and slides along the smooth rod. Two springs are attached to it and the ends of the rod as shown. If each spring has an uncompressed length of 1 m and the collar has a speed of 2 m/s when $s = 0$, determine the maximum compression of each spring due to the back-and-forth (oscillating) motion of the collar.



SOLUTION

$$T_1 + \Sigma U_{1-2} = T_2$$

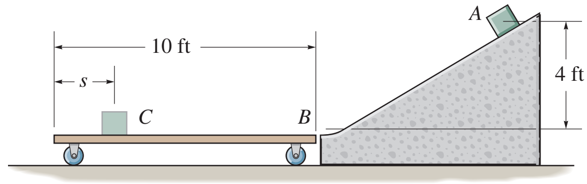
$$\frac{1}{2}(20)(2)^2 - \frac{1}{2}(50)(s)^2 - \frac{1}{2}(100)(s)^2 = 0$$



$s = 0.730$ m **Ans.**

14–30.

The 30-lb box A is released from rest and slides down along the smooth ramp and onto the surface of a cart. If the cart is *prevented from moving*, determine the distance s from the end of the cart to where the box stops. The coefficient of kinetic friction between the cart and the box is $\mu_k = 0.6$.



SOLUTION

Principle of Work and Energy: W_A which acts in the direction of the vertical displacement does *positive* work when the block displaces 4 ft vertically. The friction force $F_f = \mu_k N = 0.6(30) = 18.0$ lb does *negative* work since it acts in the opposite direction to that of displacement Since the block is at rest initially and is required to stop, $T_A = T_C = 0$. Applying Eq. 14–7, we have

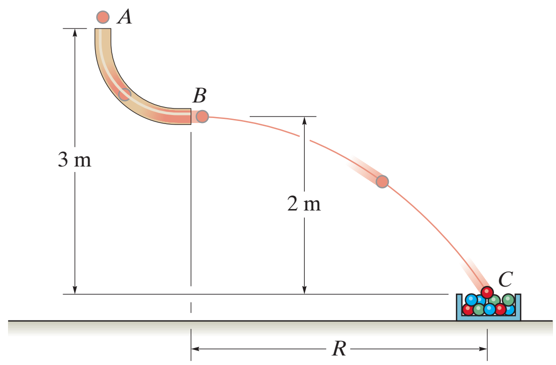
$$T_A + \sum U_{A-C} = T_C$$

$$0 + 30(4) - 18.0s' = 0 \qquad s' = 6.667 \text{ ft}$$

Thus, $$s = 10 - s' = 3.33 \text{ ft}$$ **Ans.**

14–31.

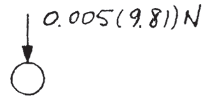
Marbles having a mass of 5 g are dropped from rest at A through the smooth glass tube and accumulate in the can at C. Determine the placement R of the can from the end of the tube and the speed at which the marbles fall into the can. Neglect the size of the can.



SOLUTION

$$T_A + \Sigma U_{A-B} = T_B$$

$$0 + [0.005(9.81)(3 - 2)] = \frac{1}{2}(0.005)v_B^2$$



$$v_B = 4.429 \text{ m/s}$$

$(+\downarrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$$2 = 0 + 0 = \frac{1}{2}(9.81)t^2$$

$$t = 0.6386 \text{ s}$$

$(\overset{+}{\rightarrow})$ $\quad s = s_0 + v_0 t$

$$R = 0 + 4.429(0.6386) = 2.83 \text{ m}$$ **Ans.**

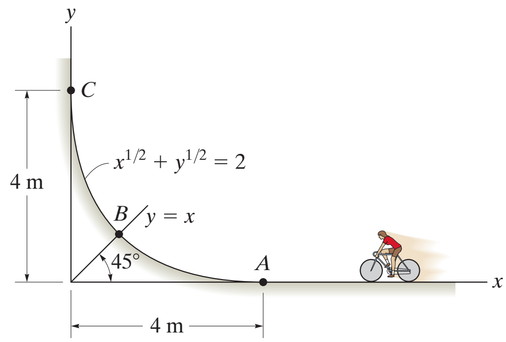
$$T_A + \Sigma U_{A-C} = T_1$$

$$0 + [0.005(9.81)(3)] = \frac{1}{2}(0.005)v_C^2$$

$$v_C = 7.67 \text{ m/s}$$ **Ans.**

***14–32.**

The cyclist travels to point A, pedaling until he reaches a speed $v_A = 8\text{ m/s}$. He then coasts freely up the curved surface. Determine the normal force he exerts on the surface when he reaches point B. The total mass of the bike and man is 75 kg. Neglect friction, the mass of the wheels, and the size of the bicycle.

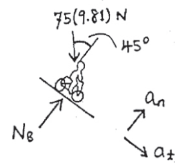


SOLUTION

$$x^{\frac{1}{2}} + y^{\frac{1}{2}} = 2$$

$$\frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}y^{-\frac{1}{2}}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-x^{-\frac{1}{2}}}{y^{-\frac{1}{2}}}$$



For $y = x$,

$$2\,x^{\frac{1}{2}} = 2$$

$x = 1, y = 1$ (Point B)

Thus,

$$\tan\theta = \frac{dy}{dx} = -1$$

$$\theta = -45°$$

$$\frac{dy}{dx} = (-x^{-\frac{1}{2}})(y^{\frac{1}{2}})$$

$$\frac{d^2y}{dx^2} = y^{\frac{1}{2}}\left(\frac{1}{2}x^{-\frac{3}{2}}\right) - x^{-\frac{1}{2}}\left(\frac{1}{2}\right)\left(y^{-\frac{1}{2}}\right)\left(\frac{dy}{dx}\right)$$

$$\frac{d^2y}{dx^2} = \frac{1}{2}y^{\frac{1}{2}}x^{-\frac{3}{2}} + \frac{1}{2}\left(\frac{1}{x}\right)$$

For $x = y = 1$

$$\frac{dy}{dx} = -1, \qquad \frac{d^2y}{dx^2} = 1$$

$$\rho = \frac{[1 + (-1)^2]^{3/2}}{1} = 2.828\text{ m}$$

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(75)(8)^2 - 75(9.81)(1) = \frac{1}{2}(75)(v_B^2)$$

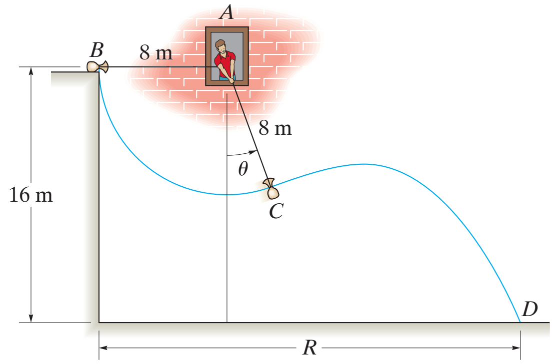
$$v_B^2 = 44.38$$

$$\nearrow + \Sigma F_n = ma_n; \qquad N_B - 9.81(75)\cos 45° = 75\left(\frac{44.38}{2.828}\right)$$

$$N_B = 1.70\text{ kN}$$ **Ans.**

14–33.

The man at the window A wishes to throw the 30-kg sack on the ground. To do this he allows it to swing from rest at B to point C, when he releases the cord at $\theta = 30°$. Determine the speed at which it strikes the ground and the distance R.



SOLUTION

$$T_B + \Sigma U_{B-C} = T_C$$

$$0 + 30(9.81)8 \cos 30° = \frac{1}{2}(30)v_C^2$$

$$v_C = 11.659 \text{ m/s}$$

$$T_B + \Sigma U_{B-D} = T_D$$

$$0 + 30(9.81)(16) = \frac{1}{2}(30)\,v_D^2$$

$$v_D = 17.7 \text{ m/s}$$ **Ans.**

During free flight:

$$(+\downarrow)\; s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$16 = 8 \cos 30° - 11.659 \sin 30° t + \frac{1}{2}(9.81)t^2$$

$$t^2 - 1.18848\, t - 1.8495 = 0$$

Solving for the positive root:

$$t = 2.0784 \text{ s}$$

$$(\overset{+}{\rightarrow})\; s = s_0 + v_0\, t$$

$$s = 8 \sin 30° + 11.659 \cos 30°(2.0784)$$

$$s = 24.985 \text{ m}$$

Thus,

$$R = 8 + 24.985 = 33.0 \text{ m}$$ **Ans.**

Also,

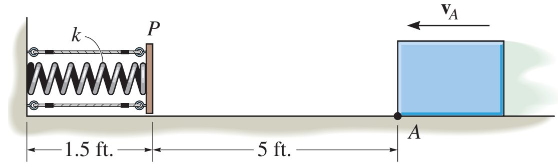
$$(v_D)_x = 11.659 \cos 30° = 10.097 \text{ m/s}$$

$$(+\downarrow)\; (v_D)_x = -11.659 \sin 30° + 9.81(2.0784) = 14.559 \text{ m/s}$$

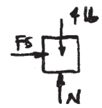
$$v_D = \sqrt{(10.097)^2 + (14.559)^2} = 7.7 \text{ m/s}$$ **Ans.**

14–34.

The spring bumper is used to arrest the motion of the 4-lb block, which is sliding toward it at $v = 9$ ft/s. As shown, the spring is confined by the plate P and wall using cables so that its length is 1.5 ft. If the stiffness of the spring is $k = 50$ lb/ft, determine the required unstretched length of the spring so that the plate is not displaced more than 0.2 ft after the block collides into it. Neglect friction, the mass of the plate and spring, and the energy loss between the plate and block during the collision.



SOLUTION



$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}\left(\frac{4}{32.2}\right)(9)^2 - \left[\frac{1}{2}(50)(s - 1.3)^2 - \frac{1}{2}(50)(s - 1.5)^2\right] = 0$$

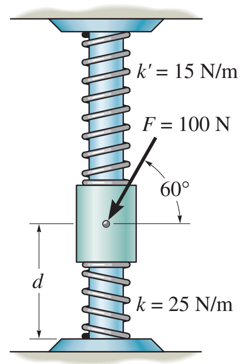
$$0.20124 = s^2 - 2.60\,s + 1.69 - (s^2 - 3.0\,s + 2.25)$$

$$0.20124 = 0.4\,s - 0.560$$

$s = 1.90$ ft **Ans.**

14–35.

The collar has a mass of 20 kg and is supported on the smooth rod. The attached springs are undeformed when $d = 0.5$ m. Determine the speed of the collar after the applied force $F = 100$ N causes it to be displaced so that $d = 0.3$ m. When $d = 0.5$ m the collar is at rest.

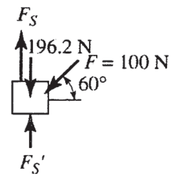


SOLUTION

$$T_1 + \sum U_{1-2} = T_2$$

$$0 + 100 \sin 60°(0.5 - 0.3) + 196.2(0.5 - 0.3) - \frac{1}{2}(15)(0.5 - 0.3)^2$$

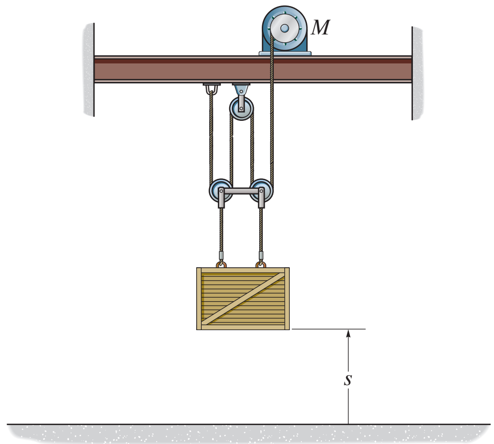
$$- \frac{1}{2}(25)(0.5 - 0.3)^2 = \frac{1}{2}(20)v_C^2$$



$v_C = 2.36$ m/s **Ans.**

***14–36.**

If the force exerted by the motor M on the cable is 250 N, determine the speed of the 100-kg crate when it is hoisted to $s = 3$ m. The crate is at rest when $s = 0$.



SOLUTION

Kinematics: Expressing the length of the cable in terms of position coordinates s_C and s_P referring to Fig. *a*,

$$3s_C + (s_C - s_P) = l$$

$$4s_C - s_P = l \qquad \textbf{(1)}$$

Using Eq. (1), the change in position of the crate and point P on the cable can be written as

$$(+\downarrow) \qquad 4\Delta s_C - \Delta s_P = 0$$

Here, $\Delta s_C = -3$ m. Thus,

$$(+\downarrow) \qquad 4(-3) - \Delta s_P = 0 \qquad\qquad \Delta s_p = -12 \text{ m} = 12 \text{ m} \uparrow$$

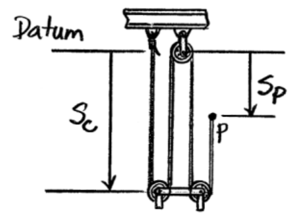
Principle of Work and Energy: Referring to the free-body diagram of the pulley system, Fig. *b*, $\mathbf{F}_1$ and $\mathbf{F}_2$ do no work since it acts at the support; however, $\mathbf{T}$ does positive work and $\mathbf{W}_C$ does negative work.

$$T_1 + \Sigma U_{1-2} = T_2$$

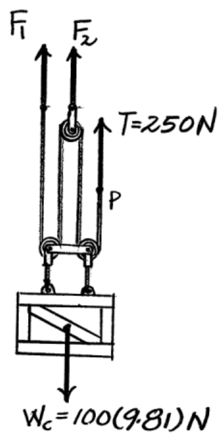
$$0 + T\Delta s_P + [-W_C \Delta s_C] = \frac{1}{2} m_C v^2$$

$$0 + 250(12) + [-100(9.81)(3)] = \frac{1}{2}(100)v^2$$

$$v = 1.07 \text{ m/s} \qquad \textbf{Ans.}$$



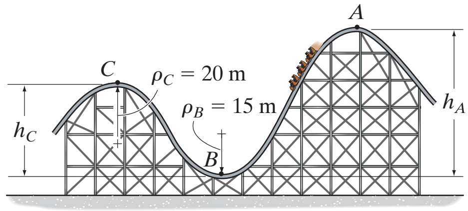
(a)



(b)

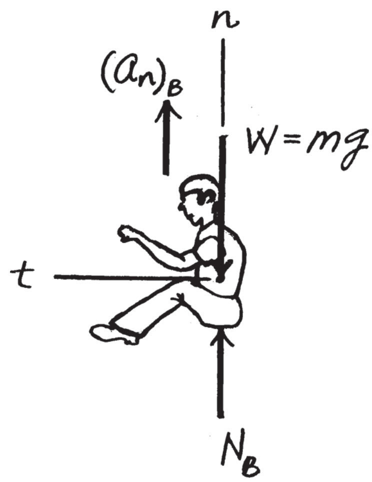
14–37.

If the track is to be designed so that the passengers of the roller coaster do not experience a normal force equal to zero or more than 4 times their weight, determine the limiting heights h_A and h_C so that this does not occur. The roller coaster starts from rest at position A. Neglect friction.



SOLUTION

Free-Body Diagram: The free-body diagram of the passenger at positions B and C are shown in Figs. a and b, respectively.



(a)

Equations of Motion: Here, $a_n = \dfrac{v^2}{\rho}$. The requirement at position B is that $N_B = 4mg$. By referring to Fig. a,

$$+\uparrow \Sigma F_n = ma_n; \qquad 4mg - mg = m\left(\frac{v_B{}^2}{15}\right)$$

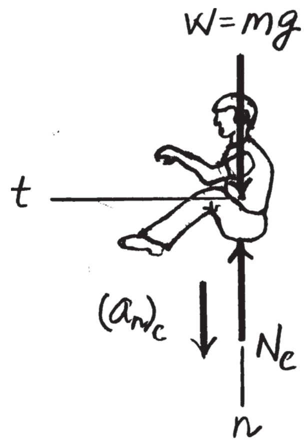
$$v_B{}^2 = 45g$$

At position C, N_C is required to be zero. By referring to Fig. b,

$$+\downarrow \Sigma F_n = ma_n; \qquad mg - 0 = m\left(\frac{v_C{}^2}{20}\right)$$

$$v_C{}^2 = 20g$$

Principle of Work and Energy: The normal reaction **N** does no work since it always acts perpendicular to the motion. When the rollercoaster moves from position A to B, **W** displaces vertically downward $h = h_A$ and does positive work.



(b)

We have

$$T_A + \Sigma U_{A-B} = T_B$$

$$0 + mgh_A = \frac{1}{2}m(45g)$$

$$h_A = 22.5 \text{ m} \qquad \textbf{Ans.}$$

When the rollercoaster moves from position A to C, **W** displaces vertically downward $h = h_A - h_C = (22.5 - h_C)$ m.

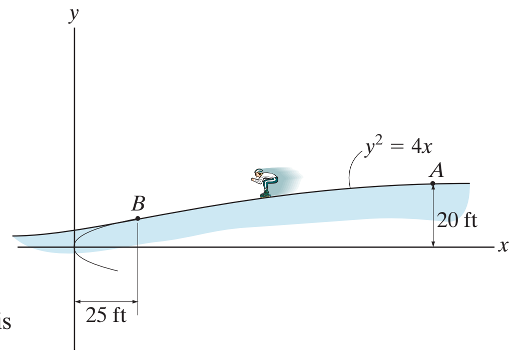
$$T_A + \Sigma U_{A-B} = T_B$$

$$0 + mg(22.5 - h_C) = \frac{1}{2}m(20g)$$

$$h_C = 12.5 \text{ m} \qquad \textbf{Ans.}$$

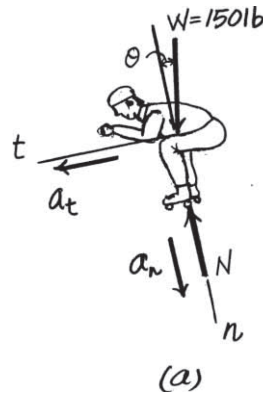
14–38.

The 150-lb skater passes point A with a speed of 6 ft/s. Determine his speed when he reaches point B and the normal force exerted on him by the track at this point. Neglect friction.



SOLUTION

Free-Body Diagram: The free-body diagram of the skater at an arbitrary position is shown in Fig. a.



Principle of Work and Energy: By referring to Fig. a, notice that **N** does no work since it always acts perpendicular to the motion. When the skier slides down the track from A to B, **W** displaces vertically downward $h = y_A - y_B = 20 - \left[2(25)^{1/2}\right] = 10$ ft and does positive work.

$$T_A + \Sigma U_{A-B} = T_B$$

$$\frac{1}{2}\left(\frac{150}{32.2}\right)(6^2) + \left[150(10)\right] = \frac{1}{2}\left(\frac{150}{32.2}\right){v_B}^2$$

$$v_B = 26.08 \text{ ft/s} = 26.1 \text{ ft/s} \qquad \textbf{Ans.}$$

Equations of Motion: Here, $a_n = \dfrac{v^2}{\rho}$. By referring to Fig. a,

$$\searrow + \Sigma F_n = ma_n; \qquad 150 \cos \theta - N = \frac{150}{32.2}\left(\frac{v^2}{\rho}\right)$$

$$N = 150 \cos \theta - \frac{150}{32.2}\left(\frac{v^2}{\rho}\right) \qquad \textbf{(1)}$$

Geometry: Here, $y = 2x^{1/2}$, $\dfrac{dy}{dx} = \dfrac{1}{x^{1/2}}$, and $\dfrac{d^2y}{dx^2} = -\dfrac{1}{2x^{3/2}}$. The slope that the track at position B makes with the horizontal is $\theta_B = \tan^{-1}\left(\dfrac{dx}{dy}\right)\Bigg|_{x=25\text{ ft}}$ $= \tan\left(\dfrac{1}{x^{1/2}}\right)\Bigg|_{x=25\text{ ft}} = 11.31°$. The radius of curvature of the track at position B is given by

$$\rho_B = \frac{\left[1 + \left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}{\left|\dfrac{d^2y}{dx^2}\right|} = \frac{\left[1 + \left(\dfrac{1}{x^{1/2}}\right)^2\right]^{3/2}}{\left|-\dfrac{1}{2x^{3/2}}\right|}\Bigg|_{x=25\text{ ft}} = 265.15 \text{ ft}$$

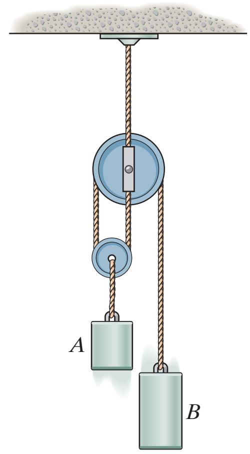
Substituting $\theta = \theta_B = 11.31°$, $v = v_B = 26.08$ ft/s, and $\rho = \rho_B = 265.15$ ft into Eq. (1),

$$N_B = 150 \cos 11.31° - \frac{150}{32.2}\left(\frac{26.08^2}{265.15}\right)$$

$$= 135 \text{ lb} \qquad \textbf{Ans.}$$

14–39.

The 8-kg cylinder A and 3-kg cylinder B are released from rest. Determine the speed of A after it has moved 2 m starting from rest. Neglect the mass of the cord and pulleys.



SOLUTION

Kinematics: Express the length of cord in terms of position coordinates s_A and s_B by referring to Fig. *a*

$$2s_A + s_B = l \qquad \textbf{(1)}$$

Thus

$$2\Delta s_A + \Delta s_B = 0 \qquad \textbf{(2)}$$

If we assume that cylinder A is moving downward through a distance of $\Delta s_A = 2$ m, Eq. (2) gives

$$(+\downarrow) \qquad 2(2) + \Delta s_B = 0 \qquad \Delta s_B = -4 \text{ m} = 4 \text{ m} \uparrow$$

Taking the time derivative of Eq. (1),

$$(+\downarrow) \qquad 2v_A + v_B = 0 \qquad \textbf{(3)}$$

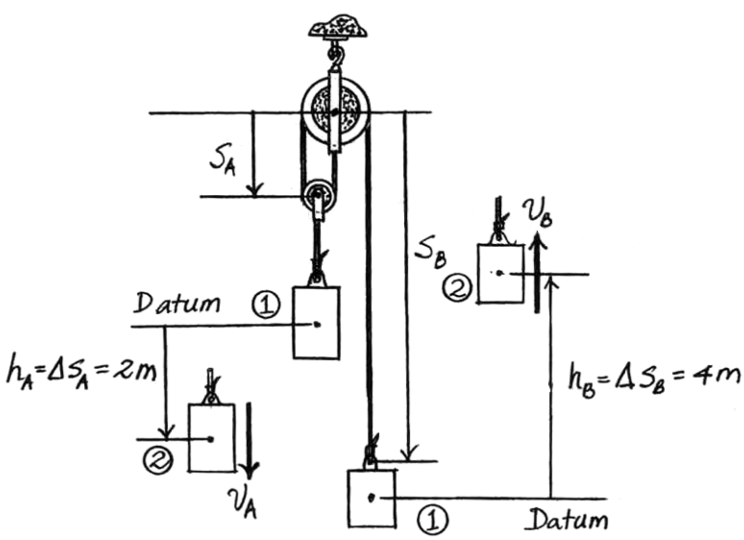
$$\Sigma T_1 + \Sigma U_{1-2} = \Sigma T_2$$

$$0 + 8(2)9.81 - 3(4)9.81 = \frac{1}{2}(8)v_A^2 + \frac{1}{2}(3)v_B^2$$

Positive net work on left means assumption of A moving down is correct. Since $v_B = -2v_A$,

$$v_A = 1.98 \text{ m/s} \downarrow \qquad \textbf{Ans.}$$

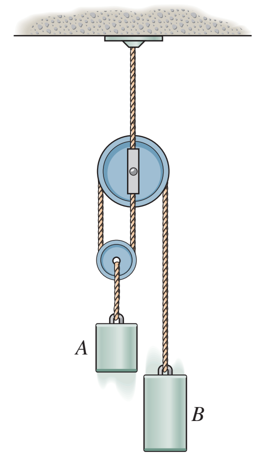
$$v_B = -3.96 \text{ m/s} = 3.96 \text{ m/s} \uparrow$$



(a)

***14–40.**

Cylinder A has a mass of 3 kg and cylinder B has a mass of 8 kg. Determine the speed of A after it moves upwards 2 m starting from rest. Neglect the mass of the cord and pulleys.



SOLUTION

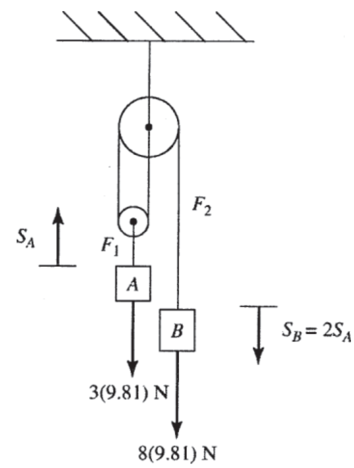
$$\sum T_1 + \sum U_{1-2} = \sum T_2$$

$$0 + 2[F_1 - 3(9.81)] + 4[8(9.81) - F_2] = \frac{1}{2}(3)v_A^2 + \frac{1}{2}(8)v_B^2$$

Also, $v_B = 2v_A$, and because the pulleys are massless, $F_1 = 2F_2$. The F_1 and F_2 terms drop out and the work-energy equation reduces to

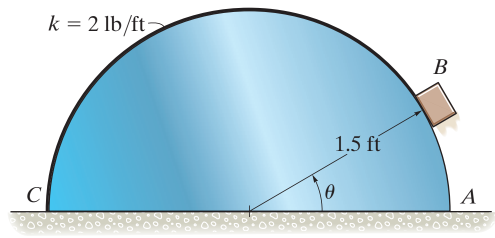
$$255.06 = 17.5v_A^2$$

$$v_A = 3.82 \text{ m/s}$$ **Ans.**

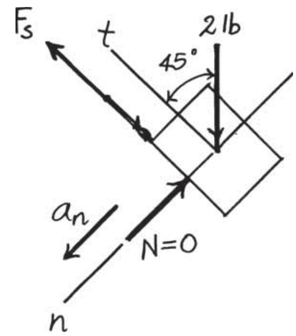


14–41.

A 2-lb block rests on the smooth semicylindrical surface. An elastic cord having a stiffness $k = 2$ lb/ft is attached to the block at B and to the base of the semicylinder at point C. If the block is released from rest at A ($\theta = 0°$), determine the unstretched length of the cord so that the block begins to leave the semicylinder at the instant $\theta = 45°$. Neglect the size of the block.



SOLUTION



$+\swarrow \Sigma F_n = ma_n; \qquad 2 \sin 45° = \frac{2}{32.2}\left(\frac{v^2}{1.5}\right)$

$$v = 5.844 \text{ ft/s}$$

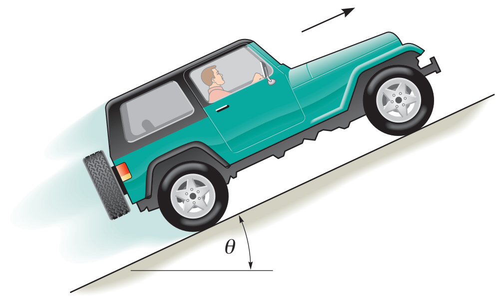
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + \frac{1}{2}(2)\left[\pi(1.5) - l_0\right]^2 - \frac{1}{2}(2)\left[\frac{3\pi}{4}(1.5) - l_0\right]^2 - 2(1.5 \sin 45°) = \frac{1}{2}\left(\frac{2}{32.2}\right)(5.844)^2$$

$l_0 = 2.77$ ft **Ans.**

14–42.

The jeep has a weight of 2500 lb and an engine which transmits a power of 100 hp to *all* the wheels. Assuming the wheels do not slip on the ground, determine the angle θ of the largest incline the jeep can climb at a constant speed $v = 30$ ft/s.

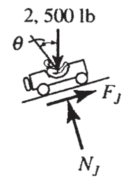


SOLUTION

$P = F_J v$

$100(550) = 2500 \sin\theta(30)$

$\theta = 47.2°$ **Ans.**



14–43.

Determine the power input for a motor necessary to lift 300 lb at a constant rate of 5 ft/s. The efficiency of the motor is $\epsilon = 0.65$.

SOLUTION

Power: The power output can be obtained using Eq. 14–10.

$$P = \mathbf{F} \cdot \mathbf{v} = 300(5) = 1500 \text{ ft} \cdot \text{lb/s}$$

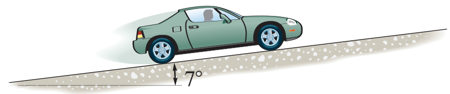
Using Eq. 14–11, the required power input for the motor to provide the above power output is

$$\text{power input} = \frac{\text{power output}}{\epsilon}$$

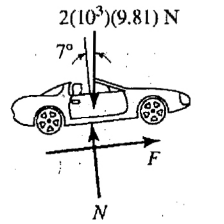
$$= \frac{1500}{0.65} = 2307.7 \text{ ft} \cdot \text{lb/s} = 4.20 \text{ hp} \qquad \textbf{Ans.}$$

***14–44.**

An automobile having a mass of 2 Mg travels up a 7° slope at a constant speed of $v = 100$ km/h. If mechanical friction and wind resistance are neglected, determine the power developed by the engine if the automobile has an efficiency $\epsilon = 0.65$.



SOLUTION



Equation of Motion: The force F which is required to maintain the car's constant speed up the slope must be determined first.

$$+\Sigma F_{x'} = ma_{x'}; \qquad F - 2(10^3)(9.81)\sin 7° = 2(10^3)(0)$$

$$F = 2391.08 \text{ N}$$

Power: Here, the speed of the car is $v = \left[\dfrac{100(10^3)\text{ m}}{\text{h}}\right] \times \left(\dfrac{1\text{ h}}{3600\text{ s}}\right) = 27.78$ m/s. The power output can be obtained using Eq. 14–10.

$$P = \mathbf{F} \cdot \mathbf{v} = 2391.08(27.78) = 66.418(10^3)\text{ W} = 66.418 \text{ kW}$$

Using Eq. 14–11, the required power input from the engine to provide the above power output is

$$\text{power input} = \frac{\text{power output}}{\varepsilon}$$

$$= \frac{66.418}{0.65} = 102 \text{ kW} \qquad \textbf{Ans.}$$

14–45.

The Milkin Aircraft Co. manufactures a turbojet engine that is placed in a plane having a weight of 13000 lb. If the engine develops a constant thrust of 5200 lb, determine the power output of the plane when it is just ready to take off with a speed of 600 mi/h.

SOLUTION

At 600 ms/h.

$$P = 5200(600)\left(\frac{88 \text{ ft/s}}{60 \text{ m/h}}\right)\frac{1}{550} = 8.32\ (10^3) \text{ hp} \qquad \textbf{Ans.}$$

14–46.

To dramatize the loss of energy in an automobile, consider a car having a weight of 5000 lb that is traveling at 35 mi/h. If the car is brought to a stop, determine how long a 100-W light bulb must burn to expend the same amount of energy. (1 mi = 5280 ft.)

SOLUTION

Energy: Here, the speed of the car is $v = \left(\frac{35\text{ mi}}{\text{h}}\right) \times \left(\frac{5280\text{ ft}}{1\text{ mi}}\right) \times \left(\frac{1\text{ h}}{3600\text{ s}}\right) =$ 51.33 ft/s. Thus, the kinetic energy of the car is

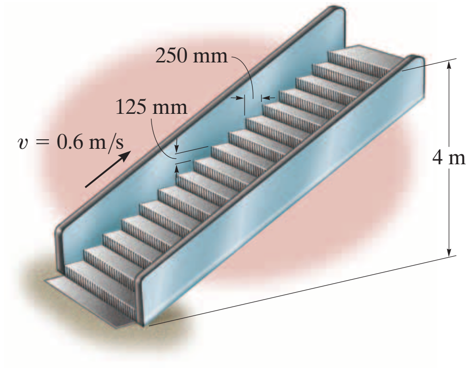
$$U = \frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{5000}{32.2}\right)\left(51.33^2\right) = 204.59\left(10^3\right)\text{ ft}\cdot\text{lb}$$

The power of the bulb is $P_{bulb} = 100\text{ W} \times \left(\frac{1\text{ hp}}{746\text{ W}}\right) \times \left(\frac{550\text{ ft}\cdot\text{lb/s}}{1\text{ hp}}\right) =$ 73.73 ft · lb/s. Thus,

$$t = \frac{U}{P_{bulb}} = \frac{204.59(10^3)}{73.73} = 2774.98\text{ s} = 46.2\text{ min} \qquad \textbf{Ans.}$$

14–47.

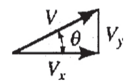
The escalator steps move with a constant speed of 0.6 m/s. If the steps are 125 mm high and 250 mm in length, determine the power of a motor needed to lift an average mass of 150 kg per step. There are 32 steps.



SOLUTION

Step height: 0.125 m

The number of steps: $\dfrac{4}{0.125} = 32$



Total load: $32(150)(9.81) = 47\,088$ N

If load is placed at the center height, $h = \dfrac{4}{2} = 2$ m, then

$$U = 47\,088\left(\frac{4}{2}\right) = 94.18 \text{ kJ}$$

$$\nu_y = \nu \sin\theta = 0.6\left(\frac{4}{\sqrt{(32(0.25))^2 + 4^2}}\right) = 0.2683 \text{ m/s}$$

$$t = \frac{h}{\nu_y} = \frac{2}{0.2683} = 7.454 \text{ s}$$

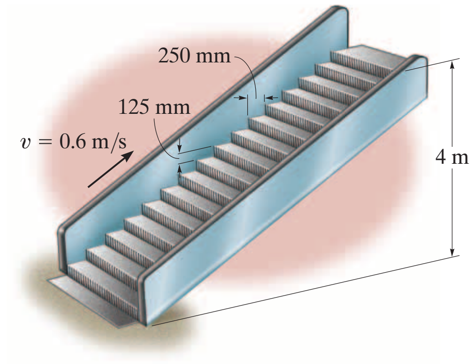
$$P = \frac{U}{t} = \frac{94.18}{7.454} = 12.6 \text{ kW}$$ **Ans.**

Also,

$$P = \mathbf{F} \cdot \mathbf{v} = 47\,088(0.2683) = 12.6 \text{ kW}$$ **Ans.**

***14–48.**

If the escalator in Prob. 14–47 is not moving, determine the constant speed at which a man having a mass of 80 kg must walk up the steps to generate 100 W of power—the same amount that is needed to power a standard light bulb.



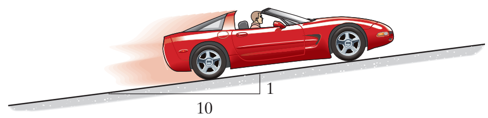
SOLUTION

$$P = \frac{U_{1-2}}{t} = \frac{(80)(9.81)(4)}{t} = 100 \qquad t = 31.4\text{ s}$$

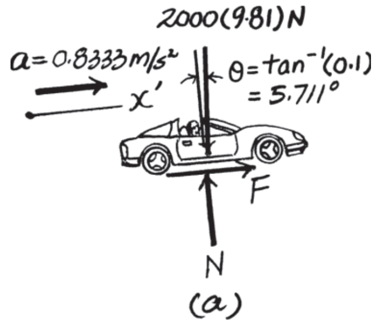
$$v = \frac{s}{t} = \frac{\sqrt{(32(0.25))^2 + 4^2}}{31.4} = 0.285\text{ m/s} \qquad \textbf{Ans.}$$

14–49.

The 2-Mg car increases its speed uniformly from rest to 25 m/s in 30 s up the inclined road. Determine the maximum power that must be supplied by the engine, which operates with an efficiency of $\epsilon = 0.8$. Also, find the average power supplied by the engine.



SOLUTION



Kinematics: The constant acceleration of the car can be determined from

$$(\overset{+}{\rightarrow}) \qquad v = v_0 + a_c t$$

$$25 = 0 + a_c(30)$$

$$a_c = 0.8333 \text{ m/s}^2$$

Equations of Motion: By referring to the free-body diagram of the car shown in Fig. *a*,

$$\Sigma F_{x'} = ma_{x'}; \qquad F - 2000(9.81)\sin 5.711° = 2000(0.8333)$$

$$F = 3618.93\text{N}$$

Power: The maximum power output of the motor can be determined from

$$(P_{\text{out}})_{\max} = \mathbf{F}\cdot\mathbf{v}_{\max} = 3618.93(25) = 90\,473.24 \text{ W}$$

Thus, the maximum power input is given by

$$P_{\text{in}} = \frac{P_{\text{out}}}{\varepsilon} = \frac{90473.24}{0.8} = 113\,091.55 \text{ W} = 113 \text{ kW} \qquad \textbf{Ans.}$$

The average power output can be determined from

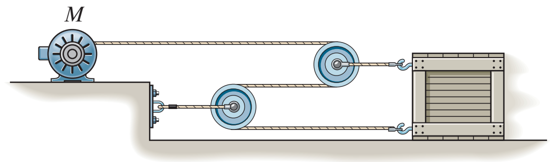
$$(P_{\text{out}})_{\text{avg}} = \mathbf{F}\cdot\mathbf{v}_{\text{avg}} = 3618.93\left(\frac{25}{2}\right) = 45\,236.62 \text{ W}$$

Thus,

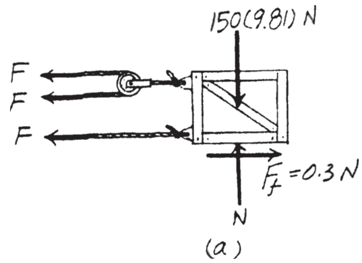
$$(P_{\text{in}})_{\text{avg}} = \frac{(P_{\text{out}})_{\text{avg}}}{\varepsilon} = \frac{45236.62}{0.8} = 56\,545.78 \text{ W} = 56.5 \text{ kW} \qquad \textbf{Ans.}$$

14–50.

The crate has a mass of 150 kg and rests on a surface for which the coefficients of static and kinetic friction are $\mu_s = 0.3$ and $\mu_k = 0.2$, respectively. If the motor M supplies a cable force of $F = (8t^2 + 20)$ N, where t is in seconds, determine the power output developed by the motor when $t = 5$ s.



SOLUTION

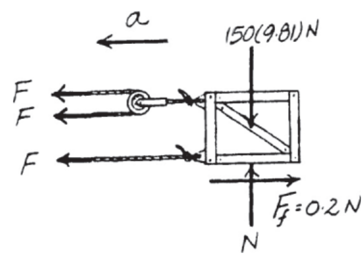


Equations of Equilibrium: If the crate is on the verge of slipping, $F_f = \mu_s N = 0.3N$. From FBD(a),

$$+\uparrow \Sigma F_y = 0; \qquad N - 150(9.81) = 0 \qquad N = 1471.5 \text{ N}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 0.3(1471.5) - 3\left(8t^2 + 20\right) = 0 \qquad t = 3.9867 \text{ s}$$

Equations of Motion: Since the crate moves 3.9867 s later, $F_f = \mu_k N = 0.2N$. From FBD(b),



$$+\uparrow \Sigma F_y = ma_y; \qquad N - 150(9.81) = 150\,(0) \qquad N = 1471.5 \text{ N}$$

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 0.2\,(1471.5) - 3\left(8t^2 + 20\right) = 150\,(-a)$$

$$a = \left(0.160t^2 - 1.562\right) \text{ m/s}^2$$

Kinematics: Applying $dv = a\,dt$, we have

$$\int_0^v dv = \int_{3.9867 \text{ s}}^{5} \left(0.160t^2 - 1.562\right) dt$$

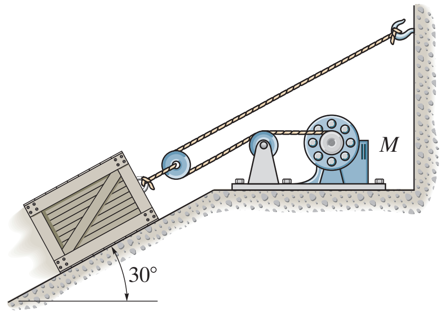
$$v = 1.7045 \text{ m/s}$$

Power: At $t = 5$ s, $F = 8\left(5^2\right) + 20 = 220$ N. The power can be obtained using Eq. 14–10.

$$P = \mathbf{F} \cdot \mathbf{v} = 3\,(220)\,(1.7045) = 1124.97 \text{ W} = 1.12 \text{ kW}$$ **Ans.**

14–51.

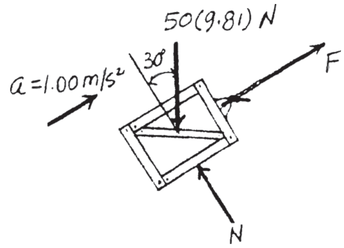
The 50-kg crate is hoisted up the 30° incline by the pulley system and motor M. If the crate starts from rest and, by constant acceleration, attains a speed of 4 m/s after traveling 8 m along the plane, determine the power that must be supplied to the motor at the instant the crate has moved 8 m. Neglect friction along the plane. The motor has an efficiency of $\epsilon = 0.74$.



SOLUTION

Kinematics: Applying equation $v^2 = v_0^2 + 2a_c(s - s_0)$, we have

$$4^2 = 0^2 + 2a(8 - 0) \qquad a = 1.00 \text{ m/s}^2$$



Equations of Motion:

$$+\Sigma F_{x'} = ma_{x'}; \qquad F - 50(9.81)\sin 30° = 50(1.00) \qquad F = 295.25 \text{ N}$$

Power: The power output at the instant when $v = 4$ m/s can be obtained using Eq. 14–10.

$$P = \mathbf{F} \cdot \mathbf{v} = 295.25\,(4) = 1181 \text{ W} = 1.181 \text{ kW}$$

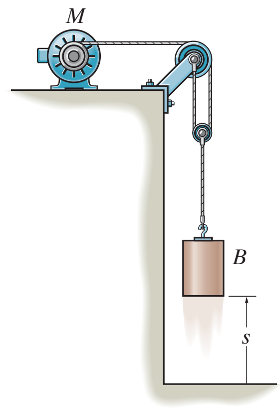
Using Eq. 14–11, the required power input to the motor in order to provide the above power output is

$$\text{power input} = \frac{\text{power output}}{\varepsilon}$$

$$= \frac{1.181}{0.74} = 1.60 \text{ kW} \qquad \textbf{Ans.}$$

***14–52.**

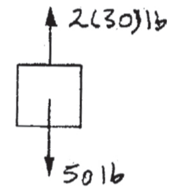
The 50-lb load is hoisted by the pulley system and motor M. If the motor exerts a constant force of 30 lb on the cable, determine the power that must be supplied to the motor if the load has been hoisted $s = 10$ ft starting from rest. The motor has an efficiency of $\epsilon = 0.76$.



SOLUTION

$+\uparrow \Sigma F_y = m\,a_y; \qquad 2(30) - 50 = \dfrac{50}{32.2} a_B$

$a_B = 6.44 \text{ m/s}^2$



$(+\uparrow)\ v^2 = v_0^2 + 2a_c\,(s - s_0)$

$v_B^2 = 0 + 2(6.44)(10 - 0)$

$v_B = 11.349 \text{ ft/s}$

$2s_B + s_M = l$

$2v_B = -\,v_M$

$v_M = -2(11.349) = 22.698 \text{ ft/s}$

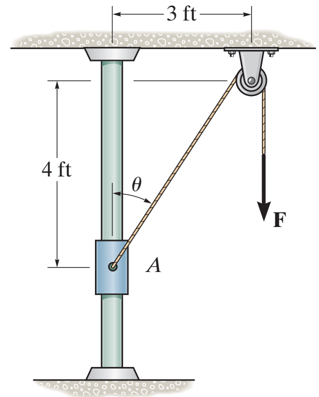
$P_o = \mathbf{F} \cdot \mathbf{v} = 30(22.698) = 680.94 \text{ ft} \cdot \text{lb/s}$

$P_i = \dfrac{680.94}{0.76} = 895.97 \text{ ft} \cdot \text{lb/s}$

$P_i = 1.63 \text{ hp}$ **Ans.**

14–53.

The 10-lb collar starts from rest at A and is lifted by applying a constant vertical force of $F = 25$ lb to the cord. If the rod is smooth, determine the power developed by the force at the instant $\theta = 60°$.



SOLUTION

Work of **F**

$U_{1-2} = 25(5 - 3.464) = 38.40 \text{ lb} \cdot \text{ft}$

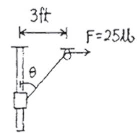
$T_1 + \Sigma U_{1-2} = T_2\text{s}$

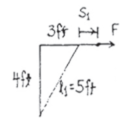
$0 + 38.40 - 10(4 - 1.732) = \frac{1}{2}(\frac{10}{32.2})v^2$

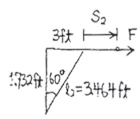
$v = 10.06 \text{ ft/s}$

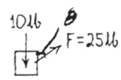
$P = \mathbf{F} \cdot \mathbf{v} = 25 \cos 60°(10.06) = 125.76 \text{ ft} \cdot \text{lb/s}$

$P = 0.229 \text{ hp}$ **Ans.**



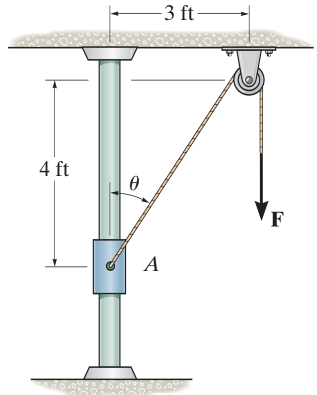






14–54.

The 10-lb collar starts from rest at A and is lifted with a constant speed of 2 ft/s along the smooth rod. Determine the power developed by the force **F** at the instant shown.



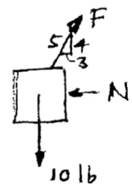
SOLUTION

$$+\uparrow \Sigma F_y = m\,a_y; \qquad F\left(\frac{4}{5}\right) - 10 = 0$$

$$F = 12.5\text{ lb}$$

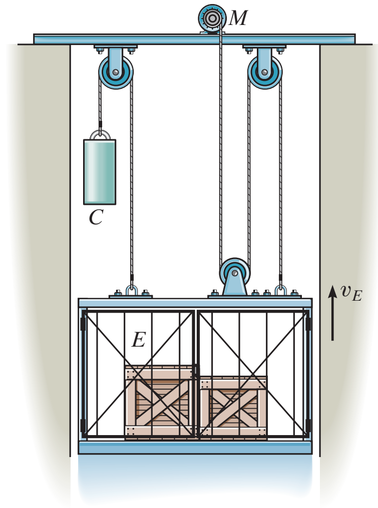
$$P = \mathbf{F}\cdot\mathbf{v} = 12.5\left(\frac{4}{5}\right)(2) = 20\text{ lb}\cdot\text{ft/s}$$

$$= 0.0364\text{ hp}$$ **Ans.**



14–55.

The elevator E and its freight have a total mass of 400 kg. Hoisting is provided by the motor M and the 60-kg block C. If the motor has an efficiency of $\epsilon = 0.6$, determine the power that must be supplied to the motor when the elevator is hoisted upward at a constant speed of $v_E = 4$ m/s.



SOLUTION

Elevator:

Since $a = 0$,

$+\uparrow \Sigma F_y = 0; \qquad 60(9.81) + 3T - 400(9.81) = 0$

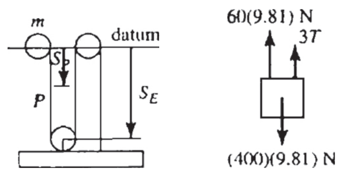
$$T = 1111.8 \text{ N}$$

$$2s_E + (s_E - s_P) = l$$

$$3v_E = v_P$$

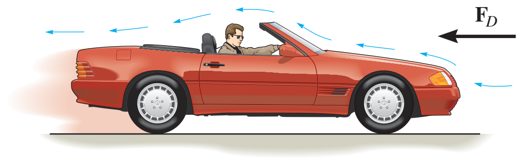
Since $v_E = -4$ m/s, $\quad v_P = -12$ m/s

$$P_i = \frac{\mathbf{F} \cdot \mathbf{v}_P}{e} = \frac{(1111.8)(12)}{0.6} = 22.2 \text{ kW}$$ **Ans.**

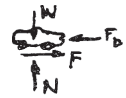


***14–56.**

The sports car has a mass of 2.3 Mg, and while it is traveling at 28 m/s the driver causes it to accelerate at 5 m/s^2. If the drag resistance on the car due to the wind is $F_D = (0.3v^2)$ N, where v is the velocity in m/s, determine the power supplied to the engine at this instant. The engine has a running efficiency of $\epsilon = 0.68$.



SOLUTION



$\overset{+}{\rightarrow} \Sigma F_x = m\,a_x;\qquad F - 0.3v^2 = 2.3(10^3)(5)$

$$F = 0.3v^2 + 11.5(10^3)$$

At $v = 28$ m/s

$$F = 11\,735.2 \text{ N}$$

$$P_O = (11\,735.2)(28) = 328.59 \text{ kW}$$

$$P_i = \frac{P_O}{e} = \frac{328.59}{0.68} = 438 \text{ kW}$$ **Ans.**

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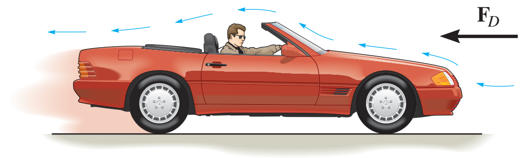
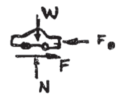
14–57.

The sports car has a mass of 2.3 Mg and accelerates at $6\ \mathrm{m/s^2}$, starting from rest. If the drag resistance on the car due to the wind is $F_D = (10v)$ N, where v is the velocity in m/s, determine the power supplied to the engine when $t = 5$ s. The engine has a running efficiency of $\epsilon = 0.68$.



SOLUTION



$\stackrel{+}{\rightarrow} \Sigma F_x = m\, a_x; \qquad F - 10v = 2.3(10^3)(6)$

$$F = 13.8(10^3) + 10\, v$$

$(\stackrel{+}{\rightarrow})\ v = v_0 + a_c\, t$

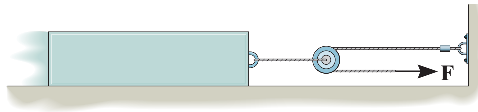
$$v = 0 + 6(5) = 30\ \mathrm{m/s}$$

$$P_O = \mathbf{F} \cdot \mathbf{v} = [13.8(10^3) + 10(30)](30) = 423.0\ \mathrm{kW}$$

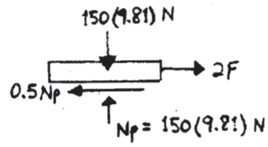
$$P_i = \frac{P_O}{e} = \frac{423.0}{0.68} = 622\ \mathrm{kW}$$ **Ans.**

14–58.

The block has a mass of 150 kg and rests on a surface for which the coefficients of static and kinetic friction are $\mu_s = 0.5$ and $\mu_k = 0.4$, respectively. If a force $F = (60t^2)$ N, where t is in seconds, is applied to the cable, determine the power developed by the force when $t = 5$ s. *Hint:* First determine the time needed for the force to cause motion.



SOLUTION



$\xrightarrow{+} \Sigma F_x = 0; \qquad 2F - 0.5(150)(9.81) = 0$

$F = 367.875 = 60t^2$

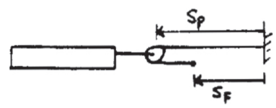
$t = 2.476$ s

$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 2(60t^2) - 0.4(150)(9.81) = 150a_p$

$a_p = 0.8t^2 - 3.924$

$dv = a\, dt$

$$\int_0^v dv = \int_{2.476}^{5} \left(0.8t^2 - 3.924\right) dt$$



$$v = \left(\frac{0.8}{3}\right)t^3 - 3.924t \Big|_{2.476}^{5} = 19.38 \text{ m/s}$$

$s_P + (s_P - s_F) = l$

$2v_P = v_F$

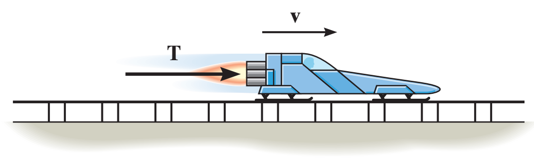
$v_F = 2(19.38) = 38.76$ m/s

$F = 60(5)^2 = 1500$ N

$P = \mathbf{F} \cdot \mathbf{v} = 1500(38.76) = 58.1$ kW **Ans.**

14–59.

The rocket sled has a mass of 4 Mg and travels from rest along the horizontal track for which the coefficient of kinetic friction is $\mu_k = 0.20$. If the engine provides a constant thrust $T = 150$ kN, determine the power output of the engine as a function of time. Neglect the loss of fuel mass and air resistance.



SOLUTION

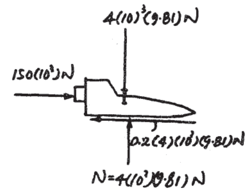
$\xrightarrow{+} \Sigma F_x = ma_x;\qquad 150(10)^3 - 0.2(4)(10)^3(9.81) = 4(10)^3\, a$

$a = 35.54 \text{ m/s}^2$

$(\xrightarrow{+})\ v = v_0 + a_c t$

$= 0 + 35.54t = 35.54t$

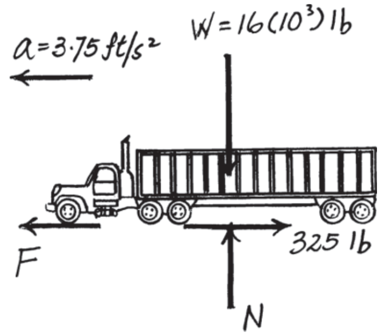
$P = \mathbf{T}\cdot\mathbf{v} = 150(10)^3(35.54t) = 5.33t \text{ MW}$ **Ans.**



***14–60.**

A loaded truck weighs $16(10^3)$ lb and accelerates uniformly on a level road from 15 ft/s to 30 ft/s during 4 s. If the frictional resistance to motion is 325 lb, determine the maximum power that must be delivered to the wheels.

SOLUTION



$$a = \frac{\Delta v}{\Delta t} = \frac{30 - 15}{4} = 3.75 \text{ ft/s}^2$$

$$\overset{+}{\leftarrow} \Sigma F_x = ma_x; \qquad F - 325 = \left(\frac{16(10^3)}{32.2}\right)(3.75)$$

$$F = 2188.35 \text{ lb}$$

$$P_{\max} = \mathbf{F} \cdot \mathbf{v}_{\max} = \frac{2188.35(30)}{550} = 119 \text{ hp}$$ **Ans.**

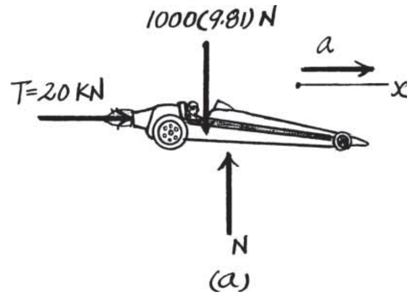
14–61.

If the jet on the dragster supplies a constant thrust of $T = 20$ kN, determine the power generated by the jet as a function of time. Neglect drag and rolling resistance, and the loss of fuel. The dragster has a mass of 1 Mg and starts from rest.

SOLUTION

Equations of Motion: By referring to the free-body diagram of the dragster shown in Fig. *a*,

$$\overset{+}{\rightarrow}\Sigma F_x = ma_x; \qquad 20(10^3) = 1000(a) \qquad a = 20\ \text{m/s}^2$$



Kinematics: The velocity of the dragster can be determined from

$$\left(\overset{+}{\rightarrow}\right) \qquad v = v_0 + a_c t$$

$$v = 0 + 20t = (20t)\ \text{m/s}$$

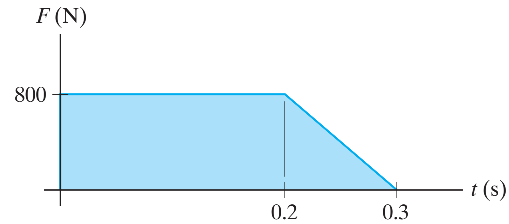
Power:

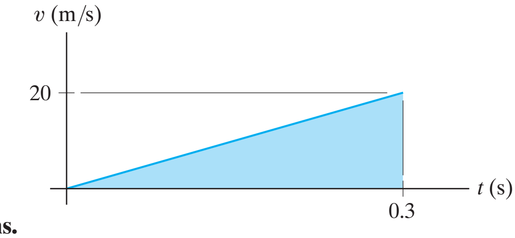
$$P = \mathbf{F}\cdot\mathbf{v} = 20(10^3)(20t)$$

$$= \left[400(10^3)t\right]\ \text{W} \qquad \textbf{Ans.}$$

14–62.

An athlete pushes against an exercise machine with a force that varies with time as shown in the first graph. Also, the velocity of the athlete's arm acting in the same direction as the force varies with time as shown in the second graph. Determine the power applied as a function of time and the work done in $t = 0.3$ s.





SOLUTION

For $0 \leq t \leq 0.2$

$F = 800$ N

$$v = \frac{20}{0.3}t = 66.67t$$

$\boldsymbol{P} = \mathbf{F} \cdot \mathbf{v} = 53.3t$ kW **Ans.**

For $0.2 \leq t \leq 0.3$

$F = 2400 - 8000t$

$v = 66.67t$

$P = \mathbf{F} \cdot \mathbf{v} = (160t - 533t^2)$ kW **Ans.**

$$u = \int_0^{0.3} P\,dt$$

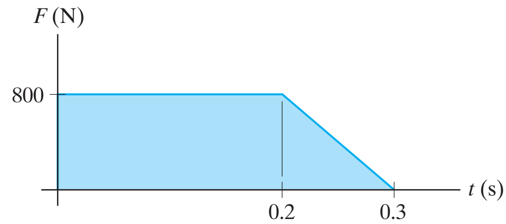
$$u = \int_0^{0.2} 53.3t\,dt + \int_{0.2}^{0.3} (160t - 533t^2)\,dt$$

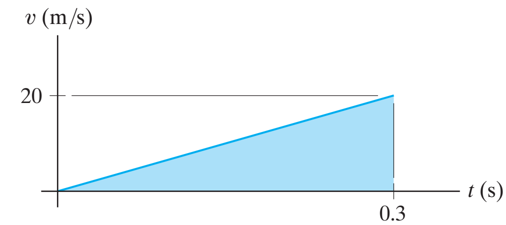
$$= \frac{53.3}{2}(0.2)^2 + \frac{160}{2}[(0.3)^2 - (0.2)^2] - \frac{533}{3}[(0.3)^3 - (0.2)^3]$$

$= 1.69$ kJ **Ans.**

14–63.

An athlete pushes against an exercise machine with a force that varies with time as shown in the first graph. Also, the velocity of the athlete's arm acting in the same direction as the force varies with time as shown in the second graph. Determine the maximum power developed during the 0.3-second time period.





SOLUTION

See solution to Prob. 14–62.

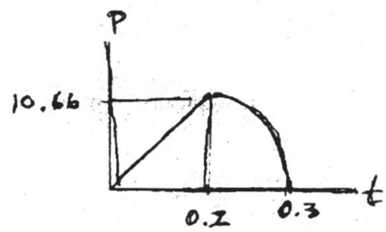
$P = 160\,t - 533\,t^2$

$\dfrac{dP}{dt} = 160 - 1066.6\,t = 0$

$t = 0.15\text{ s} < 0.2\text{ s}$

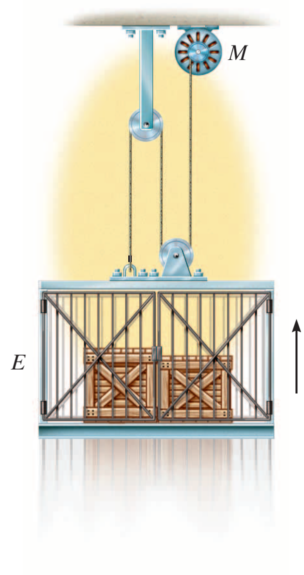
Thus maximum occurs at $t = 0.2$ s

$P_{max} = 53.3(0.2) = 10.7\text{ kW}$ **Ans.**



*14–64.

The 500-kg elevator starts from rest and travels upward with a constant acceleration $a_c = 2\ \mathrm{m/s^2}$. Determine the power output of the motor M when $t = 3$ s. Neglect the mass of the pulleys and cable.



SOLUTION

$+\uparrow \Sigma F_y = m\,a_y;\qquad 3T - 500(9.81) = 500(2)$

$$T = 1968.33\ \mathrm{N}$$

$3s_E - s_P = l$

$3\,v_E = v_P$

When $t = 3$ s,

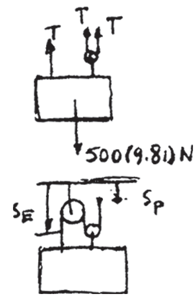
$(+\uparrow)\ \ v_0 + a_c\,t$

$$v_E = 0 + 2(3) = 6\ \mathrm{m/s}$$

$$v_P = 3(6) = 18\ \mathrm{m/s}$$

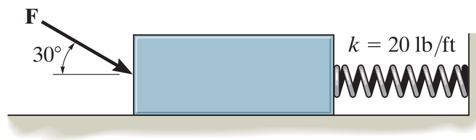
$$P_O = 1968.33(18)$$

$$P_O = 35.4\ \mathrm{kW}$$ **Ans.**

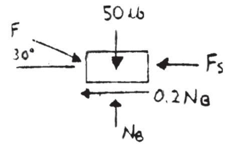


14–65.

The 50-lb block rests on the rough surface for which the coefficient of kinetic friction is $\mu_k = 0.2$. A force $F = (40 + s^2)$ lb, where s is in ft, acts on the block in the direction shown. If the spring is originally unstretched ($s = 0$) and the block is at rest, determine the power developed by the force the instant the block has moved $s = 1.5$ ft.



SOLUTION



$$+\uparrow \Sigma F_y = 0; \qquad N_B - \left(40 + s^2\right)\sin 30^\circ - 50 = 0$$

$$N_B = 70 + 0.5s^2$$

$$T_1 + \Sigma U_{1-2} + T_2$$

$$0 + \int_0^{1.5} \left(40 + s^2\right)\cos 30^\circ \, ds - \frac{1}{2}(20)(1.5)^2 - 0.2\int_0^{1.5} \left(70 + 0.5s^2\right) ds = \frac{1}{2}\left(\frac{50}{32.2}\right)v_2^2$$

$$0 + 52.936 - 22.5 - 21.1125 = 0.7764\, v_2^2$$

$$v_2 = 3.465 \text{ ft/s}$$

When $s = 1.5$ ft,

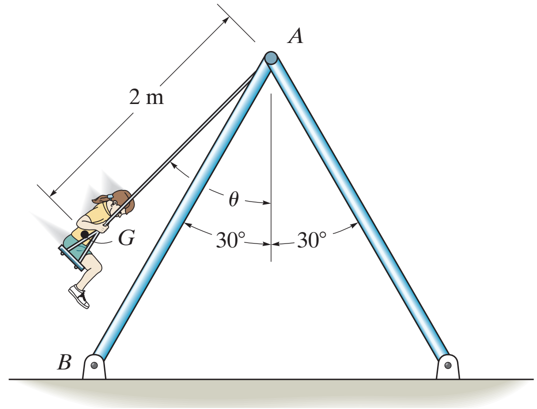
$$F = 40 + (1.5)^2 = 42.25 \text{ lb}$$

$$P = \mathbf{F}\cdot\mathbf{v} = (42.25 \cos 30^\circ)(3.465)$$

$$P = 126.79 \text{ ft}\cdot\text{lb/s} = 0.231 \text{ hp}$$ **Ans.**

14–66.

The girl has a mass of 40 kg and center of mass at G. If she is swinging to a maximum height defined by $\theta = 60°$, determine the force developed along each of the four supporting posts such as AB at the instant $\theta = 0°$. The swing is centrally located between the posts.



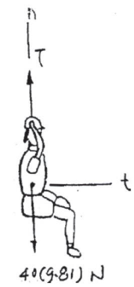
SOLUTION

The maximum tension in the cable occurs when $\theta = 0°$.

$$T_1 + V_1 = T_2 + V_2$$

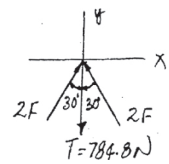
$$0 + 40(9.81)(-2\cos 60°) = \frac{1}{2}(40)v^2 + 40(9.81)(-2)$$

$$v = 4.429 \text{ m/s}$$



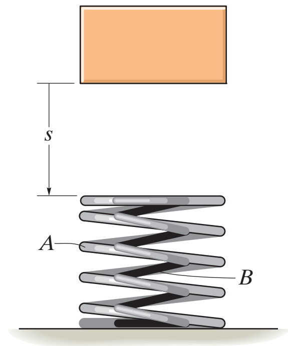
$+\uparrow \Sigma F_n = ma_n;\qquad T - 40(9.81) = (40)\left(\dfrac{4.429^2}{2}\right)\qquad T = 784.8 \text{ N}$

$+\uparrow \Sigma F_y = 0;\qquad 2(2F)\cos 30° - 784.8 = 0\qquad F = 227 \text{ N}$ **Ans.**



14–67.

Two equal-length springs are "nested" together in order to form a shock absorber. If it is designed to arrest the motion of a 2-kg mass that is dropped $s = 0.5$ m above the top of the springs from an at-rest position, and the maximum compression of the springs is to be 0.2 m, determine the required stiffness of the inner spring, k_B, if the outer spring has a stiffness $k_A = 400$ N/m.



SOLUTION

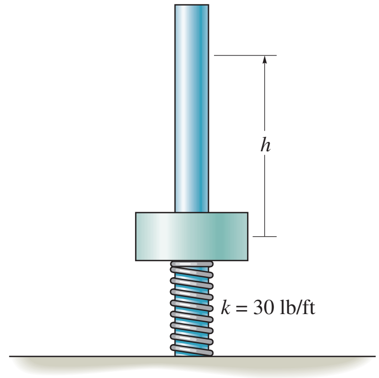
$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = 0 - 2(9.81)(0.5 + 0.2) + \frac{1}{2}(400)(0.2)^2 + \frac{1}{2}(k_B)(0.2)^2$$

$$k_B = 287 \text{ N/m}$$ **Ans.**

***14–68.**

The collar has a weight of 8 lb. If it is pushed down so as to compress the spring 2 ft and then released from rest ($h = 0$), determine its speed when it is displaced $h = 4.5$ ft. The spring is not attached to the collar. Neglect friction.



SOLUTION

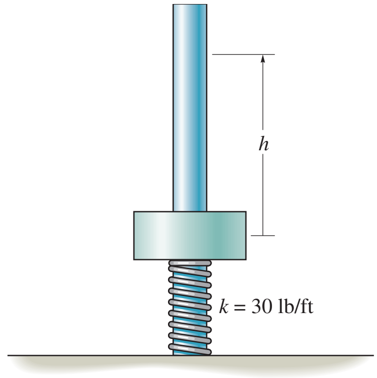
$$T_1 + V_1 = T_2 + V_2$$

$$0 + \frac{1}{2}(30)(2)^2 = \frac{1}{2}\left(\frac{8}{32.2}\right)v_2^2 + 8(4.5)$$

$$v_2 = 13.9 \text{ ft/s}$$ **Ans.**

14–69.

The collar has a weight of 8 lb. If it is released from rest at a height of $h = 2$ ft from the top of the uncompressed spring, determine the speed of the collar after it falls and compresses the spring 0.3 ft.



SOLUTION

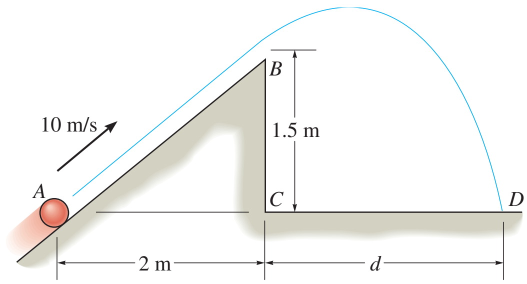
$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = \frac{1}{2}\left(\frac{8}{32.2}\right)v_2^2 - 8(2.3) + \frac{1}{2}(30)(0.3)^2$$

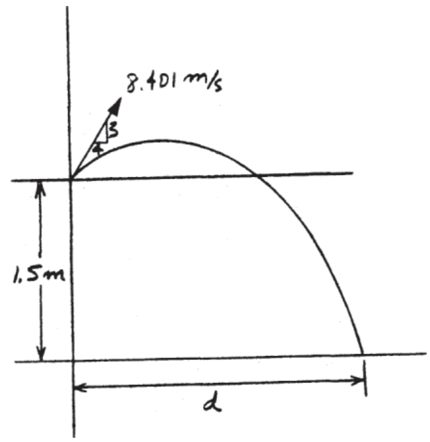
$$v_2 = 11.7 \text{ ft/s}$$ **Ans.**

14–70.

The 2-kg ball of negligible size is fired from point A with an initial velocity of 10 m/s up the smooth inclined plane. Determine the distance from point C to where it hits the horizontal surface at D. Also, what is its velocity when it strikes the surface?



SOLUTION



Datum at A:

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}(2)(10)^2 + 0 = \frac{1}{2}(2)(v_B)^2 + 2(9.81)(1.5)$$

$$v_B = 8.401 \text{ m/s}$$

$$(\xrightarrow{+}) \quad s = s_0 + v_0 t$$

$$d = 0 + 8.401\left(\frac{4}{5}\right)t$$

$$(+\uparrow) \quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$-1.5 = 0 + 8.401\left(\frac{3}{5}\right)t + \frac{1}{2}(-9.81)t^2$$

$$-4.905t^2 + 5.040t + 1.5 = 0$$

Solving for the positive root,

$$t = 1.269 \text{ s}$$

$$d = 8.401\left(\frac{4}{5}\right)(1.269) = 8.53 \text{ m}$$ **Ans.**

Datum at A:

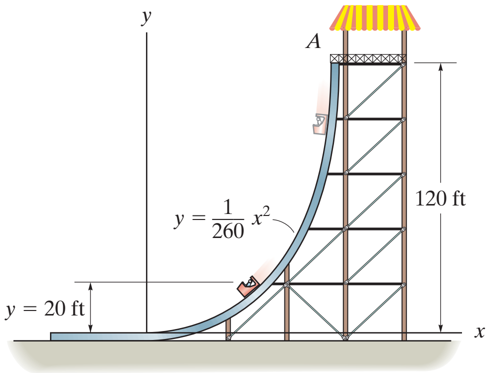
$$T_A + V_A = T_D + V_D$$

$$\frac{1}{2}(2)(10)^2 + 0 = \frac{1}{2}(2)(v_D)^2 + 0$$

$$v_D = 10 \text{ m/s}$$ **Ans.**

14–71.

The ride at an amusement park consists of a gondola which is lifted to a height of 120 ft at A. If it is released from rest and falls along the parabolic track, determine the speed at the instant $y = 20$ ft. Also determine the normal reaction of the tracks on the gondola at this instant. The gondola and passenger have a total weight of 500 lb. Neglect the effects of friction and the mass of the wheels.



SOLUTION

$$y = \frac{1}{260}x^2$$

$$\frac{dy}{dx} = \frac{1}{130}x$$

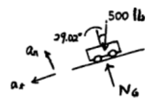
$$\frac{d^2y}{dx2} = \frac{1}{130}$$

At $y = 120 - 100 = 20$ ft

$$x = 72.11 \text{ ft}$$

$$\tan\theta = \frac{dy}{dx} = 0.555, \qquad \theta = 29.02^\circ$$

$$\rho = \frac{[1 + (0.555)^2]^{3/2}}{\frac{1}{130}} = 194.40 \text{ ft}$$



$$+\nwarrow \Sigma F_n = ma_n; \qquad N_G - 500\cos 29.02^\circ = \frac{500}{32.2}\left(\frac{v^2}{194.40}\right) \qquad \textbf{(1)}$$

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = \frac{1}{2}\left(\frac{500}{32.2}\right)v^2 - 500(100)$$

$$v^2 = 6440$$

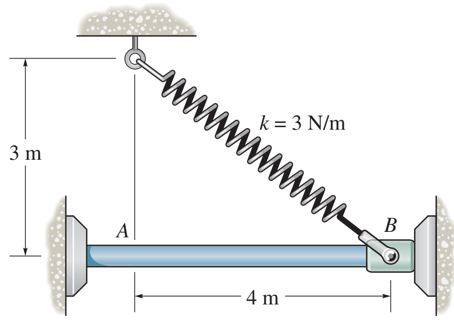
$$v = 80.2 \text{ ft/s} \qquad \textbf{Ans.}$$

Substituting into Eq. (1) yields

$$N_G = 952 \text{ lb} \qquad \textbf{Ans.}$$

***14–72.**

The 2-kg collar is attached to a spring that has an unstretched length of 3 m. If the collar is drawn to point B and released from rest, determine its speed when it arrives at point A.



SOLUTION

Potential Energy: The initial and final elastic potential energy are $\frac{1}{2}(3)\left(\sqrt{3^2 + 4^2} - 3\right)^2 = 6.00\ \mathbf{J}$ and $\frac{1}{2}(3)(3 - 3)^2 = 0$, respectively. The gravitational potential energy remains the same since the elevation of collar does not change when it moves from B to A.

Conservation of Energy:

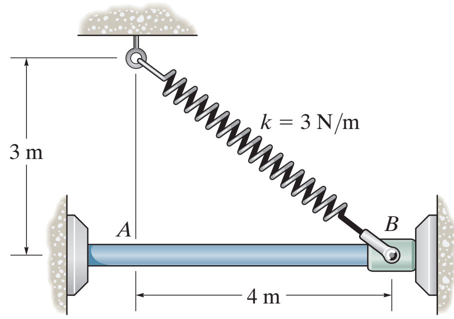
$$T_B + V_B = T_A + V_A$$

$$0 + 6.00 = \frac{1}{2}(2)\, v_A^2 + 0$$

$$v_A = 2.45\ \text{m/s}$$ **Ans.**

14–73.

The 2-kg collar is attached to a spring that has an unstretched length of 2 m. If the collar is drawn to point B and released from rest, determine its speed when it arrives at point A.



SOLUTION

Potential Energy: The stretches of the spring when the collar is at B and A are $s_B = \sqrt{3^2 + 4^2} - 2 = 3$ m and $s_A = 3 - 2 = 1$ m, respectively. Thus, the elastic potential energy of the system at B and A are

$$(V_e)_B = \frac{1}{2}ks_B^{\,2} = \frac{1}{2}(3)(3^2) = 13.5 \text{ J}$$

$$(V_e)_A = \frac{1}{2}ks_A^{\,2} = \frac{1}{2}(3)(1^2) = 1.5 \text{ J}$$

There is no change in gravitational potential energy since the elevation of the collar does not change during the motion.

Conservation of Energy:

$$T_B + V_B = T_A + V_A$$

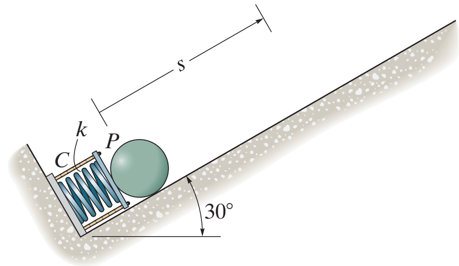
$$\frac{1}{2}mv_B^{\,2} + (V_e)_B = \frac{1}{2}mv_A^{\,2} + (V_e)_A$$

$$0 + 13.5 = \frac{1}{2}(2)v_A^{\,2} + 1.5$$

$$v_A = 3.46 \text{ m/s}$$ **Ans.**

14–74.

The 0.5-lb ball is shot from the spring device shown. The spring has a stiffness $k = 10$ lb/in. and the four cords C and plate P keep the spring compressed 2 in. when no load is on the plate. The plate is pushed back 3 in. from its initial position. If it is then released from rest, determine the speed of the ball when it travels 30 in. up the smooth plane.



SOLUTION

Potential Energy: The datum is set at the lowest point (compressed position). Finally, the ball is $\frac{30}{12}\sin 30^\circ = 1.25$ ft *above* the datum and its gravitational potential energy is $0.5(1.25) = 0.625\ \text{ft}\cdot\text{lb}$. The initial and final elastic potential energy are $\frac{1}{2}(120)\left(\frac{2+3}{12}\right)^2 = 10.42\ \text{ft}\cdot\text{lb}$ and $\frac{1}{2}(120)\left(\frac{2}{12}\right)^2 = 1.667\ \text{ft}\cdot\text{lb}$, respectively.

Conservation of Energy:

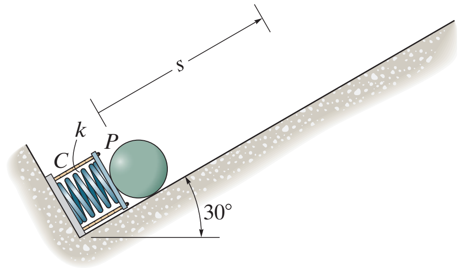
$$\Sigma T_1 + \Sigma V_1 = \Sigma T_2 + \Sigma V_2$$

$$0 + 10.42 = \frac{1}{2}\left(\frac{0.5}{32.2}\right)v^2 + 0.625 + 1.667$$

$$v = 32.3\ \text{ft/s}$$ **Ans.**

14–75.

The 0.5-lb ball is shot from the spring device shown. Determine the smallest stiffness k which is required to shoot the ball a maximum distance of 30 in. up the smooth plane after the spring is pushed back 3 in. and the ball is released from rest. The four cords C and plate P keep the spring compressed 2 in. when no load is on the plate.



SOLUTION

Potential Energy: The datum is set at the lowest point (compressed position). Finally, the ball is $\frac{30}{12}\sin 30° = 1.25$ ft *above* the datum and its gravitational potential energy is $0.5(1.25) = 0.625$ ft · lb. The initial and final elastic potential energy are $\frac{1}{2}(k)\left(\frac{2+3}{12}\right)^2 = 0.08681k$ and $\frac{1}{2}(k)\left(\frac{2}{12}\right)^2 = 0.01389k$, respectively.

Conservation of Energy:

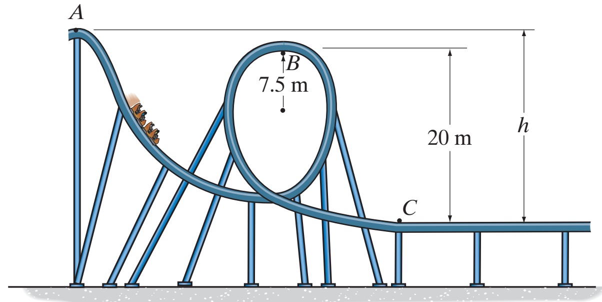
$$\Sigma T_1 + \Sigma V_1 = \Sigma T_2 + \Sigma V_2$$

$$0 + 0.08681k = 0 + 0.625 + 0.01389k$$

$$k = 8.57 \text{ lb/ft} \qquad \textbf{Ans.}$$

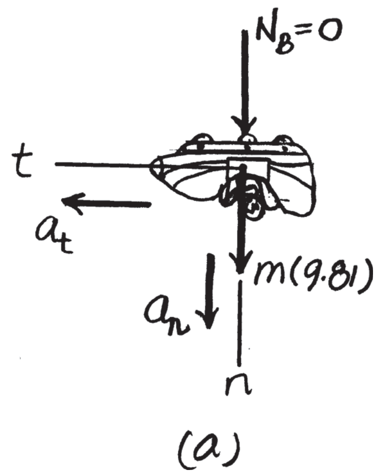
***14–76.**

The roller coaster car having a mass m is released from rest at point A. If the track is to be designed so that the car does not leave it at B, determine the required height h. Also, find the speed of the car when it reaches point C. Neglect friction.



SOLUTION

Equation of Motion: Since it is required that the roller coaster car is about to leave the track at B, $N_B = 0$. Here, $a_n = \dfrac{v_B^2}{\rho_B} = \dfrac{v_B^2}{7.5}$. By referring to the free-body diagram of the roller coaster car shown in Fig. a,



$$\Sigma F_n = ma_n; \qquad m(9.81) = m\left(\frac{v_B^2}{7.5}\right) \quad v_B^2 = 73.575 \text{ m}^2/\text{s}^2$$

Potential Energy: With reference to the datum set in Fig. b, the gravitational potential energy of the rollercoaster car at positions A, B, and C are $(V_g)_A = mgh_A = m(9.81)h = 9.81mh$, $(V_g)_B = mgh_B = m(9.81)(20) = 196.2$ m, and $(V_g)_C = mgh_C = m(9.81)(0) = 0$.

Conservation of Energy: Using the result of v_B^2 and considering the motion of the car from position A to B,

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}mv_A^2 + (V_g)_A = \frac{1}{2}mv_B^2 + (V_g)_B$$

$$0 + 9.81mh = \frac{1}{2}m(73.575) + 196.2m$$

$$h = 23.75 \text{ m}$$ **Ans.**

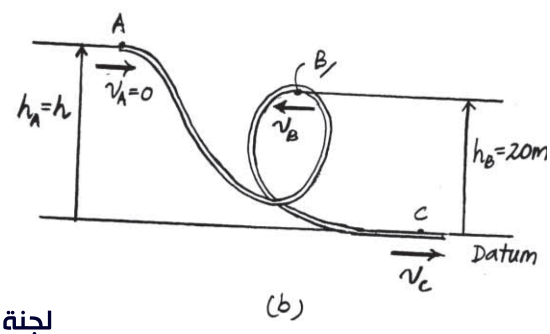
Also, considering the motion of the car from position B to C,

$$T_B + V_B = T_C + V_C$$

$$\frac{1}{2}mv_B^2 + (V_g)_B = \frac{1}{2}mv_C^2 + (V_g)_C$$

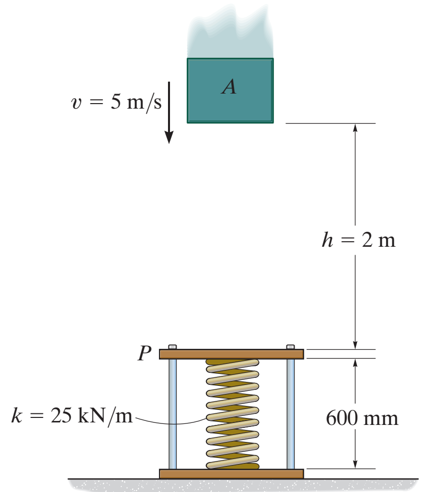
$$\frac{1}{2}m(73.575) + 196.2m = \frac{1}{2}mv_C^2 + 0$$

$$v_C = 21.6 \text{ m/s}$$ **Ans.**



14–77.

A 750-mm-long spring is compressed and confined by the plate P, which can slide freely along the vertical 600-mm-long rods. The 40-kg block is given a speed of $v = 5$ m/s when it is $h = 2$ m above the plate. Determine how far the plate moves downwards when the block momentarily stops after striking it. Neglect the mass of the plate.

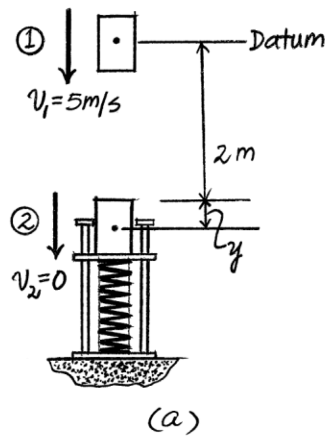


SOLUTION

Potential Energy: With reference to the datum set in Fig. a, the gravitational potential energy of the block at positions (1) and (2) are $(V_g)_1 = mgh_1 = 40(9.81)(0) = 0$ and $(V_g)_2 = mgh_2 = 40(9.81)\left[-(2+y)\right] = \left[-392.4(2+y)\right]$, respectively. The compression of the spring when the block is at positions (1) and (2) are $s_1 = (0.75 - 0.6) = 0.15$ m and $s_2 = s_1 + y = (0.15 + y)$ m. Thus, the initial and final elastic potential energy of the spring are

$$(V_e)_1 = \frac{1}{2}ks_1^2 = \frac{1}{2}(25)(10^3)(0.15^2) = 281.25\text{ J}$$

$$(V_e)_2 = \frac{1}{2}ks_2^2 = \frac{1}{2}(25)(10^3)(0.15+y)^2$$



Conservation of Energy:

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}mv_1^2 + \left[(V_g)_1 + (V_e)_1\right] = \frac{1}{2}mv_2^2 + \left[(V_g)_2 + (V_e)_2\right]$$

$$\frac{1}{2}(40)(5^2) + (0 + 281.25) = 0 + [-392.4(2+y)] + \frac{1}{2}(25)(10^3)(0.15+y)^2$$

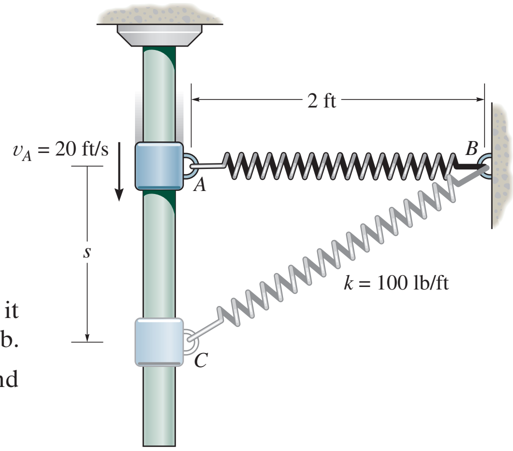
$$12500y^2 + 3357.6y - 1284.8 = 0$$

Solving for the positive root of the above equation,

$$y = 0.2133\text{ m} = 213\text{ mm} \qquad \textbf{Ans.}$$

14–78.

The 2-lb block is given an initial velocity of 20 ft/s when it is at A. If the spring has an unstretched length of 2 ft and a stiffness of $k = 100$ lb/ft, determine the velocity of the block when $s = 1$ ft.



SOLUTION

Potential Energy: Datum is set along AB. The collar is 1 ft *below* the datum when it is at C. Thus, its gravitational potential energy at this point is $-2(1) = -2.00$ ft·lb. The initial and final elastic potential energy are $\frac{1}{2}(100)(2-2)^2 = 0$ and $\frac{1}{2}(100)\left(\sqrt{2^2+1^2}-2\right)^2 = 2.786$ ft·lb, respectively.

Conservation of Energy:

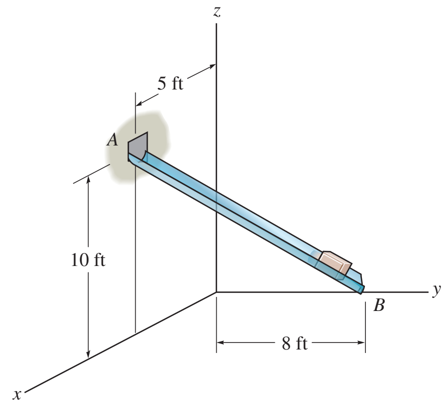
$$T_A + V_A = T_C + V_C$$

$$\frac{1}{2}\left(\frac{2}{32.2}\right)\left(20^2\right) + 0 = \frac{1}{2}\left(\frac{2}{32.2}\right)v_C^2 + 2.786 + (-2.00)$$

$$v_C = 19.4 \text{ ft/s}$$ **Ans.**

14–79.

The block has a weight of 1.5 lb and slides along the smooth chute AB. It is released from rest at A, which has coordinates of $A(5 \text{ ft}, 0, 10 \text{ ft})$. Determine the speed at which it slides off at B, which has coordinates of $B(0, 8 \text{ ft}, 0)$.



SOLUTION

Datum at B:

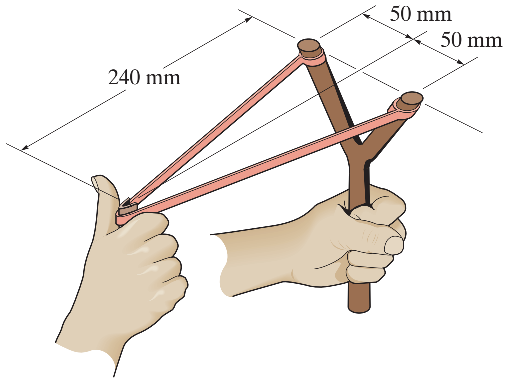
$$T_A + V_A = T_B + V_B$$

$$0 + 1.5(10) = \frac{1}{2}\left(\frac{1.5}{32.2}\right)(v_B)^2 + 0$$

$$v_B = 25.4 \text{ ft/s}$$ **Ans.**

***14–80.**

Each of the two elastic rubber bands of the slingshot has an unstretched length of 200 mm. If they are pulled back to the position shown and released from rest, determine the speed of the 25-g pellet just after the rubber bands become unstretched. Neglect the mass of the rubber bands. Each rubber band has a stiffness of $k = 50$ N/m.



SOLUTION

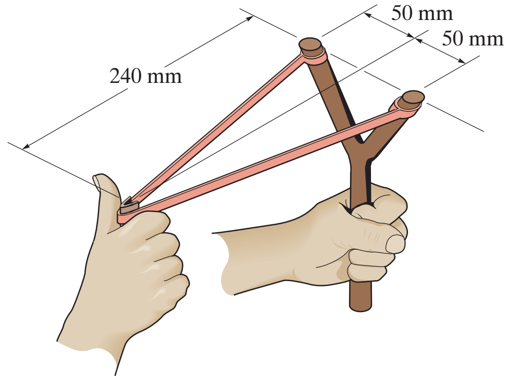
$$T_1 + V_1 = T_2 + V_2$$

$$0 + (2)\left(\frac{1}{2}\right)(50)\left[\sqrt{(0.05)^2 + (0.240)^2} - 0.2\right]^2 = \frac{1}{2}(0.025)v^2$$

$v = 2.86$ m/s **Ans.**

14–81.

Each of the two elastic rubber bands of the slingshot has an unstretched length of 200 mm. If they are pulled back to the position shown and released from rest, determine the maximum height the 25-g pellet will reach if it is fired vertically upward. Neglect the mass of the rubber bands and the change in elevation of the pellet while it is constrained by the rubber bands. Each rubber band has a stiffness $k = 50\text{ N/m}$.



SOLUTION

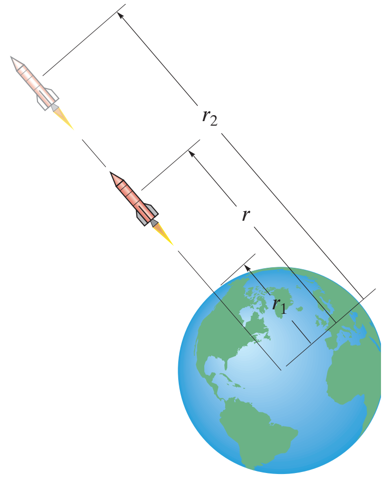
$$T_1 + V_1 = T_2 + V_2$$

$$0 + 2\left(\frac{1}{2}\right)(50)\left[\sqrt{(0.05)^2 + (0.240)^2} - 0.2\right]^2 = 0 + 0.025(9.81)h$$

$$h = 0.416\text{ m} = 416\text{ mm}$$ **Ans.**

14–82.

If the mass of the earth is M_e, show that the gravitational potential energy of a body of mass m located a distance r from the center of the earth is $V_g = -GM_em/r$. Recall that the gravitational force acting between the earth and the body is $F = G(M_em/r^2)$, Eq. 13–1. For the calculation, locate the datum at $r \rightarrow \infty$. Also, prove that F is a conservative force.



SOLUTION

The work is computed by moving F from position r_1 to a farther position r_2.

$$V_g = -U = -\int F\,dr$$

$$= -G\,M_e\,m \int_{r1}^{r2} \frac{dr}{r^2}$$

$$= -G\,M_e\,m\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

As $r_1 \rightarrow \infty$, let $r_2 = r_1$, $F_2 = F_1$, then

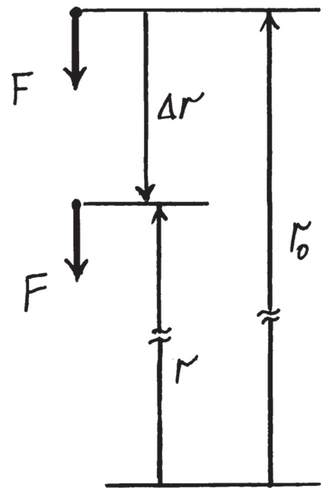
$$V_g \rightarrow \frac{-G\,M_e\,m}{r}$$

To be conservative, require

$$F = -\nabla V_g = -\frac{\partial}{\partial r}\left(-\frac{G\,M_e\,m}{r}\right)$$

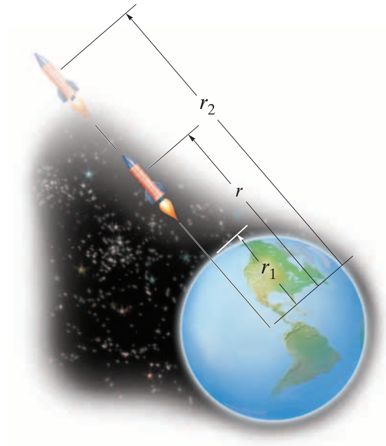
$$= \frac{-G\,M_e\,m}{r^2}$$

Q.E.D.



14–83.

A rocket of mass m is fired vertically from the surface of the earth, i.e., at $r = r_1$. Assuming no mass is lost as it travels upward, determine the work it must do against gravity to reach a distance r_2. The force of gravity is $F = GM_e m/r^2$ (Eq. 13–1), where M_e is the mass of the earth and r the distance between the rocket and the center of the earth.



SOLUTION

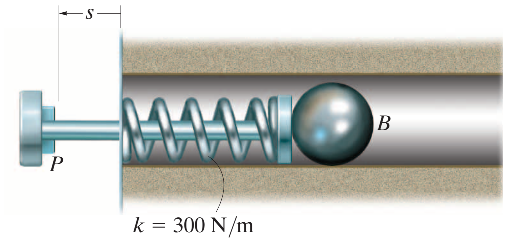
$$F = G\frac{M_e m}{r^2}$$

$$F_{1-2} = \int F\,dr = GM_e m \int_{r_1}^{r_2} \frac{dr}{r^2}$$

$$= GM_e m\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$ **Ans.**

***14–84.**

The firing mechanism of a pinball machine consists of a plunger P having a mass of 0.25 kg and a spring of stiffness $k = 300$ N/m. When $s = 0$, the spring is compressed 50 mm. If the arm is pulled back such that $s = 100$ mm and released, determine the speed of the 0.3-kg pinball B *just before* the plunger strikes the stop, i.e., $s = 0$. Assume all surfaces of contact to be smooth. The ball moves in the horizontal plane. Neglect friction, the mass of the spring, and the rolling motion of the ball.



SOLUTION

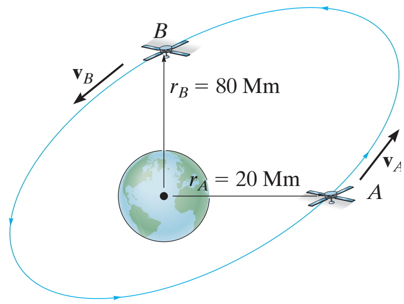
$$T_1 + V_1 = T_2 + V_2$$

$$0 + \frac{1}{2}(300)(0.1 + 0.05)^2 = \frac{1}{2}(0.25)(v_2)^2 + \frac{1}{2}(0.3)(v_2)^2 + \frac{1}{2}(300)(0.05)^2$$

$$v_2 = 3.30 \text{ m/s}$$ **Ans.**

14–85.

A 60-kg satellite travels in free flight along an elliptical orbit such that at A, where $r_A = 20$ Mm, it has a speed $v_A = 40$ Mm/h. What is the speed of the satellite when it reaches point B, where $r_B = 80$ Mm? *Hint:* See Prob. 14–82, where $M_e = 5.976(10^{24})$ kg and $G = 66.73(10^{-12})\ \mathrm{m^3/(kg \cdot s^2)}$.



SOLUTION

$v_A = 40 \text{ Mm/h} = 11\,111.1 \text{ m/s}$

Since $V = -\dfrac{GM_e m}{r}$

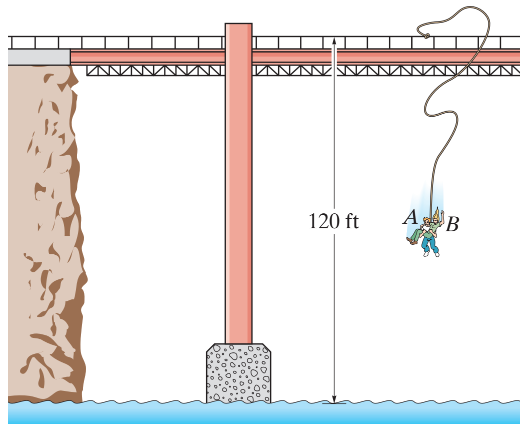
$T_1 + V_1 = T_2 + V_2$

$$\frac{1}{2}(60)(11\,111.1)^2 - \frac{66.73(10)^{-12}(5.976)(10)^{23}(60)}{20(10)^6} = \frac{1}{2}(60)v_B^2 - \frac{66.73(10)^{-12}(5.976)(10)^{24}(60)}{80(10)^6}$$

$v_B = 9672 \text{ m/s} = 34.8 \text{ Mm/h}$ **Ans.**

14–86.

Just for fun, two 150-lb engineering students A and B intend to jump off the bridge from rest using an elastic cord (bungee cord) having a stiffness $k = 80$ lb/ft. They wish to just reach the surface of the river, when A, attached to the cord, lets go of B at the instant they touch the water. Determine the proper unstretched length of the cord to do the stunt, and calculate the maximum acceleration of student A and the maximum height he reaches above the water after the rebound. From your results, comment on the feasibility of doing this stunt.

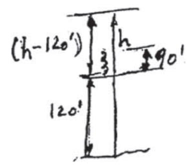


SOLUTION

$T_1 + V_1 = T_2 + V_2$

$$0 + 2(150)(120) = 0 + \frac{1}{2}(80)(x)^2$$

$x = 30$ ft



Unstretched length of cord.

$120 = l + 30$

$l = 90$ ft **Ans.**

When A lets go of B.

$T_2 + V_2 = T_3 + V_3$

$$0 + \frac{1}{2}(80)(30)^2 = 0 + (150)\,h$$

$h = 240$ ft

This is not possible since the 90 ft cord would have to stretch again, i.e., $h_{max} = 120 + 90 = 210$ ft.

Thus, $h > 120 + 90 = 210$ ft

$T_2 + V_2 = T_3 + V_3$

$$0 + \frac{1}{2}(80)(30)^2 = 0 + 150\,h + \frac{1}{2}(80)[(h - 120) - 90]^2$$

$36\,000 = 150\,h + 40(h^2 - 420\,h + 44\,100)$

$h^2 - 416.25\,h + 43\,200 = 0$

Choosing the root > 210 ft

$h = 219$ ft **Ans.**

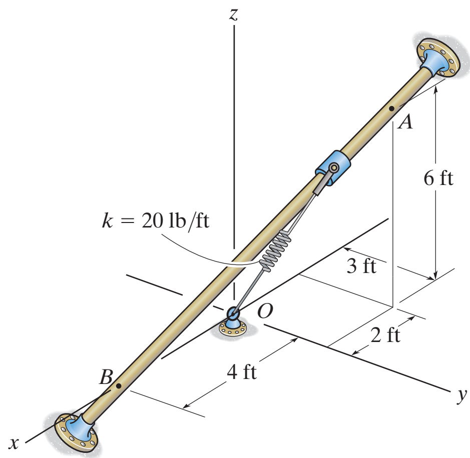
$+\uparrow \Sigma F_y = ma_y;\qquad 800(30) - 150 = \dfrac{150}{32.2}a$

$a = 483 \text{ ft/s}^2$ **Ans.**

It would not be a good idea to perform the stunt since $a = 15$ g which is excessive and A rises $219' - 120' = 99$ ft above the bridge!

14–87.

The 20-lb collar slides along the smooth rod. If the collar is released from rest at A, determine its speed when it passes point B. The spring has an unstretched length of 3 ft.



SOLUTION

$\mathbf{r}_{OA} = \{-2\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}\}$ ft, $\quad r_{OA} = 7$ ft

$\mathbf{r}_{OB} = \{4\mathbf{i}\}$ ft, $\quad r_{OB} = 4$ ft

Put datum at x–y plane

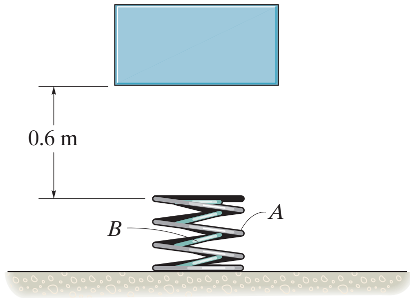
$$T_A + V_A = T_B + V_B$$

$$0 + (20\text{ lb})(6\text{ ft}) + \frac{1}{2}(20\text{ lb/ft})(7\text{ ft} - 3\text{ ft})^2 = \frac{1}{2}\left(\frac{20}{32.2}\right)v_B^2 + 0 + \frac{1}{2}(20\text{ lb/ft})(4\text{ ft} - 3\text{ ft})^2$$

$$v_B = 29.5\text{ ft/s} \qquad \textbf{Ans.}$$

***14–88.**

Two equal-length springs having a stiffness $k_A = 300$ N/m and $k_B = 200$ N/m are "nested" together in order to form a shock absorber. If a 2-kg block is dropped from an at-rest position 0.6 m above the top of the springs, determine their deformation when the block momentarily stops.



SOLUTION

Datum at initial position:

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = 0 - 2(9.81)(0.6 + x) + \frac{1}{2}(300 + 200)(x)^2$$

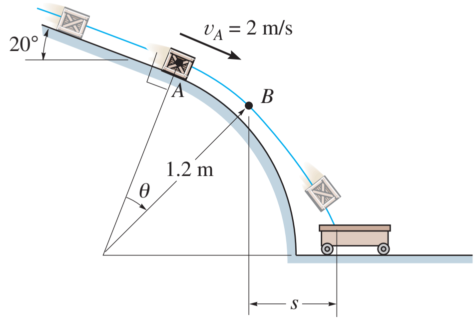
$$250x^2 - 19.62x - 11.772 = 0$$

Solving for the positive root,

$x = 0.260$ m **Ans.**

14–89.

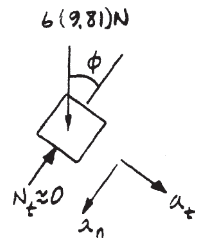
When the 6-kg box reaches point A it has a speed of $v_A = 2$ m/s. Determine the angle θ at which it leaves the smooth circular ramp and the distance s to where it falls into the cart. Neglect friction.



SOLUTION

At point B:

$$+\swarrow \Sigma F_n = ma_n; \qquad 6(9.81)\cos\phi = 6\left(\frac{v_B^2}{1.2}\right) \qquad \textbf{(1)}$$



Datum at bottom of curve:

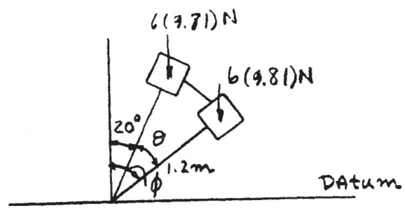
$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}(6)(2)^2 + 6(9.81)(1.2\cos 20°) = \frac{1}{2}(6)(v_B)^2 + 6(9.81)(1.2\cos\phi)$$

$$13.062 = 0.5v_B^2 + 11.772\cos\phi \qquad \textbf{(2)}$$

Substitute Eq. (1) into Eq. (2), and solving for v_B,

$$v_B = 2.951 \text{ m/s}$$



Thus, $\quad \phi = \cos^{-1}\left(\dfrac{(2.951)^2}{1.2(9.81)}\right) = 42.29°$

$\theta = \phi - 20° = 22.3°$ **Ans.**

$(+\uparrow) \qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$$-1.2\cos 42.29° = 0 - 2.951(\sin 42.29°)t + \frac{1}{2}(-9.81)t^2$$

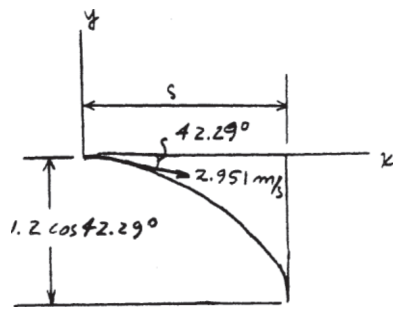
$$4.905t^2 + 1.9857t - 0.8877 = 0$$

Solving for the positive root: $t = 0.2687$ s

$(\overset{+}{\rightarrow}) \qquad s = s_0 + v_0 t$

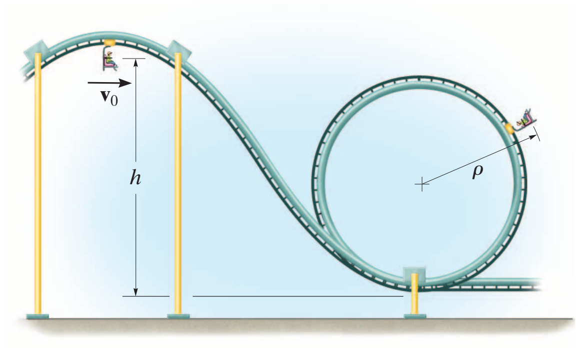
$$s = 0 + (2.951\cos 42.29°)(0.2687)$$

$s = 0.587$ m **Ans.**



14–90.

The Raptor is an outside loop roller coaster in which riders are belted into seats resembling ski-lift chairs. Determine the minimum speed v_0 at which the cars should coast down from the top of the hill, so that passengers can just make the loop without leaving contact with their seats. Neglect friction, the size of the car and passenger, and assume each passenger and car has a mass m.



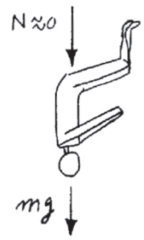
SOLUTION

Datum at ground:

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}mv_0^2 + mgh = \frac{1}{2}mv_1^2 + mg2\rho$$

$$v_1 = \sqrt{v_0^2 + 2g(h-2\rho)}$$



$$+\downarrow \Sigma F_n = ma_n; \qquad mg = m\left(\frac{v_1^2}{\rho}\right)$$

$$v_1 = \sqrt{g\rho}$$

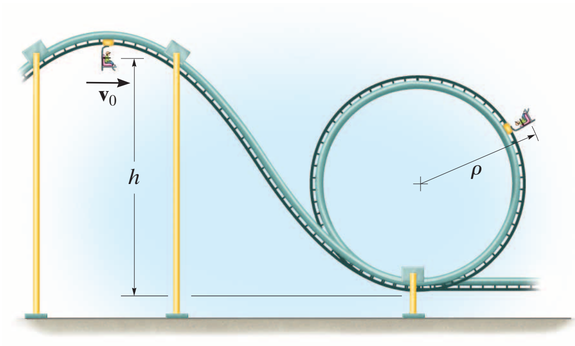
Thus,

$$g\rho = v_0^2 + 2gh - 4g\rho$$

$$v_0 = \sqrt{g(5\rho - 2h)}$$ **Ans.**

14–91.

The Raptor is an outside loop roller coaster in which riders are belted into seats resembling ski-lift chairs. If the cars travel at $v_0 = 4$ m/s when they are at the top of the hill, determine their speed when they are at the top of the loop and the reaction of the 70-kg passenger on his seat at this instant. The car has a mass of 50 kg. Take $h = 12$ m, $\rho = 5$ m. Neglect friction and the size of the car and passenger.



SOLUTION

Datum at ground:

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}(120)(4)^2 + 120(9.81)(12) = \frac{1}{2}(120)(v_1)^2 + 120(9.81)(10)$$

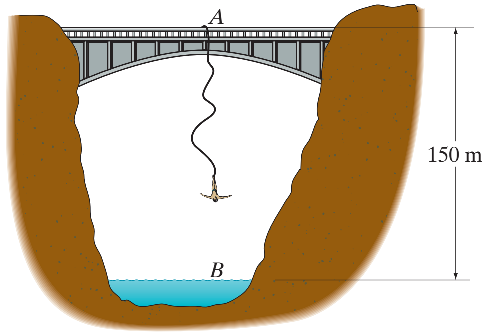
$$v_1 = 7.432 \text{ m/s}$$ **Ans.**

$$+\downarrow \Sigma F_n = ma_n; \qquad 70(9.81) + N = 70\left(\frac{(7.432)^2}{5}\right)$$

$$N = 86.7 \text{ N}$$ **Ans.**

***14–92.**

The 75-kg man bungee jumps off the bridge at A with an initial downward speed of 1.5 m/s. Determine the required unstretched length of the elastic cord to which he is attached in order that he stops momentarily just above the surface of the water. The stiffness of the elastic cord is $k = 80$ N/m. Neglect the size of the man.



SOLUTION

Potential Energy: With reference to the datum set at the surface of the water, the gravitational potential energy of the man at positions A and B are $(V_g)_A = mgh_A = 75(9.81)(150) = 110362.5$ J and $(V_g)_B = mgh_B = 75(9.81)(0) = 0$. When the man is at position A, the elastic cord is unstretched ($s_A = 0$), whereas the elastic cord stretches $s_B = (150 - l_0)$ m, where l_0 is the unstretched length of the cord. Thus, the elastic potential energy of the elastic cord when the man is at these two positions are $(V_e)_A = \frac{1}{2}ks_A{}^2 = 0$ and $(V_e)_B = \frac{1}{2}ks_B{}^2 = \frac{1}{2}(80)(150 - l_0)^2 = 40(150 - l_0)^2$.

Conservation of Energy:

$$T_A + V_A = T_B + V_B$$

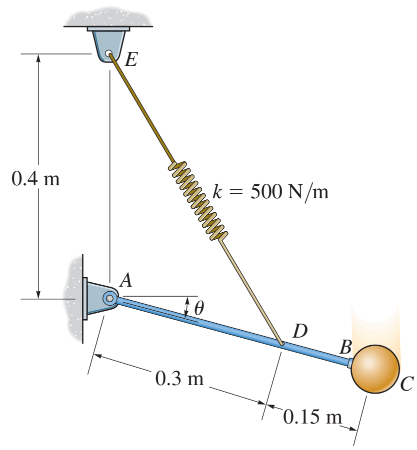
$$\frac{1}{2}mv_A{}^2 + \left[\left(V_g\right)_A + \left(V_e\right)_A\right] = \frac{1}{2}mv_B{}^2 + \left[\left(V_g\right)_B + \left(V_e\right)_B\right]$$

$$\frac{1}{2}(75)(1.5^2) + \left(110362.5 + 0\right) = 0 + \left[0 + 40(150 - l_0)^2\right]$$

$$l_0 = 97.5 \text{ m}$$ **Ans.**

14–93.

The 10-kg sphere C is released from rest when $\theta = 0°$ and the tension in the spring is 100 N. Determine the speed of the sphere at the instant $\theta = 90°$. Neglect the mass of rod AB and the size of the sphere.



SOLUTION

Potential Energy: With reference to the datum set in Fig. *a*, the gravitational potential energy of the sphere at positions (1) and (2) are $(V_g)_1 = mgh_1 = 10(9.81)(0.45) = 44.145$ J and $(V_g)_2 = mgh_2 = 10(9.81)(0) = 0$. When the sphere is at position (1), the spring stretches $s_1 = \dfrac{100}{500} = 0.2$ m. Thus, the unstretched length of the spring is $l_0 = \sqrt{0.3^2 + 0.4^2} - 0.2 = 0.3$ m, and the elastic potential energy of the spring is $(V_e)_1 = \frac{1}{2}ks_1^2 = \frac{1}{2}(500)(0.2^2) = 10$ J. When the sphere is at position (2), the spring stretches $s_2 = 0.7 - 0.3 = 0.4$ m, and the elastic potential energy of the spring is $(V_e)_2 = \frac{1}{2}ks_2^2 = \frac{1}{2}(500)(0.4^2) = 40$ J.

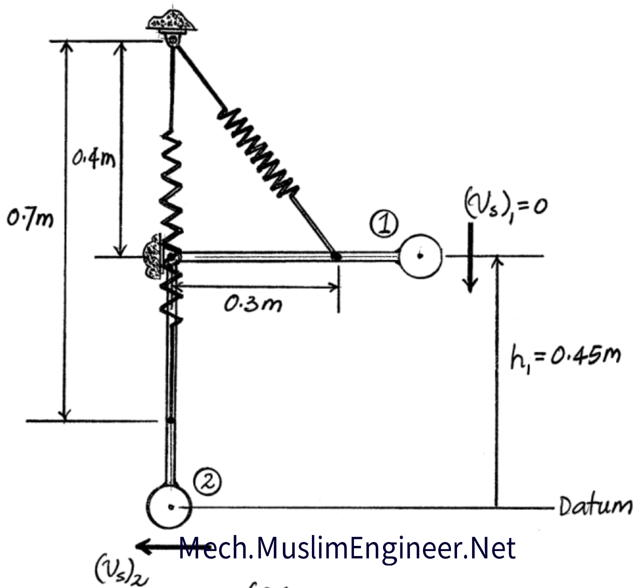
Conservation of Energy:

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}m_s(v_s)_1^2 + \left[(V_g)_1 + (V_e)_1\right] = \frac{1}{2}m_s(v_s)_2^2 + \left[(V_g)_2 + (V_e)_2\right]$$

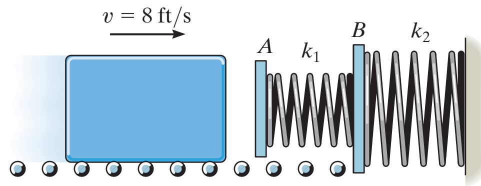
$$0 + (44.145 + 10) = \frac{1}{2}(10)(v_s)_2^2 + (0 + 40)$$

$$(v_s)_2 = 1.68 \text{ m/s} \qquad \textbf{Ans.}$$



14–94.

The double-spring bumper is used to stop the 1500-lb steel billet in the rolling mill. Determine the maximum displacement of the plate A if the billet strikes the plate with a speed of 8 ft/s. Neglect the mass of the springs, rollers and the plates A and B. Take $k_1 = 3000$ lb/ft, $k_2 = 45\,000$ lb/ft.



SOLUTION

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}\left(\frac{1500}{32.2}\right)(8)^2 + 0 = 0 + \frac{1}{2}(3000)s_L^2 + \frac{1}{2}(4500)s_2^2 \qquad \textbf{(1)}$$

$$F_s = 3000s_1 = 4500s_2; \qquad \textbf{(2)}$$

$$s_1 = 1.5s_2$$

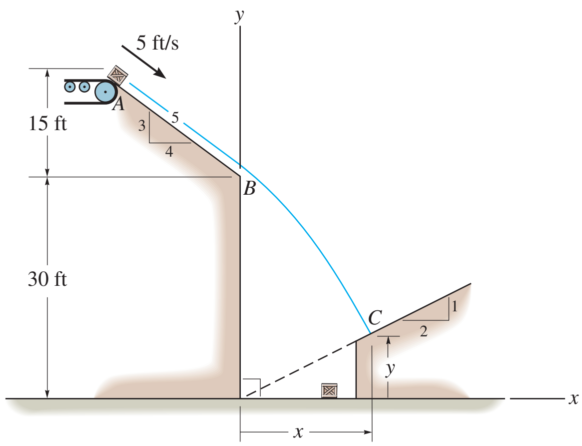
Solving Eqs. (1) and (2) yields:

$$s_2 = 0.5148 \text{ ft} \qquad s_1 = 0.7722 \text{ ft}$$

$$s_A = s_1 + s_2 = 0.7722 + 0.5148 = 1.29 \text{ ft} \qquad \textbf{Ans.}$$

14–95.

The 2-lb box has a velocity of 5 ft/s when it begins to slide down the smooth inclined surface at A. Determine the point C (x, y) where it strikes the lower incline.



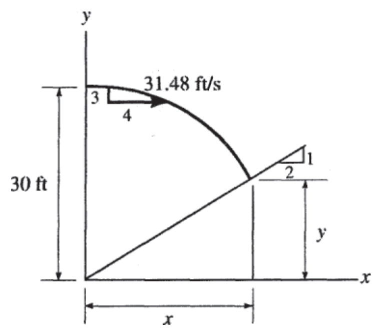
SOLUTION

Datum at A:

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}\left(\frac{2}{32.2}\right)(5)^2 + 0 = \frac{1}{2}\left(\frac{2}{32.2}\right)v_B^2 - 2(15)$$

$$v_B = 31.48 \text{ ft/s}$$



$(\xrightarrow{+})$ $\quad s = s_0 + v_0 t$

$$x = 0 + 31.48\left(\frac{4}{5}\right)t \qquad \textbf{(1)}$$

$(+\uparrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$$y = 30 - 31.48\left(\frac{3}{5}\right)t + \frac{1}{2}(-32.2)t^2$$

Equation of inclined surface:

$$\frac{y}{x} = \frac{1}{2}; \qquad y = \frac{1}{2}x \qquad \textbf{(2)}$$

Thus,

$$30 - 18.888t - 16.1t^2 = 12.592t$$

$$-16.1t^2 - 31.480t + 30 = 0$$

Solving for the positive root,

$$t = 0.7014 \text{ s}$$

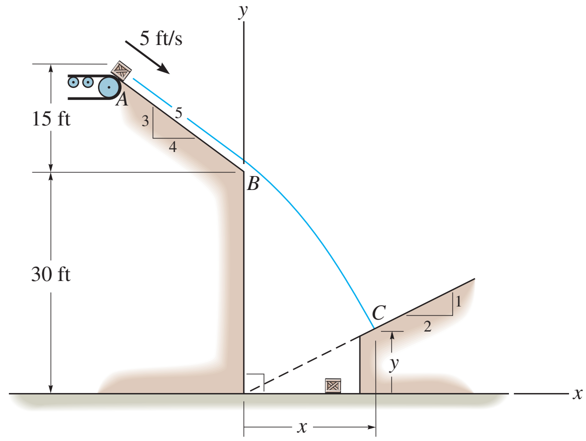
From Eqs. (1) and (2):

$$x = 31.48\left(\frac{4}{5}\right)(0.7014) = 17.66 = 17.7 \text{ ft} \qquad \textbf{Ans.}$$

$$y = \frac{1}{2}(17.664) = 8.832 = 8.83 \text{ ft} \qquad \textbf{Ans.}$$

***14–96.**

The 2-lb box has a velocity of 5 ft/s when it begins to slide down the smooth inclined surface at A. Determine its speed just before hitting the surface at C and the time to travel from A to C. The coordinates of point C are $x = 17.66$ ft, and $y = 8.832$ ft.



SOLUTION

Datum at A:

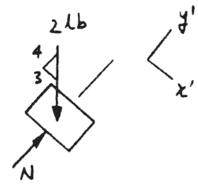
$$T_A + V_A = T_C + V_C$$

$$\frac{1}{2}\left(\frac{2}{32.2}\right)(5)^2 + 0 = \frac{1}{2}\left(\frac{2}{32.2}\right)(v_C)^2 - 2[15 + (30 - 8.832)]$$

$v_C = 48.5$ ft/s **Ans.**

$$+\searrow \Sigma F_{x'} = ma_{x'}; \qquad 2\left(\frac{3}{5}\right) = \left(\frac{2}{32.2}\right)a_{x'}$$

$$a_{x'} = 19.32 \text{ ft/s}^2$$



$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}\left(\frac{2}{32.2}\right)(5)^2 + 0 = \frac{1}{2}\left(\frac{2}{32.2}\right)v_B^2 - 2(15)$$

$$v_B = 31.48 \text{ ft/s}$$

$(+\searrow)$ $\qquad v_B = v_A + a_c t$

$$31.48 = 5 + 19.32t_{AB}$$

$$t_{AB} = 1.371 \text{ s}$$

$(\xrightarrow{+})$ $\qquad s = s_0 + v_0 t$

$$x = 0 + 31.48\left(\frac{4}{5}\right)t \qquad \textbf{(1)}$$

$(+\uparrow)$ $\qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$$y = 30 - 31.48\left(\frac{3}{5}\right)t + \frac{1}{2}(-32.2)t^2$$

Equation of inclined surface:

$$\frac{y}{x} = \frac{1}{2}; \qquad y = \frac{1}{2}x \qquad \textbf{(2)}$$

Thus

$$30 - 18.888t - 16.1t^2 = 12.592t$$

$$-16.1t^2 - 31.480t + 30 = 0$$

Solving for the positive root:

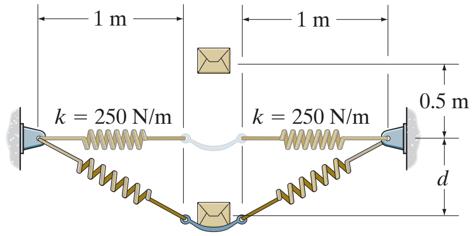
$$t = 0.7014 \text{ s}$$

Total time is

$t = 1.371 + 0.7014 = 2.07$ s **Ans.**

14–97.

A pan of negligible mass is attached to two identical springs of stiffness $k = 250$ N/m. If a 10-kg box is dropped from a height of 0.5 m above the pan, determine the maximum vertical displacement d. Initially each spring has a tension of 50 N.



SOLUTION

Potential Energy: With reference to the datum set in Fig. *a*, the gravitational potential energy of the box at positions (1) and (2) are $(V_g)_1 = mgh_1 = 10(9.81)(0) = 0$ and $(V_g)_2 = mgh_2 = 10(9.81)\left[-(0.5 + d)\right] = -98.1(0.5 + d)$. Initially, the spring stretches $s_1 = \dfrac{50}{250} = 0.2$ m. Thus, the unstretched length of the spring is $l_0 = 1 - 0.2 = 0.8$ m and the initial elastic potential of each spring is $(V_e)_1 = (2)\dfrac{1}{2}ks_1{}^2 = 2(250/2)(0.2^2) = 10$ J. When the box is at position (2), the spring stretches $s_2 = \left(\sqrt{d^2 + 1^2} - 0.8\right)$ m. The elastic potential energy of the springs when the box is at this position is

$$(V_e)_2 = (2)\frac{1}{2}ks_2{}^2 = 2(250/2)\left[\sqrt{d^2+1} - 0.8\right]^2 = 250\left(d^2 - 1.6\sqrt{d^2+1} + 1.64\right).$$

Conservation of Energy:

$$T_1 + V_1 + T_2 + V_2$$

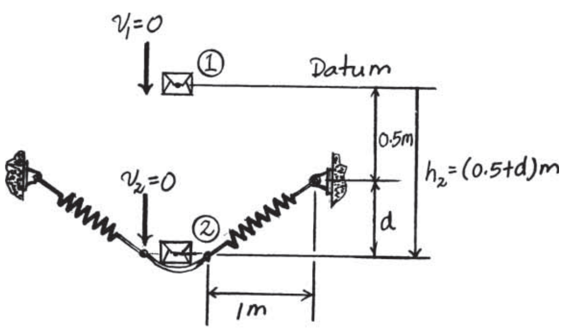
$$\frac{1}{2}mv_1{}^2 + \left[\left(V_g\right)_1 + (V_e)_1\right] = \frac{1}{2}mv_2{}^2 + \left[\left(V_g\right)_2 + (V_e)_2\right]$$

$$0 + (0 + 10) = 0 + \left[-98.1(0.5 + d) + 250\left(d^2 - 1.6\sqrt{d^2+1} + 1.64\right)\right]$$

$$250d^2 - 98.1d - 400\sqrt{d^2+1} + 350.95 = 0$$

Solving the above equation by trial and error,

$$d = 1.34 \text{ m} \qquad \textbf{Ans.}$$



(a)